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ULTRASONIC DELAY LINES. I.*

BY

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ABSTRACT.

This paper, the first of two dealing with ultrasonic delay lines, is concerned mainly with the theoretical considerations involved in the general process of ultrasonic propagation. The transducer action is given considerable attention and an equivalent circuit employing a transmission line as an element is demonstrated. The diffraction spreading of the beam and absorptive losses in the transmitting medium are also discussed.

A. INTRODUCTION.

This paper and its sequel give an account of that research on ultrasonics which was carried out at the Radiation Laboratory at M. I. T. on the development of ultrasonic delay lines. In this paper will be presented some of the general theoretical considerations which are involved in such a device and in the second paper the construction and operation of the actual delay line is treated from the experimental standpoint.

An ultrasonic delay line functions by transforming an electrical signal into a sonic one by an electromechanical transducer, propagating it through a fixed path in the transmitting medium and then converting the sonic signal back into an electrical signal by a second transducer. Very long delays can be obtained by this method, in comparison to purely electrical devices, because of the greatly decreased velocity of sound as compared to the velocity of electromagnetic propagation. Since the velocity of sound in liquids is about a third of that in solids,

* This paper is based on work done for the Office of Scientific Research and Development under Contract OEMsr-262 with the Massachusetts Institute of Technology.

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When the crystal is electrically excited at a frequency $\omega/2\pi$, nearly plane waves are propagated outward from both interfaces with wavelengths of $2\pi/\beta_1$ and $2\pi/\beta_2$, respectively (see Fig. 1). In the adjacent media the stress is given by $S = Y_{1 \text{ or } 2} (\partial\xi/\partial x)$. Inside the crystal, waves will be moving in both directions. (It will be our practice throughout these articles to consider that the time dependent factor, $e^{j\omega t}$, is implicitly contained in the amplitude coefficient.) One is able to assign a definite wavelength to the propagation inside the crystal by virtue of the fact that the quantity $hD/4\pi$ in (1) is independent of x.

The boundary conditions are that the mechanical displacement and pressure be continuous. This gives four equations for the four k unknown amplitude coefficients:

$$A = B + C, \tag{3a}$$

$$Be^{-j\beta d} + Ce^{j\beta d} = A'e^{-j\beta_2 d}, \tag{3b}$$

$$j\beta_1 Y_1 A = -j\beta YB + j\beta YC + hD/4\pi, \tag{3c}$$

$$-j\beta YBe^{-j\beta d} + j\beta YCe^{j\beta d} + hD/4x = -j\beta_2 Y_2 A'e^{-j\beta_2 d}, \tag{3d}$$

where (1) is used to obtain the last two equations.

Taking S as the area of the interface, one writes for the current

$$I = S\frac{d\sigma}{dt} = j\omega\sigma S, \tag{4}$$

where (σ) is free change per unit area of the electrodes. For the voltage across the crystal measured in the sense of Ohm's Law, one obtains from equation (2)

$$V = +\int_0^d Edx = h(\xi_d - \xi_0) + \frac{4\pi\sigma d}{K}. \tag{5}$$

The equations (3) can be solved for displacements in the usual way and give

$$\begin{aligned}\xi_d &= A'e^{-j\beta_2 d} \\ &= jh\sigma\frac{\beta Y\{\cos(\beta d) - 1\} + jY_1\beta_1 \sin\beta d}{(\beta Y)(\beta_1 Y_1 + \beta_2 Y_2)\cos\beta d + j[(\beta Y)^2 + \beta_1 Y_1\beta_2 Y_2]\sin\beta d} \\ &= jh\sigma F_2(\beta d),\end{aligned} \tag{6a}$$

$$\begin{aligned}-\xi_0 &= -A \\ &= jh\sigma\frac{\beta Y\{\cos(\beta d) - 1\} + jY_2\beta_2 \sin\beta d}{(\beta Y)(\beta_1 Y_1 + \beta_2 Y_2)\cos\beta d + j[(\beta Y)^2 + \beta_1 Y_1\beta_2 Y_2]\sin\beta d} \\ &= jh\sigma F_1(\beta d).\end{aligned} \tag{6b}$$

The expression for the electrical impedance of the crystal can now be

written

$$\frac{V}{I} = \frac{4\pi d}{j\omega KS} + \frac{h^2}{\omega S} F(\beta d), \tag{6c}$$

where

$$F = F_1 + F_2. \tag{6d}$$

The first term in (6c) is always much larger than the second, provided that at least one of the media adjoining the crystal is either a

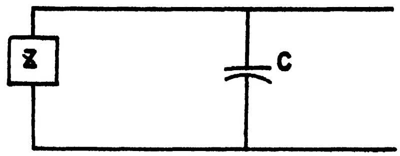

FIG. 2. Shunt circuit.

liquid or a solid. This makes it feasible to express the transducer as a simple parallel combination.

$$I/V = j\omega C + (Z)^{-1} \tag{7}$$

Here C is the electrostatic capacity of the crystal and

$$Z = \frac{16\pi^2 d^2}{h^2 K^2 \omega S F(\beta d)} \tag{7a}$$

is the equivalent electrical impedance of the crystal under the above conditions of acoustical loading (see Fig. 2). The approximation implicit in (7) greatly simplifies the subsequent treatment and the results check closely with the more exact calculation. For example, the

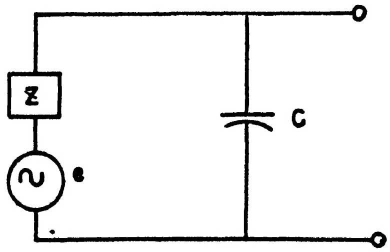

FIG. 3. Circuit with generator.

resonant frequency of the transducer as developed here differs by less than 1 per cent. from the exact value.

From (6c) it is apparent that the electrostatic capacity of the crystal acts as a series element. This is a consequence of the fact that the thickness vibration is being considered. (Where the direction of the supersonic waves is at right angles to the electric field the electrostatic capacity appears as a shunt element.)

If one had developed the case of the receiving rather than the transmitting crystal, an impinging acoustical wave would have been introduced. Then the equivalent electrical circuit would have involved a generator, with voltage amplitude proportional to the amplitude of the incoming sonic wave (see Fig. 3).

The resonant condition for the crystal is that its width be an odd number of half wavelengths, or $\beta d = n\pi$ where n is odd. At these values F (see 6) reduces to $4(\beta_1 Y_1 + \beta_2 Y_2)^{-1}$. Since $Y = v^2\rho$ where ρ is the density and v the acoustical velocity of the excited mode in the medium, the impedance of Z at resonance becomes

$$Z(n\pi) = \tfrac{1}{4}(Z_1 + Z_2), \tag{8}$$

$$Z_1 = k\rho_1 v_1 \qquad \text{and} \qquad Z_2 = k\rho_2 v_2.$$

The quantity k is given by

$$k = \left(\frac{4\pi d}{hK}\right)^2 \frac{1}{S}. \tag{8a}$$

Since ρv is the acoustical impedance of a medium, k is the proportionality factor between electrical and acoustical impedance. The quantity k depends entirely on the characteristics of the crystal alone, i.e. its dimensions and its electric and piezoelectric properties. For an X-cut quartz crystal

$$K = 4.58,$$
$$h = 14.3 \times 10^4 \text{ c.g.s. e.s.u.}$$

and

$$d \text{ in mm.} = \frac{2.86 \times 10^6}{f},$$

where f is the fundamental resonant frequency. For a 30 mc./sec. crystal with an area of 1 cm². we have

$$Z_1 = 0.030(\rho_1 v_1) \text{ ohms for } (\rho v \text{ in c.g.s.}).$$

For mercury ρv is 19.8×10^5 in c.g.s. so that each quartz mercury interface contributes 15,000 ohms to the impedance of the transducer at resonance. It is apparent from the form of equation (8) that at resonance the two bounding media act as series elements.

From the direct approach employed in the preceding paragraphs, an expression for the impedance of the transducer, looking in from the electrical side, has been obtained. Its behavior at resonance has been investigated. A proportionality factor between electrical and acoustical impedance has been evolved and some actual numbers inserted. Also, expressions have been derived for the displacement amplitudes at the interfaces (equation 6a and 6b) in terms of the free charge on the crystal. A somewhat more general approach to the fundamental

equations will now be employed with a view to obtaining a compact equivalent circuit from which any specific result could be derived.

One starts from equations (3) using parts (a) and (b) to eliminate B and C from parts c and d. The new equations are

$$\omega\rho_1 v_1 A = j\omega\rho v(A \cot \beta d - A' e^{-i\beta_2 d} \csc \beta d) - \frac{h}{\omega S} I, \tag{9a}$$

$$\omega\rho_2 v_2 A' e^{-i\beta_2 d} = j\omega\rho v(-A \csc \beta d + A' e^{-i\beta_2 d} \cot \beta d) + \frac{h}{\omega S} I. \tag{9b}$$

Next one identifies the displacements and pressures at the interfaces with fictitious voltages and currents respectively, as follows:

$$A = -e_1/h, \qquad A' e^{-i\omega_2 d} = e_2/h,$$
$$\omega\rho_1 v_1 A = \frac{h}{\omega S} i_1, \qquad \omega\rho_2 v_2 A' e^{-i\beta_2 d} = \frac{h}{\omega S} i_2. \tag{10}$$

The substitution of these quantities into 9a, 9b and 5 gives the following three equations:

$$\frac{h^2}{j\omega^2 \rho v S}(i_1 - I) = e_1 \cot(\beta d) + e_2 \csc(\beta d),$$
$$\frac{h^2}{j\omega^2 \rho v S}(i_2 - I) = e_1 \csc(\beta d) + e_2 \cot(\beta d), \tag{11}$$
$$V = e_1 + e_2 + \frac{4\pi d I}{j k \omega S}.$$

These equations give the relations between the voltages and currents in a six-terminal network which could serve as an exact equivalent circuit for the transducer. As before, it will be expedient to change from a circuit where the electrostatic condenser is in series to one in which the condenser is in parallel and to neglect terms of the order of $(\omega c Z)^{-2}$, which is permissible when the crystal is damped. In the equations for the new network the roles of current and voltage in equation (11) are interchanged at each terminal and the impedance factors, such as $h^2/\omega^2\rho v S$, are multiplied by $(kS/4\pi d)^2$ and become admittances. Consequently, the new network equations are

$$(e_1 - V) = jZ_0[i_1 \cot(\beta d) + i_2 \csc(\beta d)],$$
$$(e_2 - V) = jZ_0[i_1 \csc(\beta d) + i_2 \cot(\beta d)], \tag{12}$$
$$I = i_1 + i_2 = \frac{j\omega k S}{4\pi d} V,$$

where $Z_0 = k\rho v$, the equivalent electrical impedance of the quartz.

Equations (12) can be represented by the equivalent circuit shown in Fig. 4, which first appeared in a report by H. Grayson.[2] Here the crystal is represented as a section of a transmission line (see the two parallel heavy lines) of electrical length βd and characteristic impedance Z_0. By tying on impedances Z_1 and Z_2, to the appropriate terminals, one can now rederive the specific results obtained in the first part of this section.

The next problem is the investigation of the band-pass characteristics of the crystal. From the analytical expression for Z (6 and 7) one obtains the following formula for conductance,

$$G = Re\left(\frac{1}{Z}\right) = \frac{\{Z_0^2(\cos(\beta d) - 1)^2 + Z_1Z_2\sin^2\beta d\}(Z_1 + Z_2)}{Z_0^2(Z_1 + Z_2)^2\cos^2(\beta d) + [(Z_1Z_2 + Z_0^2)\sin\beta d]^2}. \quad (13)$$

This quantity (13) is proportional to the power (V^2G) absorbed by one crystal from a constant voltage source. The frequency response is obtained by varying βd. Figure 5 shows the behavior of (13) under four different conditions. All curves are periodic with period 2π for βd. When the crystal is acoustically matched on both sides, the curve is sinusoidal (A). The curve is narrowed somewhat (B) as the result of allowing one surface to vibrate freely ($Z_2 = 0$). If one employs a material of higher acoustical impedance on both sides ($Z_1 = Z_2 = \sqrt{2}Z_0$), a broader curve is obtained (C). Curve D is obtained when one of these surfaces is allowed to vibrate freely ($Z_2 = 0$) and shows a zero curvature at resonance, nearly the case of quartz with mercury on one side. This final result is somewhat unexpected and is of practical interest.

The expression (13) gives the *total* power absorbed by the crystal. For the two special cases considered above ($Z_1 = Z_2$, and $Z_2 = 0$) there is no question how the power is divided between the two media. Though these are two situations of main practical concern, it is of interest to determine how the power is divided in the general case. To do this, we must return to equation (6) to evaluate the ratio

$$Y_1\left|\xi_0\frac{\partial\xi_0}{\partial x}\right| : \qquad Y_2\left|\xi_d\frac{\partial\xi_d}{\partial x}\right|.$$

This turns out to be

$$\frac{\text{Power transmitted to medium 1}}{\text{Power transmitted to medium 2}}$$

$$= \frac{Z_1Z_2^2\sin^2(\beta d) + Z_0^2Z_1[\cos(\beta d) - 1]^2}{Z_1^2Z_2\sin^2(\beta d) + Z_0^2Z_2[\cos(\beta d) - 1]^2}. \quad (14)$$

It follows that the power absorbed by the medium 1 from a voltage

[2] "The Quartz Crystal as a Generator and Receiver of Supersonic Vibration," H. Grayson, TRE Report T-1358.

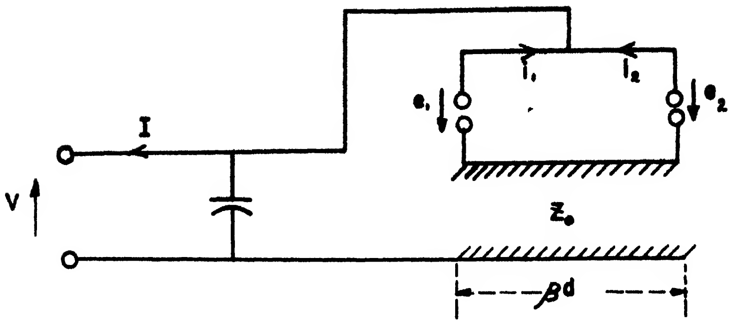

FIG. 4. Transmission-line equivalent circuit.

		"Q"
A	$\rho_1 v_1 = \rho v = \rho_2 v_2$	.79
B	$\rho_1 v_1 = \rho v;\ \rho_2 v_2 = 0$	1.11
C	$\rho_1 v_1 = \sqrt{2}\,\rho v = \rho_2 v_2$	.56
D	$\rho_1 v_1 = \sqrt{2}\,\rho v;\ \rho_2 v_2 = 0$	0

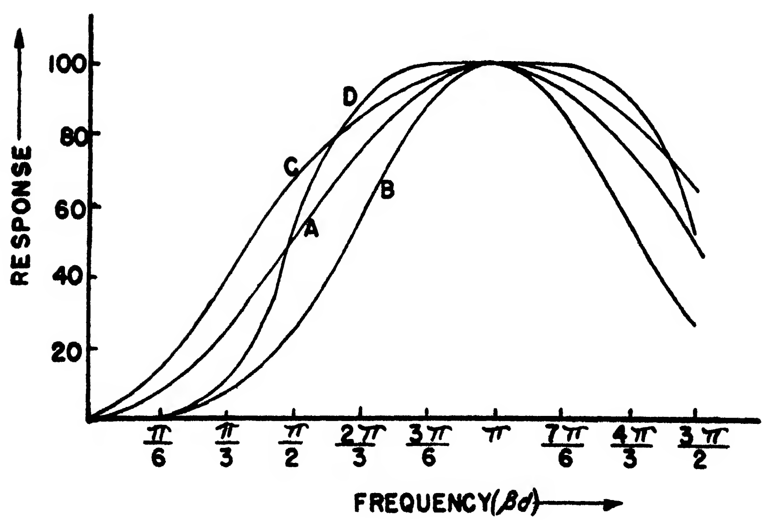

FIG. 5. Acoustic output power of a crystal as a function of frequency. The crystal is assumed to be subject to a constant voltage. Fig. 1 gives meaning of subscripts. Resonant frequency of free crystal occurs for $\beta d = \pi$. "Q" refers to curvature at resonance.

source is proportional to

$$G_1 = \frac{Z_0^2 Z_1 [\cos(\beta d) - 1]^2 + Z_1 Z_2^2 \sin^2(\beta d)}{[Z_0(Z_1 + Z_2)\cos(\beta d)]^2 + [(Z_1 Z_2 + Z_0^2)\sin(\beta d)]^2}. \quad (15)$$

There is a similar expression for G_2 so that $G_1 + G_2 = G$.

As an example the case of a quartz crystal with mercury on one side and water on the other has been worked out. In Fig. 7 the power

		"Q"
A	$\rho v = 2\rho_1 v_1$	.39
B	$\rho v = \rho_1 v_1$	.79
C	$\rho v = \frac{1}{2}\rho_1 v_1$	1.57

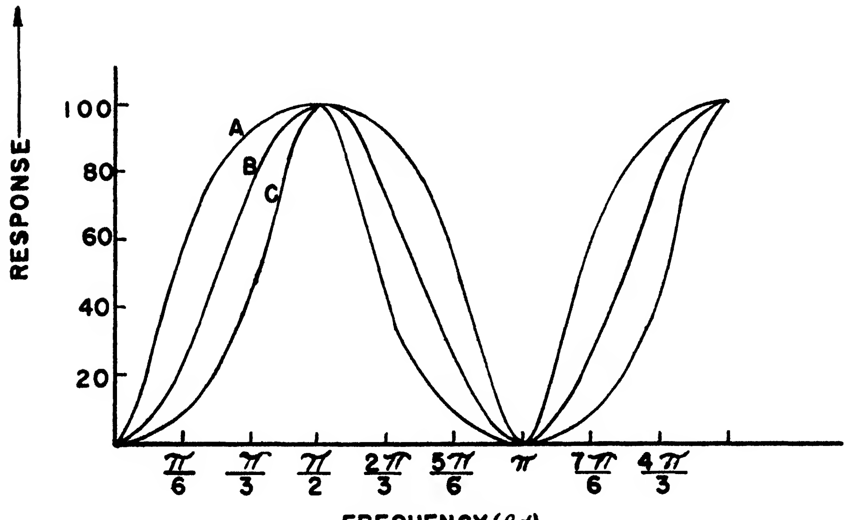

FIG. 6. Output power of crystal when backed by high impedance material.

absorbed by the mercury, by the water, and the sum of these two quantities are plotted on a relative logarithmic scale as a function of frequency.

In some applications the behavior of the frequency response in the neighborhood of resonance is of particular interest, One cannot use a simple series resonant circuit to represent both the amplitude and phase characteristics accurately here even over a limited frequency range. However, it is convenient to introduce a parameter, "Q," which is the Q value for the simple series resonant circuit whose amplitude response matches that of the transducer in curvature at resonance.

For all but the lowest values, the parameter "Q" corresponds satisfactorily with the usual ratio of mid-frequency to distance between half power points. The formula for "Q" is given below and applies to

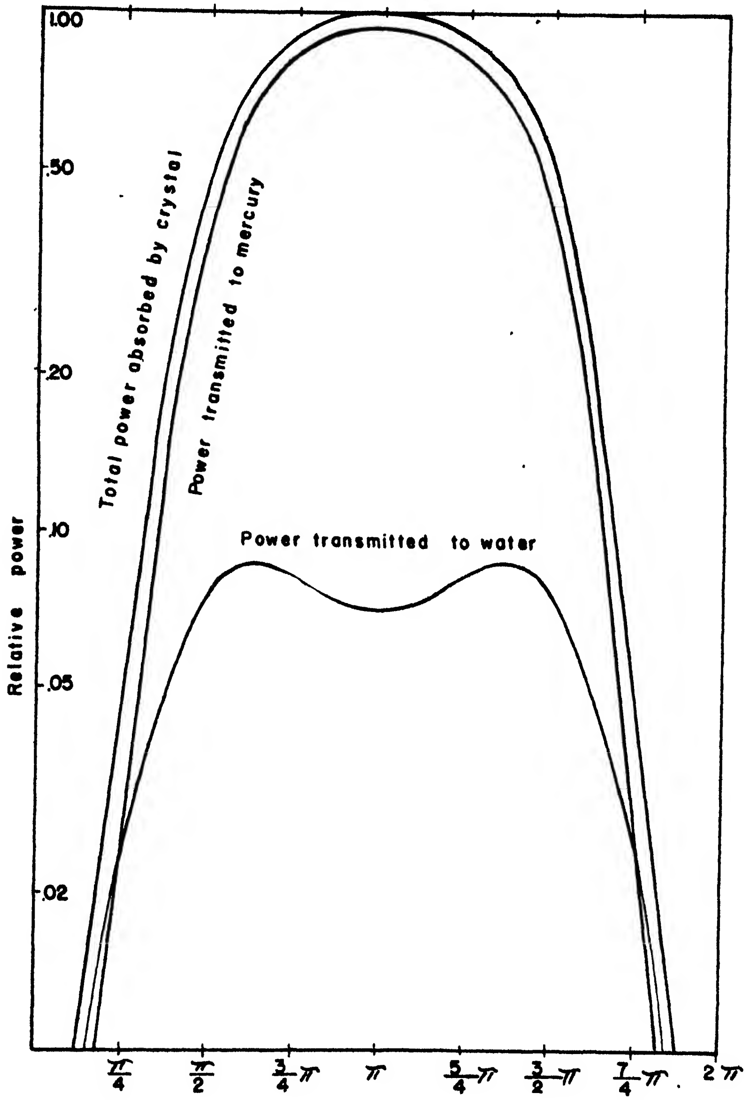

FIG. 7. Frequency response; quartz crystal loaded with mercury on one side, water on the other.

the curve for power absorbed by the transmitting medium when the transmitter crystal is driven from a low impedaance.

$$\text{"}Q\text{"} = \frac{n\pi}{4}\left[\frac{4(Z_1Z_2 + Z_0^2)^2 - (Z_1 + Z_2)^2(Z_2^2 + 2Z_0^2)}{Z_0^2(Z_1 + Z_2)^2}\right]^{\frac{1}{2}}, \tag{16}$$

where medium 1 is the transmitting medium. Here n is the order of the harmonic frequency of the crystal. For special cases the following simplifications occur:

For $Z_1 = Z_2$,

$$``Q" = \frac{n\pi}{4}\frac{Z_0}{Z_1}, \tag{17a}$$

and for $Z_2 = 0$,

$$``Q" = \frac{n\pi}{4Z_1}[2(2Z_0^2 - Z_1^2)]^{\frac{1}{2}}. \tag{17b}$$

From (17b) it is evident for one free surface the response becomes antiresonant at frequencies $\beta d = n\pi$ (n odd), if $Z_1 > \sqrt{2}Z_0$.

An interesting extreme occurs when one adjacent medium has a very large acoustical impedance ($Z_2 \to \infty$). The resonant points then occur at $\beta d = n\pi/2$ for n odd, with a response characteristic (Fig. 6).

$$G = \frac{Z_1 \sin^2 (\beta d)}{Z_0^2 \cos^2 (\beta d) + Z_1^2 \sin^2 (\beta d)} \tag{18}$$

and

$$``Q" = \frac{n\pi}{4}\frac{Z_0}{Z_1}.$$

It is apparent that the acoustical impedance of the piezoelectric material affects the bandwidth even though it does not enter into the expression for the equivalent electrical impedance at resonance (8). For quartz in mercury the response is broad. However, consider a 10 mc. crystal driven at 30 mc. in contact with water on each side.

$$\rho v \text{ for water} = 1.5 \times 10^5 \text{ c.g.s.},$$
$$\rho v \text{ for quartz} = 15 \times 10^5 \text{ c.g.s.},$$

then

$$``Q" = \frac{3\pi}{4} \times 10 = 23.5.$$

Under the assumption of low-impedance electrical circuits used in conjunction with the crystals the shunt capacity of the crystals does not enter directly into the analysis of the preceding paragraphs. This assumption is not as drastic as might at first appear. Actually for many applications, particularly for delay lines where a broad and flat passband is desired, the crystal capacity is an element in a low-Q shunt resonant circuit. The impedance of this circuit is kept low in comparison with Z_1, Z_2, etc. by the bandwidth requirements. Under these circumstances the results of the last paragraphs are quite valid.

It remains to calculate the actual loss through mismatch that occurs where a signal is transmitted through a delay line as depicted in Fig. 8. The equivalent circuit is given in Fig. 9. The load resistors shunting

the crystals at the transmitting and receiving ends are R_t and R_r, respectively. Consider the crystals' capacities as tuned out. (The resolution of Z into two series elements $\frac{1}{4}Z_1$ and $\frac{1}{4}Z_2$, is possible only at resonance.)

If a current pulse of magnitude I is applied to the transmitting crystal, the voltage which is propagated down the line is given by

$$V = IR_t \frac{Z_1}{Z_1 + Z_2 + 4R_t}. \tag{19}$$

Here the transit time of the line is long compared with the pulse duration, so that the pulse sees the characteristic line impedance.

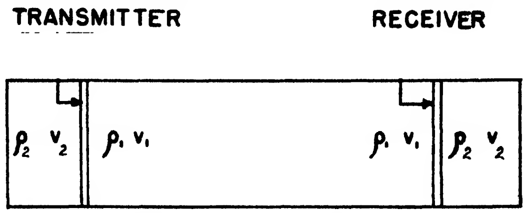

FIG. 8. Delay line schematic.

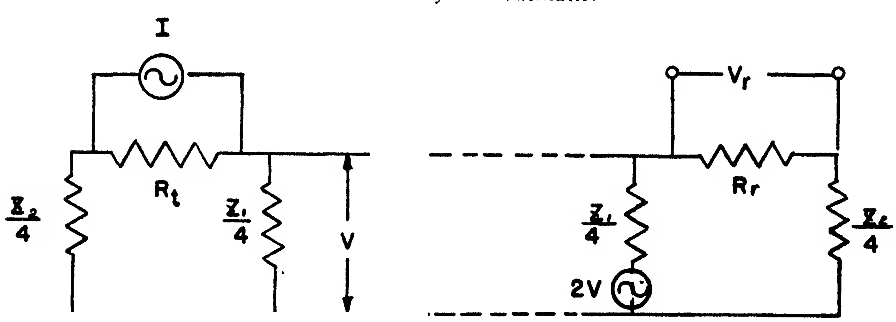

FIG. 9. Equivalent circuit of delay line at resonance.

If the other end of the line were terminated in its characteristic impedance, $Z_1/4$ then V would appear across it. Therefore, looking back at the line, one sees a pulse generator of $2V$ in series with the line impedance.[3] The voltage V_r across the resistor R_r is given by

$$V_r = 2V \frac{R_r}{\frac{1}{4}(Z_1 + Z_2) + R_r}$$
$$= IR_t \frac{8Z_1R_r}{(Z_1 + Z_2 + 4R_t)(Z_1 + Z_2 + 4R_r)}. \tag{20}$$

[3] If the crystals at the two ends of the line differ with respect to any of the quantities upon which k (eq. 8a) depends, then Z_1 will have different values at the two ends, say Z_1' and Z_1''. If we require that the available power be transformed without reflection by the line, then equation (20) is valid (with appropriate superscripts in the denominator) if Z_1 in the numerator is replaced by the geometric mean of Z_1' and Z_1''.

C. WAVE PROPAGATION.

It is of considerable importance to determine in as great detail as possible the way in which energy travels after leaving the transmitter of a supersonic system. To this end some calculations are first included on the radiation pattern to be expected from a piezoelectric crystal transmitting single-frequency compressional vibrations into a semi-infinite medium. Secondly, we shall make some remarks on the wave pattern present in a semi-infinite cylindrical tube at the end of which a similar crystal is vibrating. The second case corresponds more closely to the actual physical set-up for a delay line, though results obtained from the first case are also of value in determining mechanical tolerances for delay line construction.

Two assumptions in particular which are made in connection with both cases should be noted. First, as is customary in such calculations,

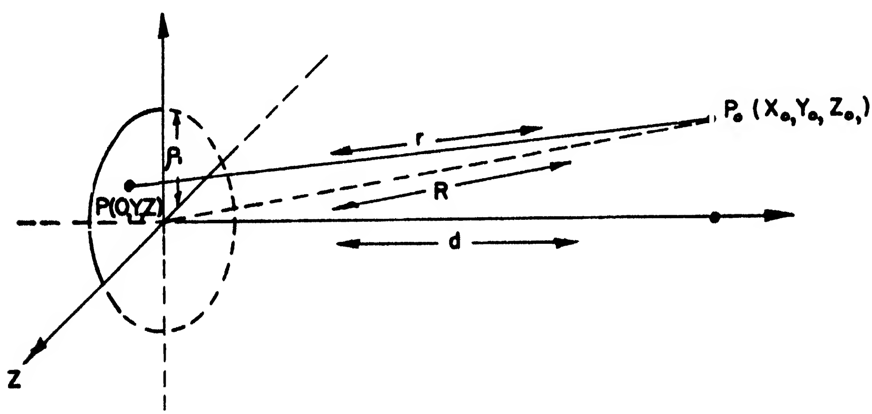

FIG. 10 Geometry of the radiation problem

we determine the beam pattern in a certain region of "empty" space and then assume that the wave function will remain unchanged if the receiver is placed in this region. This is of course valid for pulse transmission which was our main concern at the Radiation Laboratory. Secondly, we consider that out medium is non-viscous. In the next section of this part, certain effects of viscosity will be considered.

Case I: The Radiation Pattern in a Semi-infinite Medium.

General and Fresnel Case

The nomenclature for the problem is indicated in Fig. 10. A crystal with a circular cross section of radius ρ_1 and with its center at (0, 0, 0) is executing small amplitude, single-frequency vibrations in the direction of the x-axis. The portion of the YZ plane for which $\rho > \rho_1$ is assumed to be a rigid baffle. We wish to know the radiation pattern as

observed at an arbitrary point (X_0, Y_0, Z_0) to the right of the YZ plane.

It is well known that the velocity potential, W, for sound waves of small amplitude in a perfect compressible fluid satisfies the scalar wave equation.[4]

$$\frac{\partial^2 W}{\partial t^2} = v^2 \nabla^2 W, \tag{22a}$$

where v is the velocity. The boundary conditions on the plane $X = 0$ are:

$$\frac{\partial S}{\partial X} = \begin{cases} \text{constant} \neq 0 & (\rho \leq \rho_0) \\ 0 & (\rho > \rho_0). \end{cases} \tag{22b}$$

These boundary conditions neglect the fact that the face of the crystal does not always lie in the same plane. But we shall always calculate the amplitude of the sound wave at a distance from the circular opening which is large compared with the wavelength, and in this case this is a valid approximation. We use the Kirchhoff condition which states:[5]

$$V_0 = \int_{\text{opening}} \frac{\frac{\partial W}{\partial n}}{r} \sin 2\pi \left(\frac{t}{T} - \frac{r}{\lambda} \right) \cos (nr) d\sigma, \tag{23}$$

where, referring to Fig. 10,

V_0 = velocity at P_0, the observation point,
$\frac{\partial W}{\partial n} = \frac{A}{\lambda}$ in which A is a constant,
λ = wavelength,
T = period.
(nr) = angle between direction of X-axis and r,
t = time variable,
$d\sigma$ = element of area of surface $x = 0$.

Referring to Fig. 10 we see

$$r = R\sqrt{1 + \frac{Y^2 + Z^2 - 2YY_0 - 2ZZ_0}{R^2}}.$$

By analogy to light diffraction problems we distinguish two cases. If it is assumed that r is expanded as

$$r = R\left(1 + \frac{Y^2 + Z^2}{2R^2} - \frac{YY_0 + ZZ_0}{R^2}\right),$$

[4] A. G. Webster, "Dynamics"; Stechert, pp. 542–3 (1922).

[5] For discussion and partial derivation see Stratton's "Electromagnetic Theory," McGraw-Hill, pp. 460–463 (1941). It should be pointed out that our radiation problem resembles that of determining the diffraction pattern to the right of a circular opening upon which a plane wave of light is incident from the left.

we are dealing with the Fresnel case. If

$$r = R\left(1 - \frac{YY_0 + ZZ_0}{R^2}\right)$$

we are dealing with the Fraunhofer case. We consider first the Fresnel case. Hence (23) can be simplified to (24).

$$V_0 = A' \int_{\text{opening}} \sin 2\pi \left(\frac{t}{T} - \frac{r}{\lambda}\right) d\sigma, \tag{24}$$

where

$$A' = \frac{A}{\lambda} \frac{\cos (nR)}{R}.$$

This integral is now in a form which has been evaluated. With the omission of the factor A' the integral can be written as

$$\int_0^{2\pi} \int_0^{\rho_0} \sin 2\pi \left(\frac{t}{T} - \frac{r}{\lambda}\right) \rho d\phi d\rho = M \sin \left(\theta + 2\pi \frac{t}{T}\right), \tag{25}$$

where ρ and ϕ are polar coordinates for the circular opening, M and θ are functions of X_0, Y_0, and Z_0. The elaborate and laborious process of computing M for several values of (X_0, Y_0, Z_0) has been carried through in a paper by E. Lommel on "The Diffraction Phenomena of a Circular Opening and a Circular Screen." [6] These calculations check the experimental data on diffraction of light to within about 1 or 2 per cent.

Fraunhofer Case

The solution of the Fraunhofer problem is well known,[7]

$$M^2 = 4 \left[\frac{J_1 \left(\frac{2\pi}{\lambda} \rho_1 x\right)}{\left(\frac{2\pi}{\lambda} \rho_1 x\right)} \right]^2, \tag{26}$$

where J_1 is a Bessel function of the first kind and x is the sine of the angle which R makes with the X axis.

On the Axis

An analytical expression for the disturbance on the X-axis for the Fresnel case can be obtained quite readily. By (23) for a plane wave incident normally.

$$V_0 = \frac{A}{\lambda} \int_{\text{opening}} \frac{\sin 2\pi \left(\frac{t}{T} - \frac{r}{\lambda}\right) d\sigma}{r}. \tag{27}$$

[6] E. Lommel, *Abhandlungen der Bayerischen Akademie der Wissenchaften*, 15, 233 (1886).

[7] M. Born, "Optik," pp. 159–161 (J. Springer, 1933); or P. Morse, "Vibration and Sound," p. 257 (McGraw-Hill, 1936).

This expression is valid so long as $r > \rho_1$. It can be integrated directly with respect to r and gives

$$V_0 = 2A \sin\left(\frac{\pi\rho_1^2}{2\lambda d}\right) \sin 2\pi\left(\frac{t}{T} - \frac{d}{\lambda}\right), \tag{28a}$$

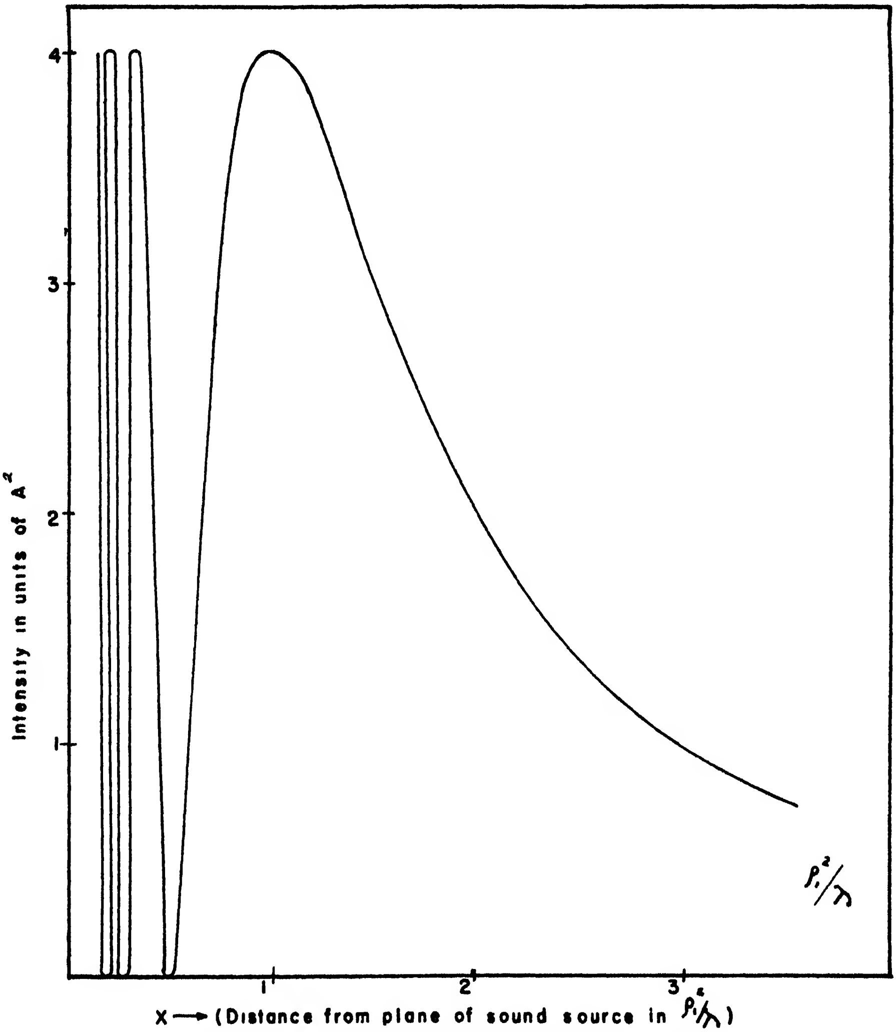

FIG. 11. Intensity of sound on X-axis vs. distance from sound source.

where A is as before the amplitude of the velocity at the crystal. This equation is plotted in Fig. 11. At large distances the amplitude of the velocity reduces to the well-known formula:

$$V_0 = \frac{A_0}{\lambda d}(\pi\rho_1^2). \tag{28b}$$

Total Force on Receiving Crystals

Since the wave incident on our receiver crystal is not a plane wave, pressures of different amplitude and phase will fall at different points of the receiver crystal. It appears that the factor of importance in determining the response of the receiver crystal is the total force on it. This quantity F can be determined by integrating over the area of the receiving crystal.

$$F = P \int_{\text{area}} M \sin\left(\theta + 2\pi \frac{t}{T}\right) d\sigma, \tag{29a}$$

$$|F| = P\left[\left[\int_{\text{area}} M \cos\theta d\sigma\right]^2 + \left[\int_{\text{area}} M \sin\theta d\sigma\right]^2\right], \tag{29b}$$

where P is the pressure amplitude at the transmitting crystal.

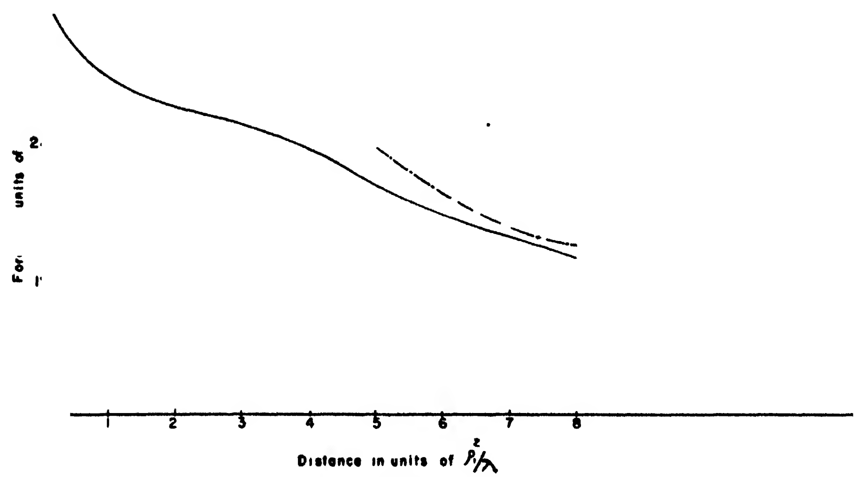

FIG. 12. Force on receiver crystal ($\rho_2 = \rho_1$) vs. distance between crystals. The two crystals have equal radii and are aligned parallel. Dotted —·—·—· gives asymptotic behaviour at large distances.

Two cases are shown plotted in Figs. 12 and 13. At large distances the force on the receiver crystal approaches asymptotically to the hyperbola

$$|F| = P\frac{\pi^2 \rho_1^2 \rho_2^2}{\lambda d}, \tag{30}$$

which is also drawn in Figs. 12 and 13 (ρ_2 is radius of receiver crystal). Fig. 12 applies to the usual case of two crystals of the same size, or one crystal and a plane reflector. For Fig. 13 the radius of the receiver crystal is $\frac{1}{2}$ that of the transmitter. The curve shows an oscilla-

tory motion at small separation and a pronounced maximum near $d = 1.2\rho_1^2/\lambda$, at which point presumably it is being effectively irradiated by all parts of the transmitter crystal. The value of the average pressure here is nearly 60 per cent. greater than the value, P, at the transmitter.

Case II: The Radiation Pattern in a Semi-Infinite Tube with a Vibrating Crystal at One End.

The analysis of ultrasonic propagation in a tube of semi-infinite length depends on the choice of boundary conditions at the wall of the tube which correspond to physical reality. If the boundary condition taken is that the normal component of the velocity at the wall of the tube shall be zero, then for a system with a transmitter equal to the tube radius a nearly plane wave can be made to travel down the tube.

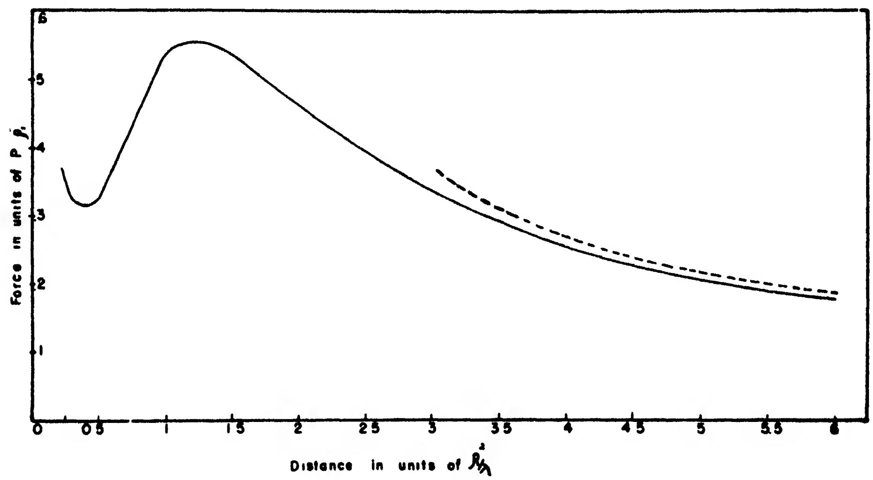

FIG. 13. Force on receiver crystal ($\rho_2 = \frac{1}{3}\rho_1$) vs. distance between crystals. The radius of the receiving crystal is one-third of the transmitter.

If the radius of the transmitter is less than the radius of the tube, the disturbance in the tube can be considered as the sum of the disturbances due to an infinite number of cylindrical modes travelling down the tube with differing velocities.

Another boundary condition which may be reasonable in some cases is that the pressure at the wall of the tube is zero. Such a boundary condition may be approximately valid when there are air spaces between the fluid and the tube (as often occur for mercury in a steel tube). With this boundary condition it is found that a series of cylindrical modes will travel down the tube with varying velocities. We are indebted to Mr. H. J. McSkimin of the Bell Telephone Laboratories for allowing us to see his unpublished analysis of this situation. His results show that the intensity of the ultrasonic beam falls off very

slightly at large distances in the tube in contrast to the inverse square law spreading of the free-space beam.

In Section D where viscosity effects also are considered there will be presented some experimental data on the question of propagation of waves in tubes.

D. ATTENUATION.

Free Space Beam

A "free space" beam in a liquid decreases in two distinct ways. There is first the spreading of the beam due to diffraction, which gives rise to an inverse square law for the intensity as large distances from the transmitting crystal (see eq. 28b of preceding section). The amplitude behavior can be described by

$$a = a_0 \frac{S}{d\lambda}, \tag{31}$$

where a_0 is the amplitude near the transmitting crystal, S the area of the crystal, d the distance from the crystal and λ the wavelength. Eq. (31) holds provided $d\lambda > S$.

The second kind of decrease is an absorption due to viscosity and thermal conduction in the liquid, which results in an exponential law for amplitude versus distance of the form $a = a_0 e^{-\alpha d}$. The theoretical formula [8] for α is given by the expression.

$$\alpha = \frac{2\eta^2 f^2}{\rho v^3}\left[\frac{4}{3}\eta + \frac{(\gamma - 1)K}{c_p}\right], \tag{32}$$

where η is the viscosity coefficient, f the frequency, ρ the density and v the acoustic velocity. In the second term γ is the ratio of the specific heats, K the thermal conductivity and c_p the specific heat at constant pressure. For non-metallic liquids the effect of thermal conduction is negligible. For many liquids this simple theory predicts an attenuation much smaller than that found by experiment. For mercury, however, which was widely used in delay line operation, agreement is good. Theory predicts an attenuation of about 1.1 db. per ft. at 10 mc., whereas the experimental values indicate $1.2 \pm .2$ db. per ft. Here the heat conduction loss accounts for 80 per cent. of the attenuation.

Since the viscosity coefficient decreases with rising temperature, attenuation generally decreases also. The effect is very marked in the case of water.[9]

[8] W. P. Mason, "Electromechanical Transducers and Wave Filters," Van Nostrand, p. 305 (1942).

[9] C. E. Teeter, Jr., *J. Acous. Soc. Am.*, 18, 488 (1946).

Guided Beam

It seems to be possible to avoid at least in part the decrease due to spreading of the beam by confining the beam within a tube of the same diameter as the transmitting crystal. This is what is meant by a guided beam. If the liquid could slip freely over the tube surface, a piston-like motion would take place corresponding to perfect plane-wave propagation, without any loss due to spreading. Actually, the conditions at the wall are not as simple as this and depend on the nature of the surface. A possible assumption is that the liquid sticks to the surface instead of sliding freely over it. Crandall [10] has shown that the viscous losses take place in a very thin layer, of the order of 10^{-6} cm. in thickness, across which a major portion of the velocity change occurs. The thickness of the layer is given by

$$= \left(\frac{\eta}{\pi \rho f}\right)^{\frac{1}{2}}, \tag{33}$$

where η is the viscosity, ρ the density, f the frequency. The drag produced by the layer on the liquid column causes an exponential attenuation with an amplitude absorption coefficient per unit length given by

$$\beta = \frac{2\pi\delta}{b\lambda}, \tag{34}$$

where b is the radius of tube and λ the wavelength. For 10 mc. waves in mercury contained in a tube of radius 0.3 cm. we have $\delta = 2.85 \times 10^{-6}$ cm. and $\beta = 0.0040$ cm^{-1}. Some experiments have been carried out at 10 mc. which indicate that the attenuation of ultrasonic waves in mercury shows a dependence on tube diameter of this order of magnitude. The effect was, however, influenced by the character of the tube's inner surface. It has already been mentioned in Section C that the boundary conditions may have a profound effect. For example, if, instead of a high acoustic impedance in the steel wall, the air occluded in the irregularities of wall surface offers a low impedance, there will be no energy absorption at the tube wall and the propagation will always consist of many cylindrical waves travelling with different velocities down the tube.

Since the guided beam is not subject to the attenuation from spreading given by equation (31), total attenuation is generally less in this case than for the free space beam. Only for long distances can the tubular attenuation give an effect comparable with the spreading. Of course both beams are subject to the absorption varying as the square of the frequency (eq. 32).

[10] I. B. Crandall, "Theory of Vibrating Systems and Sound," Van Nostrand, p. 229 (1927).

Science Captures a Chinese Art.—In China the making of soya sauce is mainly a household art, with grandmothers passing along to daughters and granddaughters the details of recipes and the strains of ferments used in the process.

In the United States, soya sauce—as is true of many other food products—is chiefly a factory product. Each manufacturer is concerned not only with producing a sauce of high quality, but also a standardized sauce in which each batch will have the flavor, color, and taste of every other batch. This calls for standardization of the raw materials and for use of standard strains of microorganisms in the fermentation of the sauce.

The U. S. Department of Agriculture has announced that strains of four organisms desirable in preparing soya sauce have been added to the culture collection of industrial ferments at the Northern Regional Research Laboratory at Peoria, Ill. These include two molds, a yeast, and a bacterium. They will be maintained as pure cultures and will be available to industrial users. This makes it possible for a fermenter to make a fresh start with new and pure cultures, if at any time his stock cultures become contaminated with "wild" molds or yeasts that injure the quality or uniformity of his product.

The Fermentation Division of the Bureau of Agricultural and Industrial Chemistry credits Mr. Pei Sung King of the National Bureau of Industrial Research, Chungking, China, with aid in selecting the strains of organisms and standardizing a process of fermentation that yields a high-quality sauce. Mr. King has been a guest worker at the Northern Laboratory, and suggested various methods of fermentation which the scientists tested and compared in working out the method they now recommend to manufacturers. It is not practical for home use or for small quantities of sauce.

The preparation of soya sauce calls for a brine fermentation of the beans for from 30 to 90 days. But the "starter" used in this process is a mixture of three previously prepared cultures; (1) of a mold that develops on cooked rice; (2) of a yeast working on soya broth; and (3) of a bacterial fermentation of soya broth. For a quality product these should be mixed at the right stage of development in suitable proportions.

R. H. O.

AN INTEGRAL-EQUATION APPROACH TO PROBLEMS OF VIBRATING BEAMS.*

BY

WALTER T. WHITE, Sc.D.†

PART I.

ABSTRACT.

In this paper integral equations are applied for the calculation of the normal modes of vibrating beams. Both exact and approximate methods of solving the integral equation are considered. The Green's function, or kernel, of the integral equation is constructed for both uniform and nonuniform beams. Solutions for the normal modes of a uniform cantilever are given. A nonuniform, naturally-twisted turbine blade is studied in detail and the first and second normal modes are calculated by the integral-equation method.

1. INTRODUCTION.

In many mechanical designs, it is important to eliminate vibrations associated with mechanical resonance. Calculations of the normal modes of the system often reveal objectionable resonances, and when they are known, methods can be devised to reduce or eliminate them. In some cases, damping or appropriate shock mounting may achieve the desired result. But in others, it is necessary to change the inertia and stiffness of the system. The theory of vibrating beams, when applicable, is the most effective basis for such calculations.

Vibrating members treated as beams are rotating shafts, turbine blades, hulls of ships, airplanes, missiles, and other complex structures. The beam and its supports may be sufficiently simple for straightforward analysis by classical methods with differential equations. If the problem is complicated by variations of physical properties, the solution by integral equations gives results more conveniently than solution by differential equations. The formulation of an integral equation is based upon a simple picture of the beam and the application of the superposition principle. Variations of physical properties and such effects as rotary inertia, longitudinal inertia, and gyroscopic moment cause no particular difficulty with the integral-equation formulation.

The purpose of this paper is to outline the integral-equation approach to the calculation of normal modes of vibrating beams. Certain properties of integral equations required for the solution of practical problems are given. A method is developed for constructing the kernels for the integral equations of vibrating beams. To demonstrate the integral-equation method and its accuracy, an application is made to a

* This paper is from a thesis, "Integral-Equation and Approximate Solutions for Normal Modes of Vibration," submitted to the department of Electrical Engineering, M.I.T., 1941.

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simple cantilever. A second application is made to a nonuniform naturally-twisted turbine blade.

2. FORMULATION OF THE INTEGRAL EQUATION.

Two procedures generally are available for the integral-equation formulation of a vibration problem: (1) obtain the differential equation, and then make a transformation to the integral equation; (2) formulate the integral equation directly from the physical nature of the problem. Evidently the second procedure has distinct advantages. Indeed, one of the principal advantages of the integral-equation treatment is the closeness with which it is tied in with the physical picture.

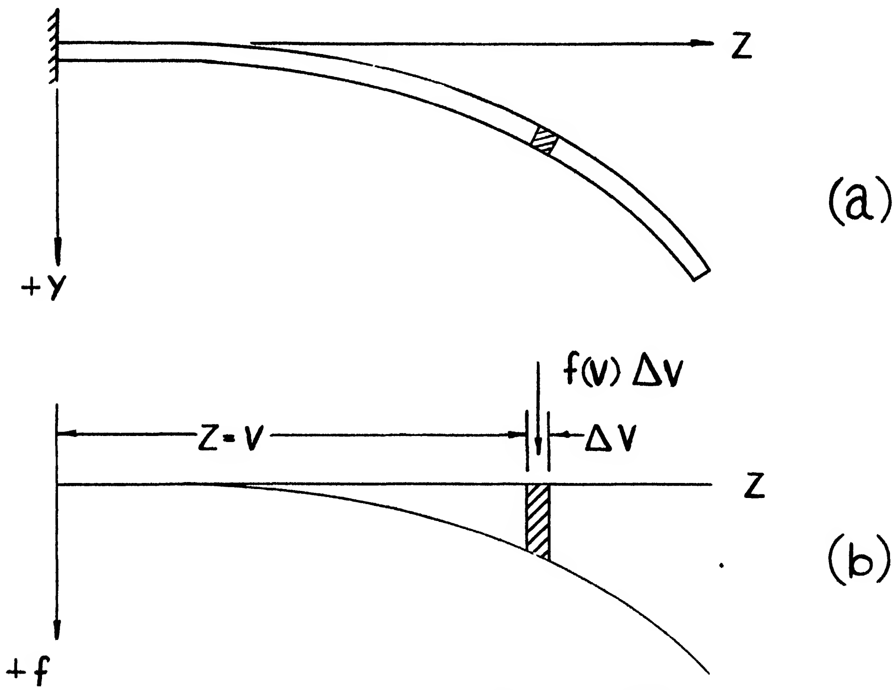

FIG. 1. Inertial loading on vibrating cantilever.

As an example, consider the cantilever. To formulate the integral equation, it is necessary to consider the inertial loading while the cantilever is freely vibrating. If the cantilever is observed at the instant of its maximum downward deflection, as in Fig. 1a, each elemental mass has a deflection $+y(z, t)$ and an upward acceleration $-a(z, t)$. The product of mass m per unit length and acceleration gives the upward inertial force which the beam exerts upon its mass. By the principle of action and reaction, the mass exerts a downward force or loading $f(z, t)$ per unit length that is given by

$$f(z, t) = -ma(z, t). \tag{1}$$

The loading on an elemental length Δv is equivalent to a concentrated load $f(v)\Delta v$ (see Fig. 1b). From texts on strength of materials,*[1] the static deflection of a uniform cantilever caused by a unit-concentrated load applied at $z = v$ is

$$G(z, v) = \left.\begin{aligned} &= z^2(3v - z)/6EI, \quad && z \leq v \\ &= v^2(3z - v)/6EI, && v \leq z \end{aligned}\right\}, \tag{2}$$

where EI is the flexural rigidity of the cantilever. The static deflection caused by a unit-concentrated load generally is called a *Green's function*. The deflection caused by the loading $f\Delta v$ is

$$\Delta y(z, v) = G(z, v)f(v)\Delta v. \tag{3}$$

By superposition, the deflection at z is given as the summation of the deflections caused by each elemental loading along the beam, or

$$y(z, t) = \int_0^L G(z, v)f(v, t)dv. \tag{4}$$

The time variable t is introduced, since the loading f in Eq. 1 is a time-varying quantity.

Since undamped oscillations of small amplitude are harmonic functions of time, $y(z, t)$ can be written as

$$y(z, t) = y(z) \sin \omega t, \tag{5}$$

where ω is the frequency of oscillation in radians per second. The acceleration is obtained by differentiation of Eq. 5, and is

$$a(z, t) = - \omega^2 y(z) \sin \omega t. \tag{6}$$

Substitutions from Eqs. 1, 5 and 6 in Eq. 4 gives

$$y(z) \sin \omega t = m\omega^2 \int_0^L G(z, v)y(v) \sin \omega t dv. \tag{7}$$

Since the time function is not dependent upon the variable of integration, it is cancelled to give

$$y(z) = \lambda \int_0^L G(z, v)y(v)dv, \tag{7a}$$

where the constants are lumped in the parameter λ.

Equation 7a is the *integral equation* for the freely vibrating cantilever. The *kernel*, or Green's function, $G(z, v)$, is given by Eq. 2, and y and λ are unknowns. The problem is to find values of λ for which non-zero functions y exist. Equation 7a is called a *homogeneous integral equation;* the λ's are called *eigenvalues* and the corresponding y's are called *eigenfunctions.* The eigenvalues and eigenfunctions determine the *normal modes* of vibration for the cantilever.

* For numbered references see Bibliography.

3. PROPERTIES OF HOMOGENEOUS INTEGRAL EQUATIONS.

Before completing the solution of Eq. 7a for the cantilever, it is well to consider certain properties of homogeneous integral equations[2] which are useful in the solution of practical problems.

A general form of the homogeneous integral equation is

$$y(z) = \lambda \int_a^b K(z, v)y(v)dv, \tag{8}$$

where a and b are constants and $K(z, v)$ is the kernel corresponding to the Green's function in Eq. 7a. In vibrations and related problems, the kernel is often a symmetric function of z and v:

$$K(z, v) = K(v, z). \tag{9}$$

If the kernel is asymmetric, it can be made symmetric by a simple transformation, as will be shown later. In practical problems the kernel is real, continuous and not zero on the interval (a, b).

An obvious solution of Eq. 8 is

$$y(z) = 0.$$

Although this result yields no useful information, useful solutions can be found. For certain eigenvalues λ there exist corresponding solutions or eigenfunctions y which are written

$$\left.\begin{array}{l} y_1(z) = \lambda_1 \int_a^b K(z, v)y_1(v)dv \\ y_2(z) = \lambda_2 \int_a^b K(z, v)y_2(v)dv \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ y_n(z) = \lambda_n \int_a^b K(z, v)y_n(v)dv \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \end{array}\right\}. \tag{10}$$

Clearly, each eigenfunction is determined only within a constant multiplying factor, for, if $y_n(z)$ is a solution of Eq. 8, substitution shows that $cy_n(z)$ is a solution where c is a constant. Solutions in Eqs. 10, therefore, only determine the form of the eigenfunctions. The eigenvalues, however, are uniquely determined.

A general property for eigenfunctions of symmetric kernels is their orthogonality. This property is expressed as

$$\int_a^b y_m(v)y_n(v)dv = 0, \qquad (m \neq n). \tag{11}$$

If $m = n$, the integral has a value N_n called the *norm*; that is,

$$\int_a^b [y_n(v)]^2 dv = N_n. \tag{12}$$

A useful property of the kernel is its *bilinear expansion*. For an infinity of eigenvalues and eigenfunctions, the bilinear expansion is given as

$$K(z, v) = \sum_{n=1}^{\infty} y_n(z) y_n(v)/\lambda_n N_n. \tag{13}$$

This series expansion for the kernel must be uniformly convergent for all values of z and v on the interval (a, b). The expansion is Hilbert's bilinear formula.[2] It gives the kernel in the manner of a Fourier series in terms of the eigenfunctions. When applied to physical problems the uniform convergence is assured.

Often the kernel is asymmetric. For example, if the cantilever which led to Eq. 7a has a nonuniform cross section and loading, there is associated with the Green's function a function $\mu(v)$ which takes care of the variations of mass and flexural rigidity along the length of the cantilever. The integral equation then has the form

$$y(z) = \lambda \int_0^L G(z, v)\mu(v) y(v) dv, \tag{14}$$

and the kernel is

$$K(z, v) = G(z, v)\mu(v). \tag{15}$$

Obviously, the kernel is asymmetric, since

$$K(z, v) \neq K(v, z).$$

By a simple transformation, the asymmetric kernel can be made symmetric. Multiply Eq. 14 by $\sqrt{\mu(z)}$ and define a new eigenfunction

$$Y(z) = y(z)\sqrt{\mu(z)}, \tag{16}$$

and a new kernel

$$K_s(z, v) = G(z, v)\sqrt{\mu(z)\mu(v)}. \tag{17}$$

With these substitutions Eq. 14 is written

$$Y(z) = \lambda \int_0^L K_s(z, v) Y(v) dv. \tag{18}$$

Since the kernel of Eq. 17 is symmetric, Eq. 18 can be treated as an integral equation with a symmetric kernel. The orthogonality condition from Eqs. 11 and 16 is

$$\int_0^L \mu(v) y_m(v) y_n(v) dv = 0, \qquad (m \neq n). \tag{19}$$

The foregoing properties are useful for the solution of practical integral equations. However, complex kernels arise in certain problems and such problems require special orthogonality conditions and a more complicated bilinear expansion. After the formulation of the problem, an inspection will reveal whether the integral equation is similar to Eq. 8 or 14, or whether it is more complex and requires the derivation of a special orthogonality condition or bilinear expansion.

4. SOLUTION OF INTEGRAL EQUATIONS.

Practical methods for solving integral equations seldom are exact. More often solutions are obtained by approximations, and with reasonable effort such solutions can be calculated with an accuracy comparable to the physical data. Two methods frequently used for solving integral equations are:

1) Obtain a solution by an intuitive guess, and prove it by substitution.
2) Obtain a solution by *successive approximations*.

As an example of solution by the first method, consider the cantilever of Eq. 7a. Substitution of the Green's function from Eq. 2 gives

$$y(z) = \lambda \int_0^z v^2(3z - v)y(v)dv + \lambda \int_z^L z^2(3v - z)y(v)dv,$$

where

$$\lambda = m\omega^2/6EI.$$

As a guess for the solution, try the equation

$$y(z) = B \cosh kz + C \sinh kz + D \cos kz + E \sin kz.$$

Substitution in the integral equation gives the eigenvalue

$$\lambda = k^4/6,$$

and the eigenfunction

$$y(z) = \cosh kz - \cos kz + b (\sinh kz - \sin kz).$$

The constant b is

$$\left.\begin{aligned} b &= -(\sinh kL - \sin kL)/(\cosh kL + \cos kL) \\ &= -(\cosh kL + \cos kL)/(\sinh kL + \sin kL) \end{aligned}\right\},$$

where kL is a root of

$$\cosh kL \cos kL + 1 = 0.$$

A few values of kL are

$$\begin{aligned} k_1L &= 1.8751, \\ k_2L &= 4.6941, \\ k_3L &= 7.8548. \end{aligned}$$

From the foregoing the first three natural frequencies of the canti-

lever are obtained from the eigenvalues as

$$\omega_1 = (3.516/L^2)\sqrt{EI/m},$$
$$\omega_2 = (22.03/L^2)\sqrt{EI/m},$$
$$\omega_3 = (61.70/L^2)\sqrt{EI/m} \text{ radians per second.}$$

Generally, a guess as in the foregoing will not lead to the solution in a closed form. One reason for this is that the Green's function may be known only as a family of curves. For problems not easily solved in closed form, the method of successive approximations is appropriate. When used with numerical or machine integration, the method of successive approximations is applicable to problems in which the kernel is known from tabulated data or plotted curves.

In the method of successive approximations, the normal modes are calculated singly beginning with the first, or lowest mode, and continuing in order to the higher modes. For the first mode, an approximation to the eigenfunction is used as a *generating function.* It is necessary only that the generating function be *not orthogonal* to the first eigenfunction. Although it is improbable that a function arbitrarily selected would be orthogonal to the unknown eigenfunction, it can be shown that a Green's function is certain of meeting the non-orthogonality condition. Accordingly, the generating function $g_0(z)$ is taken as

$$g_0(z) = G(z, a), \tag{20}$$

where a is an arbitrary value which gives a Green's function not identically zero.

For the first eigenfunction, select for a the value where judgment indicates a concentrated load to be most nearly equivalent to the distributed inertial load. For the uniform cantilever the value is approximately $a = 0.8L$. (The best value can be shown to be $a = 0.8021L$.) When the generating function is selected by this method, solutions are obtained with a small number of successive approximations. Therefore, unless some other function obviously has advantages, a Green's function is used as the generating function.

From the generating function $g_0(z)$, a set of equations is formed by successive iterations with the kernel. For the cantilever the iterations follow from Eq. 14 as

$$\left.\begin{aligned} g_1(z) &= \int_0^L G(z, v)\mu(v)g_0(v)dv \\ g_2(z) &= \int_0^L G(z, v)\mu(v)g_1(v)dv \\ &\cdots\cdots \\ &\cdots\cdots \\ &\cdots\cdots \\ g_{n+1}(z) &= \int_0^L G(z, v)\mu(v)g_n(v)dv \end{aligned}\right\}. \tag{21}$$

As n becomes large, these successive iterations give the solution of the integral equation [2] as

$$y_1(z) = g_n(z). \tag{22}$$

This is the eigenfunction of the first mode. The first eigenvalue is calculated by substitution of Eq. 22 in Eq. 14 which gives

$$g_n(z) = \lambda_1 \int_0^L G(z, v)\mu(v)g_n(v)dv. \tag{23}$$

Substitution from Eqs. 21 gives

$$g_n(z) = \lambda_1 g_{n+1}(z). \tag{23a}$$

The eigenvalue λ_1 is calculated from Eq. 23a for any value of z. However, where approximate methods are used, a more accurate value of λ_1 is calculated as the average value

$$\lambda_1 = \frac{\int_0^L g_n(v)dv}{\int_0^L g_{n+1}(v)dv}. \tag{24}$$

Usually the first eigenvalue is calculated after one to three iterations. The first eigenfunction generally is calculated after two to four iterations. Since the convergence is very rapid, the calculation of the eigenfunction is complete when the form of $g_{n+1}(z)$ is similar to $g_n(z)$.

Calculation of the second eigenvalue and eigenfunction by approximate methods is possible only after calculation of the first eigenfunction. A generating function is selected which is orthogonal to the first eigenfunction and not orthogonal to the second eigenfunction. The first condition is met by a procedure to be developed below. As for the second condition, it is improbable that an arbibrary function would be orthogonal to the unknown eigenfunction; yet, where certainty is desired, a Green's function given by Eq. 20 is selected. A good value to choose for a is the estimated coordinate where the maximum deflection of the beam occurs when it is vibrating in the second mode.

To construct a generating function $g_0(z)$ orthogonal to the first eigenfunction $y_1(z)$, form the equation [4]

$$g_0(z) = f_0(z) - \alpha y_1(z), \tag{25}$$

where α is a constant and $f_0(z)$ is an arbitrary function, preferably a Green's function $G(z, a)$. Multiplication of Eq. 25 by $\mu(z)y_1(z)$ and integration over the length of the beam give

$$\int_0^L \mu(z)y_1(z)g_0(z)dz = \int_0^L \mu(z)y_1(z)f_0(z)dz - \alpha \int_0^L \mu(z)y_1^2(z)dz. \tag{26}$$

The integral on the left is the required orthogonality condition and is

zero if

$$\alpha = \frac{\int_0^L \mu(z)y_1(z)f_0(z)dz}{\int_0^L \mu(z)y_1^2(z)dz}. \tag{27}$$

Accordingly, the generating function for the second eigenfunction can be constructed from Eq. 25 with α evaluated by Eq. 27.

Successive iterations of the generating function for the second eigenfunction give

$$\left.\begin{array}{l} g_1(z) = \int_0^L G(z, v)\mu(v)g_0(v)dv \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ g_{n+1}(z) = \int_0^L G(z, v)\mu(v)g_n(v)dv \end{array}\right\}. \tag{28}$$

If each integration is *exact*, the nth iteration approaches the second eigenfunction as n becomes large. However, exactness is possible only with formal integration, and if approximate integration is used, a special procedure is required to assure the convergence of Eqs. 28 to the second eigenfunction. The procedure simply is to make certain that each successive generating function is orthogonal to the first eigenfunction. This requires a construction similar to Eq. 25 after each iteration to obtain an orthogonalized generating function, $g'_n(z)$, given by

$$g'_n(z) = g_n(z) - \alpha_n y_1(z). \tag{29}$$

The $(n + 1)$ iteration gives

$$g_{n+1}(z) = \int_0^L G(z, v)\mu(v)g'_n(v)dv. \tag{30}$$

As n becomes large, the second eigenfunction is the orthogonalized $g'_{n+1}(z)$ and is given by

$$y_2(z) = g_{n+1}(z) - \alpha_{n+1}y_1(z). \tag{31}$$

The second eigenvalue is given by a relation similar to Eq. 24 as

$$\lambda_2 = \frac{\int_0^L g'_n(v)dv}{\int_0^L g'_{n+1}(v)dv}. \tag{32}$$

Another method for calculation of the second eigenfunction makes use of the bilinear expansion, Eq. 13. Since Eq. 13 is for a symmetric kernel, the integral equation must be in the form of Eq. 18:

$$Y(z) = \lambda \int_0^L K_s(z, v)Y(v)dv. \tag{18}$$

The bilinear formula gives

$$K_s(z, v) = \sum_{n=1}^{\infty} Y_n(z) Y_n(v)/\lambda_n N_n. \tag{13a}$$

If the first eigenfunction and eigenvalue are known, they are subtracted from $K_s(z, v)$ to give the ***perturbated kernel*** $K_p(z, v)$,

$$K_p(z, v) = K_s(z, v) - Y_1(z) Y_1(v)/\lambda_1 N_1. \tag{33}$$

Since the remaining eigenfunction components in $K_p(z, v)$ are orthogonal to $Y_1(z)$, no error results in the calculation of the second eigenfunction if the pertrubated kernel, $K_p(z, v)$, is used instead of $K_s(z, v)$. Hence, with a suitable generating function, which need not be orthogonal to the first eigenfunction, the second eigenfunction is calculated by successive iterations to give

$$\left.\begin{aligned} g_1(z) &= \int_0^L K_p(z, v) g_0(v) dv \\ &\cdots\cdots \\ &\cdots\cdots \\ &\cdots\cdots \\ g_{n+1}(z) &= \int_0^L K_p(z, v) g_n(v) dv \end{aligned}\right\}. \tag{34}$$

There is no tendency to converge to the first eigenfunction and successive generating functions are not required to be made orthogonal to the first eigenfunction. For large values of n the second eigenfunction is given by

$$Y_2(z) = g_n(z), \tag{35}$$

and the second eigenvalue is

$$\lambda_2 = \frac{\int_0^L g_n(v) dv}{\int_0^L g_{n+1}(v) dv}. \tag{36}$$

The extension of the foregoing method to the calculation of higher modes is straightforward. A new perturbated kernel is formed by subtraction of previously calculated components from the bilinear formula, and the next higher mode is calculated by relations similar to Eqs. 34, 35 and 36. Alternatively, the higher modes are calculated as in Eqs. 28 and 29 provided the generating function for each successive iteration is made orthogonal to the previously calculated lower modes.

An application of the foregoing methods is made for the calculation of the first two normal modes of the uniform cantilever. Although the complete solution was obtained for the integral equation, the method of successive approximations is used for purposes of illustration. A comparison of numerical results from the approximate method with the exact solution illustrates the accuracy of the method.

Several means are available for the numerical evaluation of the integrals. A standard calculating machine can be used with Simpson's rule. Machines, such as the cinema integraph [5] or the differential analyzer,[6] if available, can be used for evaluation of the integrals. In this example the differential analyzer is used.

The integral equation to be solved is

$$y(z) = \lambda \int_0^L K(z, v) y(v) dv,$$

where $K(z, v)$ is a symmetric kernel and is obtained from Eq. 2 as

$$\left.\begin{aligned} K(z, v) &= z^2(3v - z), & z \leq v \\ &= v^2(3z - v), & v \leq z \end{aligned}\right\}.$$

λ is the eigenvalue which includes the unknown natural frequency and the constants of the equation. In this example

$$\lambda = m\omega^2/6EI.$$

If the integral equation is non-dimensionalized by the substitutions

$$\left.\begin{aligned} z &= z'L \\ v &= v'L \end{aligned}\right\},$$

it is written in non-dimensional form (with the primes omitted for convenience) as

$$y(z) = \lambda \int_0^1 K(z, v) y(v) dv,$$

and

$$\lambda = mL^4\omega^2/6EI.$$

Numerical values for the kernel of the integral equation are in Table I. From the estimated form of the freely vibrating cantilever, the generat-

TABLE I.

Symmetric Kernel for Uniform Cantilever.

$K(z, v) = z^2(3v - z),\ z \leq v,\ K(z, v) = K(v, z)$

(Multiply by 10^{-3})

v \ z	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0										
0.1	0	2									
0.2	0	5	16								
0.3	0	8	28	54							
0.4	0	11	40	81	128						
0.5	0	14	52	108	176	250					
0.6	0	17	64	135	224	325	432				
0.7	0	20	76	162	272	400	540	686			
0.8	0	23	88	189	320	475	648	833	1024		
0.9	0	26	100	216	368	550	756	980	1216	1458	
1.0	0	29	112	243	416	625	864	1127	1408	1701	2000

ing function is taken as

$$g_0(z) = kz^2,$$

where k is any convenient constant. Two successive iterations of the generating function, as in Eqs. 21, are sufficient to evaluate the eigenfunction. The results from the second iteration are in Table II, as are the exact values. The eigenvalue is calculated from Eq. 24, and the natural frequency, calculated from the eigenvalue, is given in Table II. The errors between the exact and calculated values for both the eigenfunction and natural frequency are negligible. The normalization constant is calculated from Eq. 12 and listed in Table II.

For the evaluation of the second mode, the perturbated kernel is calculated from Eq. 33 as

$$K_p(z, v) = z^2(3v - z) - y_1(z)y_1(v)/\lambda_1 N_1, \qquad z \leq v.$$

The values of $y_1(z)$, λ_1 and N_1 tabulated in Table II for the second

TABLE II.

Normal Modes of Uniform Cantilever.

z	$y_1(z)$: First Mode		$y_2(z)$: Second Mode	
	Exact	Second Iteration	Exact	Third Iteration
0.0	0.0000	0.0000	0.000	0.000
0.1	0.0168	0.0168	−0.092	−0.092
0.2	0.0639	0.0639	−0.301	−0.300
0.3	0.1365	0.1366	−0.526	−0.525
0.4	0.2299	0.2302	−0.683	−0.685
0.5	0.3395	0.3391	−0.714	−0.721
0.6	0.4611	0.4612	−0.589	−0.600
0.7	0.5908	0.5905	−0.317	−0.323
0.8	0.7255	0.7247	0.070	0.075
0.9	0.8624	0.8635	0.524	0.509
1.0	1.0000	0.9997	1.000	0.986
ω: Natural frequency in radians per second				
	$\frac{3.5160\sqrt{EI/m}}{L^2}$	$\frac{3.5176\sqrt{EI/m}}{L^2}$	$\frac{22.03\sqrt{EI/m}}{L^2}$	$\frac{22.04\sqrt{EI/m}}{L^2}$
N: Normalization constant				
	2.001	2.001		

iteration are used for calculation of $K_p(z, v)$. The generating function for the second mode is

$$g_0(z) = kK_p(z, 0.6),$$

where k is a convenient constant multiplier. The results from the third iteration of the generating function are compared with the exact values in Table II. The errors for the approximate calculation of the second mode are almost as small as the errors for the first mode.

(*To be continued in next issue.*)

VISUAL MEASUREMENTS OF SHORT PULSES OF DIRECT CURRENT.

BY

PHILIP F. ORDUNG.*, †

Yale University,
1946.

SYNOPSIS.

Within recent years, engineering applications using pulses of direct current as short in duration as a microsecond and repeated several thousand times per second have become important. Such pulses may have amplitudes that are greater than 100 amperes. This paper discusses the techniques and precautions which are essential for oscillographic observations of such pulses. Particular consideration is given to the problem of determination of the bandwidths required for measurements of such pulses, of the construction and the calibration of resistors for metering such currents, and of properly terminating the coaxial transmission line used for connecting the metering element to the oscillograph.

INTRODUCTION.

With the advances and developments of recent years in the application of electron tubes has come an increasing need for accurate measurements of short pulses of direct current. The information which must be supplied by the measurements is the shape, amplitude, duration, and repetition rate of the pulse. Methods have been developed using meters for measuring the amplitude and the repetition rate of the pulse. However, visual observation of the pulse by means of an oscillograph is the only method which can give accurate data concerning the shape and the duration of the pulse. The general arrangement of the components for the oscillographic method with which this paper is concerned is shown in Fig. 1. The pulse current, shown as "i" on the figure, flows through the metering element. The corresponding voltage pulse developed across the metering element is transmitted to and viewed on the oscillograph. For this type of measurement, the sweep of the oscillograph must be synchronized to the pulse being viewed and the interval of duration of the sweep trace must be slightly greater than that of the pulse.

The types of pulses that may be encountered in the applications of pulse techniques are of a great variety. Typical pulses of current have intervals of duration between a fraction of microsecond and several hundred microseconds and have amplitudes ranging from several amperes to greater than one hundred amperes. One such pulse is shown in Fig. 5A. The shape of such pulses is approximately trapezoidal with

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† Formerly with the Naval Research Laboratory where this work was begun.

periods of rise and decay that are short in comparison to the pulse-duration. The repetition rates for these pulses range from several pulses per second to several thousand pulses per second. The problem of measuring such widely differing pulses is great, but the precautions which must be observed in their measurement are similar.

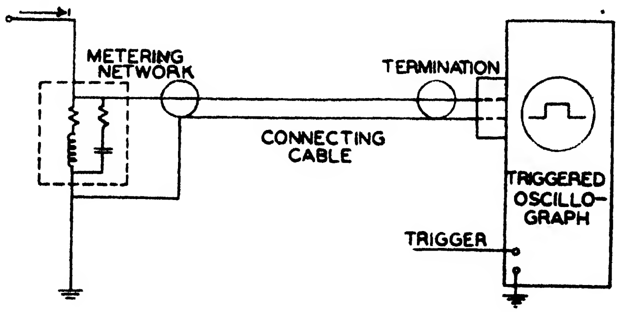

FIG. 1. Circuit for visual measurement of repeated pulses of current.

BANDWIDTH FOR PULSE MEASUREMENTS.

The measuring equipment in order to faithfully reproduce the shape of a repeated pulse must represent all of the frequency components contained in the pulse. For the purpose of determining the necessary bandwith for representing the pulse, Fourier Analysis is applied to the repeated square pulse shown in Fig. 2. T_R is the period of the pulse,

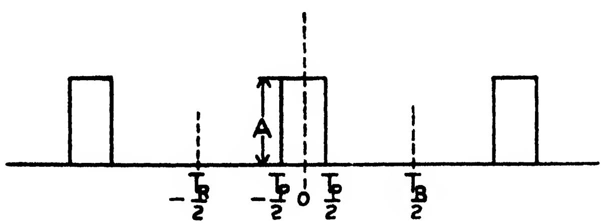

FIG. 2. Wave of repeated square pulses of amplitude, A, period, T_R, and pulse width T_P.

and T_P is the duration of the pulse. In functional form, the shape of one cycle of this pulse wave may be represented as follows:

$$f(t) = A \qquad -\frac{T_P}{2} \leq t \leq \frac{T_P}{2}$$

$$f(t) = 0 \qquad -\frac{T_R}{2} < t < -\frac{T_P}{2} \qquad \frac{T_P}{2} < t < \frac{T_R}{2}. \tag{1}$$

By conventional Fourier Analysis, the pulse wave may be represented as

$$f(t) = \tfrac{1}{2}a_0 + \sum_{k=1}^{\infty} a_k \operatorname{Cos} \frac{2\pi kt}{T_R}, \qquad k = 1, 2, 3, 4 \cdots \tag{2}$$

where

$$a_k = \frac{2}{T_R}\int_{-T_R/2}^{T_R/2} f(t)\,\mathrm{Cos}\,\frac{2\pi kt}{T_R}\,dt. \tag{2A}$$

The amplitudes of the frequencies required to represent the pulse are determined by substituting the functional form of the pulse, Eq. (1), into Eq. (2A). For the case of the repeated pulse being discussed, the Fourier series representation is

$$f(t) = A\,\frac{T_P}{T_R} + \sum_{k=1}^{\infty} \frac{2A}{\pi k}\,\mathrm{Sin}\,\pi k\,\frac{T_P}{T_R}\,\mathrm{Cos}\,\frac{2\pi kt}{T_R}. \tag{3}$$

The angular frequency components contained in the repeated pulse are given in this series by the term $2\pi k/T_R$, where k is the order of the harmonic of the repetition frequency F_R. Using Eq. (3), the frequency spectrum shown in Fig. 3 is plotted. The envelope of the spectrum is

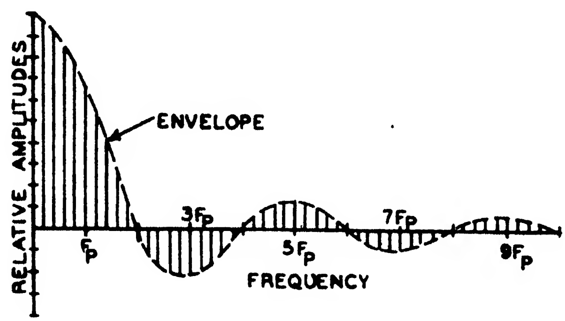

FIG. 3. Frequency spectrum corresponding to a repeated pulse wave in which $T_R = 10T_P$ and $F_P = 5F_R$.

shown by the dashed lines. Significant points in the envelope occur at frequencies that correspond to values of k such that

$$2kT_P/T_R = m. \qquad m = 1, 2, 3, 4, 5 \cdots \text{etc.} \tag{4}$$

Since the term $2\pi k/T_R$ in Equation (3) expresses angular frequency, k/T_R is frequency. The particular value of this frequency, k/T_R, that is obtained when $m = 1$, is defined to be the pulse-width frequency F_P and has a period twice that of the pulse width, $F_P = 1/2T_P$. For the even integer values of m, the magnitude of the envelope is zero. For the odd harmonics of the pulse-width frequency (frequencies that correspond to the odd integer values of m), the amplitudes of the minor lobes of the spectral envelope are approximately maximum. Thus, changing the repetition rate of a repeated pulse has no effect on the shape of the envelope of the frequency spectrum; however the number of frequencies represented in each lobe of the envelope is changed.

For a square wave, $T_P = T_R/2$, the frequencies representing the pulse are odd harmonics of the pulse-width frequency. When the pulse-

width frequency is a multiple of the repetition frequency (T_R/T_P an integer), the spectrum includes the frequencies required to represent a square wave with the given pulse width and additional frequencies located symmetrically in each lobe of the spectral envelope about the square-wave frequencies. The additional frequencies eliminate the pulses in the square wave that are not contained in the wave having the reduced repetition rate. This suggests that the number of lobes in the envelope of the frequency spectrum required to approximate a pulse of given width to a desired accuracy is the same for a square wave and for repetition rates that are submultiples of the pulse-width frequency. Generalized, this argument extends to all repetition rates less than the pulse-width frequency provided that the pulse separation is such that $\omega_1(T_R - T_P) > 18$. (See the Appendix.)

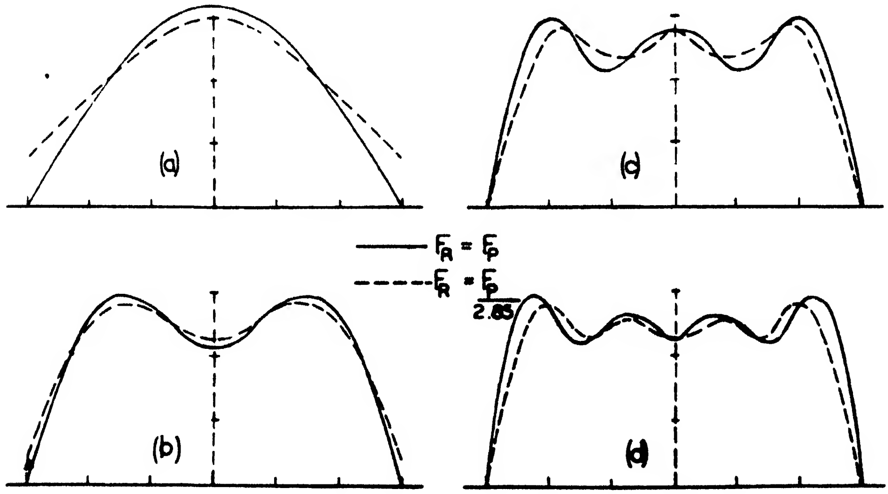

FIG. 4. Approximations to a square pulse of given width for two repetition rates—$F_R = F_P$, and $F_R = F_P/2.85$. The maximum frequencies included in the pass band for the approximations are as follows: (a) $2F_P$; (b) $4F_P$; (c) $6F_P$; and (d) $8F_P$.

Approximations of a given square pulse for two repetition rates, one of which is that for a square wave, are shown in Fig. 4. For both repetition rates, the inclusion of all frequencies to the eighth harmonic of the pulse-width frequency gives similar trapezoidal approximations having rise and decay periods less than 12 per cent. of the duration of the pulse and having less than 12 per cent. deviation in the tops of the pulses from the postulated shape. In the Appendix, the ratio of the rise time to the pulse duration is shown to be inversely proportional to the number of harmonics of the pulse-width frequency included in the reproducible band. Practically, all pulses that may be encountered have finite rise and decay periods which are seldom less than 5 per cent.

of the pulse width. Therefore, the bandwidth need seldom be greater than the twentieth harmonic of the pulse-width frequency.

CURRENT-METERING RESISTOR.

For an oscillographic measurement of a current pulse, the shape of the voltage pulse developed across the metering resistor should be exactly similar to that of the current pulse. Therefore the metering resistor must have the same phase shift and amplitude characteristics

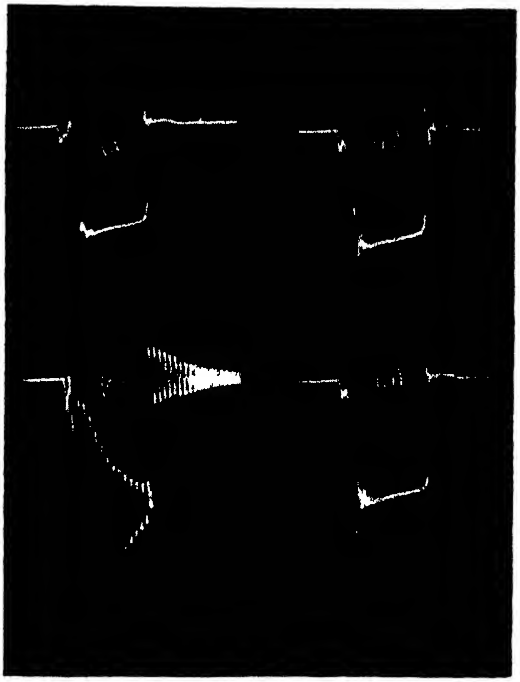

FIG. 5. Photographs of a 6 ampere, 1 microsecond current pulse repeated 1000 pulses per second.

(a) Pulse obtained with the corrected current-metering network connected with very short leads to the oscillograph.

(b) Pulse obtained with the inductive current-metering network connected with very short leads to the oscillograph.

(c) Pulse obtained with the corrected current-metering network connected with a 4-meter length of unterminated coaxial cable to the oscillograph.

(d) Pulse obtained with the corrected current-metering network connected with a 4-meter length of unterminated coaxial cable to the oscillograph.

for all frequencies contained in the band from zero to infinity. Such a resistor is physically impossible; but a network that approximates a perfect resistor over a bandwidth sufficiently great to represent the current pulse shapes to within few per cent. error can be constructed. In the measurement of large pulses of current, several watts of average

power are dissipated in the metering network. For permanency of calibration and the dissipation of power, the noninductively wound type of wire resistor should be employed as the basic element in the metering network. The frequency characteristics of a typical commercially available noninductively wound 10-watt resistor are shown in Fig. 6B. The series residual inductance exhibited by the resistor is constant over the band of frequencies of immediate importance in pulse current measurements. The wave form of the voltage across this resistor when the trapezoidal pulse of current shown in Fig. 5A is passed through it is shown in Fig. 5B. The residual inductance is responsible for the occurrence of the voltage spikes during the intervals that the pulse current has a high rate of change.

The effects of the residual inductance are corrected in the network shown in Fig. 6A. When $R_1 = R_2$, and $R_1R_2 = L_1/C$, the circuit is purely resistive with a resistance magnitude R_1 that is independent of frequency. The slight residual inductance in the resistance-capacitance

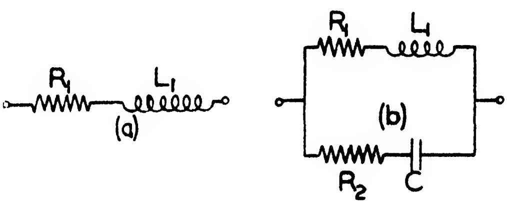

FIG. 6A. Equivalent circuits of (a) the inductive metering resistor, and (b) the same resistor connected in a frequency correcting circuit.

branch is the factor that limits the bandwidth for which the network approximates a pure resistance. The frequency characteristics for a typical network in which the inductive resistor previously discussed was used as the basic element are also shown in Fig. 6B. With this network the shape of current pulse shown in Fig. 5A was obtained. The extension of the bandwidth for the network beyond that for the inductive resistor affected elimination of the voltage spike shown in Fig. 5B, but had no effect on the remainder of the pulse. Since the bandwidth extends to 30 megacycles, this particular network may be used to view pulses of current as short as $\frac{1}{5}$ microsecond with a rise time that is 8 per cent. of the pulse width.

Practical application of this type of network is made feasible by the relative power dissipation requirements of the inductive and the capacitive branches under pulse conditions. During the pulse rise the condenser in the network charges with an efficiency of 50 per cent. through its series resistor R_2, and during the pulse decay discharges through this resistor. The energy dissipated by R_2 during the pulse rise is equal to the maximum energy that is stored in the condenser. An equal quantity

of energy is dissipated in R_2 during the pulse decay. Thus the total energy dissipated in R_2 per pulse is twice the maximum energy stored in the condenser. The average power loss in R_2 is the product of the repetition rate and twice the maximum energy stored in the condenser during the pulse. If the network (b) shown in Fig. 6A were used to view a 20-ampere 1-microsecond square pulse having a repetition rate of 1000 pulses per second, the power loss in R_2 would be $(CE^2) \times$ (Repeti-

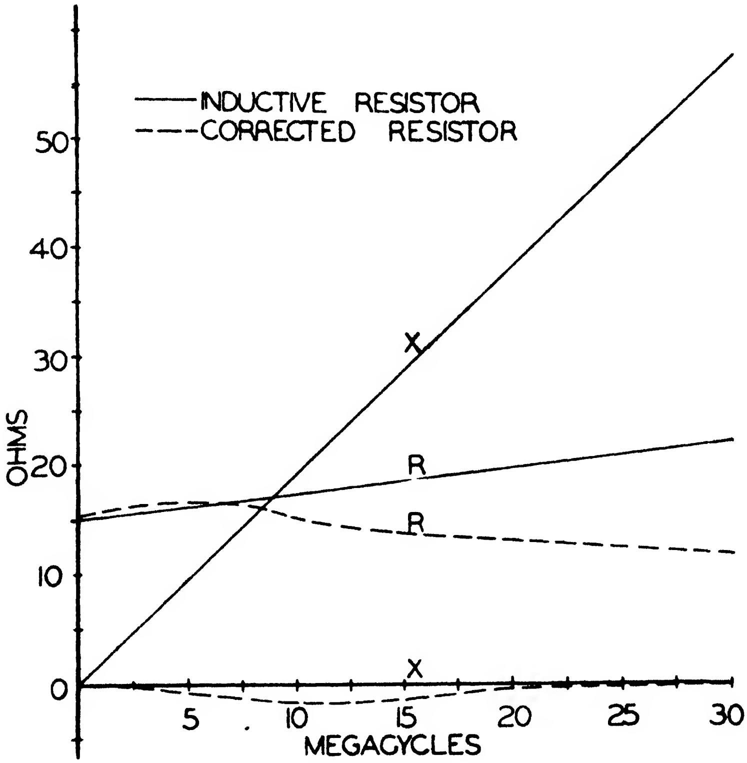

FIG. 6B. Equivalent series-circuit frequency characteristics of the inductive metering resistor and of the corrected resistor.

tion Rate) or 0.122 watt. The remainder of the power input to the network, 5.87 watts, would be dissipated in the inductive resistor, R_1. For such a low power loss R_2 may be a 1-watt composition resistor having a sufficiently small residual inductance that the $R_2 - C$ branch of the network may effectively compensate for the inductance in R_1.

TRANSMISSION OF THE PULSE FROM THE RESISTOR TO THE OSCILLOGRAPH.

From the standpoint of measurement technique the leads carrying the pulse current to the metering resistor should be short, and one terminal of the metering resistor should be as near to ground potential

as possible. Therefore lengthy connections from the metering resistor to the deflection plates of the oscillograph are usually necessary. To keep the viewing circuit as free as possible from extraneous effects, the connections should be made with coaxial cable. When measurements are to be made on short pulses, considerable care must be observed in the load termination of the coaxial cable. The efficacy of the termination of a given cable may be evaluated from measurements of its input

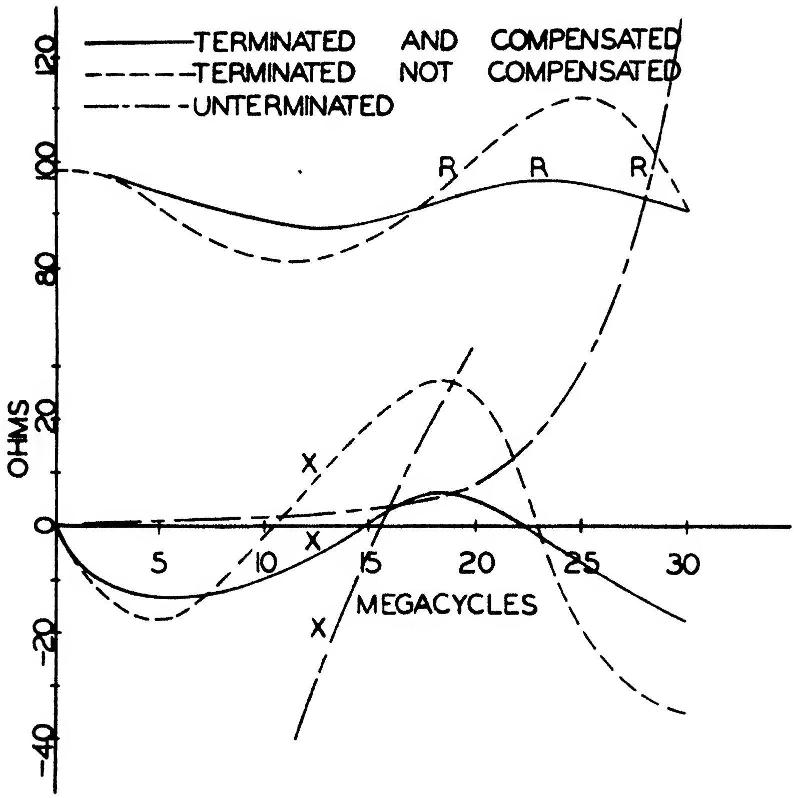

FIG. 7. Frequency characteristics of a 4-meter length of a 90-ohm coaxial cable with different terminations—(a) unterminated; (b) terminated with a single resistance; (c) terminated with the type of network shown in Figure 8.

impedance. When properly terminated, the input impedance is the characteristic impedance. When improperly terminated, the cable causes transient reflections which are superimposed upon the observed pulse and tend to obscure its shape. The amplitude of the reflections are indicated by and may be evaluated from the deviation in phase and magnitude of the input impedance from the characteristic impedance of the cable. The observed pulse when a 4-meter length of coaxial cable without termination (see Fig. 7 for the input impedance of

the cable) is used in the measurement of a trapezoidal current pulse as shown in Fig. 5C. The transient reflections make the true shape of the one microsecond pulse completely indistinguishable.

When the coaxial cable is properly terminated in its own characteristic impedance, reflections of this type are completely eliminated. In coaxial cables suited for very high frequency transmission, the attenuation at the frequencies used in pulse measurements is sufficiently low that the characteristic impedance is nearly constant in magnitude and practically resistive in nature. Therefore the termination should be purely resistive. However, the input capacitance of the oscillograph, $C_0 = 19\ \mu\mu f$ in this case, is part of the termination. For the frequencies needed to represent pulses one microsecond or greater in duration, the shunting effects of the input capacitance are sufficiently small so that a simple resistance termination may be used (see Fig. 7). For shorter pulse lengths a network which compensates for the input capacitance, C_0, of the oscillograph is necessary. A typical network electrically simi-

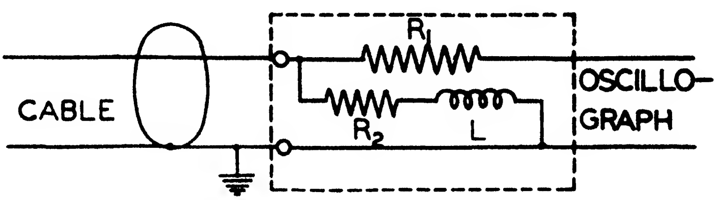

FIG. 8. Coaxial cable termination network for pulse lengths that the input capacitance, C_0, of the oscillograph is important.

lar to the type used in the current metering resistor is shown in Fig. 8. When $R_1 = R_2$ and $R_1R_2 = L/C_0$, the impedance of the network is purely resistive and of magnitude R_1. Since the power dissipated in this network is very small, low wattage components that differ little from the ideal over the pulse bandwidths may be used. The improvement in the transmission bandwidth for the pulse cable when this network is used for termination is shown by the solid line in Fig. 7. Since the bandwidth extends to 30 megacycles, the cable may be used in the measurement of pulses are short as $\frac{1}{5}$ microsecond.

SYSTEM CALIBRATION.

For measurement of the amplitude of the pulse, the deflection sensitivity of the oscillograph and the equivalent resistance of the metering network and terminated coaxial cable are required. The deflection sensitivity is measured directly at the oscillograph by observing the deflection of the beam for a known d-c. voltage. The equivalent resistance of the network and cable that is presented to the pulse is

measured at the pulse-current terminals of the network. This value measured with a Wheatstone Bridge is accurate for the bandwidth of the metering system and is equivalent to the resistance of the metering network and characteristic impedance of the terminated cable considered in parallel. When short lengths of coaxial cable are used, the attenuation of the pulse between the metering network and the oscillograph is negligible. Therefore the current sensitivity of the metering system in amperes per unit of cathode-ray deflection is the quotient of the deflection sensitivity of the oscillograph and the equivalent resistance of the metering system.

Measurement of the duration and specification of the shape of a pulse requires that the sweep of the oscillograph be calibrated against time. A circuit for calibration is shown in Fig. 9. The synchronizing pulse-generator starts the pulse generator and triggers the sweep of the oscillograph. The output of the calibration pulse generator is a series of

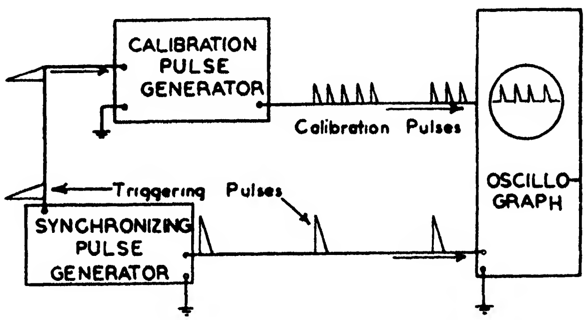

FIG. 9. Circuit for calibration of the sweep time of the oscillograph.

pulses that appear on the screen of the oscillograph and that are short in duration compared to the sweep. The time interval between the pulses shown on the screen of the oscillograph is the period of the fundamental frequency of the calibration pulse generator. If the repetition frequency of the synchronizing pulse generator is low, less than several hundred impulses per second, and if the interval of operation of the calibration pulse generator is greater than half of the interval between synchronizing impulses from the synchronizing pulse generator, the frequency of the calibration pulse generator may be measured in a conventional manner with a calibrated radio receiver. Thus the sweep of the oscillograph may be calibrated against time.

APPENDIX.

The bandwidth required for a pulse to be represented with a given accuracy may be obtained more rigorously by the use of Fourier Transforms. In this derivation the fundamental Fourier Transform rela-

tionships [1] are assumed as follows:

$$g(\omega) = \frac{1}{2\pi}\int_{\infty}^{\infty} f(t)e^{-i\omega t}dt, \tag{5}$$

$$f(t) = \int_{-\infty}^{\infty} g(\omega)e^{i\omega t}d\omega. \tag{6}$$

The time function $f(t)$ defines the disturbance that is to be analyzed. $g(\omega)$ is the corresponding frequency function that reproduces the disturbance. In this derivation $f(t)$, shown in Fig. 10A, is a repeated

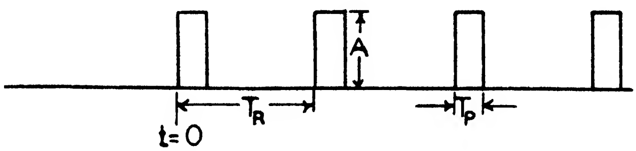

FIG. 10A. Repeated pulse wave $f(t)$ initiated at $t = 0$ and continuing thereafter.

FIG. 10B. Response function $F(t)$ showing the first two pulses of $f(t)$ for the particular case that $T_R = 20$, $T_P = 10$, and the bandwith, $\omega_1 = \pi$.

pulse wave initiated at $t = 0$. In the mathematical form which is used in the transforms, the time function is defined as follows:

$$\begin{aligned} f(t) &= 0, \quad t < 0, \quad rT_R + T_P < t < (r+1)T_R, \\ f(t) &= Ae^{-\alpha t}, \quad rT_R < t < rT_R + T_P, \quad r = 0, 1, 2, 3 \cdots. \end{aligned} \tag{7}$$

The integrand in the time integral, Eq. (5), must be convergent in order for the integral to exist. The factor $e^{-\alpha t}$ was introduced into the amplitude function of the wave to satisfy the convergence requirement of the time integral. Since the constant α may be chosen small, the effect of the $e^{-\alpha t}$ term on the first few pulses in the repeated wave is negligible. The frequency function $g(\omega)$ that reproduces this wave is obtained when the function $f(t)$ defined in Eq. (7) is substituted into the infinite time integral as follows:

$$g(\omega) = \frac{1}{2\pi}\int_0^{T_P} Ae^{-(\alpha+j\omega)t}dt + \frac{1}{2\pi}\sum_{r=1}^{\infty}\int_{rT_R}^{rT_R+T_P} Ae^{-(\alpha+j\omega)t}dt$$

$$= \frac{A}{2\pi}\left[\frac{1}{\alpha+j\omega} - \frac{e^{-(\alpha+j\omega)T_P}}{\alpha+j\omega}\right] + G(\omega), \quad (8)$$

where

$$G(\omega) = \frac{1}{2\pi}\sum_{r=1}^{\infty}\int_{rT_R}^{rT_R+T_P} Ae^{-(\alpha+j\omega)t}dt.$$

Here the frequency function is written as the sum of two parts, that which arises from integration over the first pulse and that which arises from integration over the remainder of the pulses. This form of the frequency function is retained throughout the derivation.

If this frequency function were transmitted through a perfect network, one which has either zero or linear phase shift and unity transmission for all frequencies, the time function would be reproduced but with the exception of a Gibbs effect at the extremities of each pulse. For a practical network, the pass band is finite and may be defined as $-\omega_1 < \omega < \omega_1$. In the pass band the network is presumed to have ideal characteristics—linear phase shift and unity transmission. Outside of the pass band the network has zero transmission. In practical networks used for pulse measurements the frequency characteristics are sufficiently similar to the idealized characteristics that calculations based upon the ideal are useful for obtaining the response of an actual network to a pulse.

The response $F(t)$ of the network to the frequency function $g(\omega)$ when the pass band is limited to $-\omega_1 < \omega < \omega_1$ is determined by substituting the product of $g(\omega)$ and the frequency characteristics of the network into the frequency integral, Eq. (6), with the limits of integration restricted to the pass band. Thus

$$F(t) = \int_{-\omega_1}^{\omega_1} g(\omega)e^{j\omega(t-t_0)}d\omega$$

$$= \int_{-\omega_1}^{\omega_1}\frac{A}{2\pi}\left[\frac{1}{\alpha+j\omega} - \frac{e^{-(\alpha+j\omega)T_P}}{\alpha+j\omega}\right]e^{j\omega(t-t_0)}d\omega$$

$$+ \int_{-\omega_1}^{\omega_1} G(\omega)e^{j\omega(t-t_0)}d\omega. \quad (9)$$

The term, $e^{-j\omega t_0}$, where t_0 is a constant, that appears in the integral expresses the linear phase-shift and unity-response characteristics for the frequencies in the pass band of the network. By a substitution of $t - t_0$ for t in Eq. (6), the linear phase shift term $e^{-j\omega t_0}$ is shown to have the effect of delaying the network response function $F(t)$ by a time t_0. Since the magnitude of the time delay is of no importance in the remainder of the derivation, t_0 is not evaluated. In Eq. (9), the integrand for negative frequencies is conjugate to the integrand for corresponding positive frequencies. Thus by writing the integral in Eq. (9) in the form $\int_{-\omega_1}^{\omega_1} = \int_{-\omega_1}^{0} + \int_0^{\omega_1}$ and by substituting $-\omega$ for ω in the integral for the negative frequencies, the response of the network may be written in the form

$$F(t) = \int_0^{\omega_1} \frac{A}{2\pi}\left(\frac{1}{\alpha + j\omega} - \frac{e^{-(\alpha+j\omega)T_P}}{\alpha + j\omega}\right) e^{j\omega(t-t_0)}d\omega$$
$$+ \int_0^{\omega_1} \frac{A}{2\pi}\left(\frac{1}{\alpha - j\omega} - \frac{e^{-(\alpha-j\omega)T_P}}{\alpha - j\omega}\right) e^{-j\omega(t-t_0)}d\omega$$
$$+ \int_{-\omega_1}^{\omega_1} G(\omega)e^{j\omega(t-t_0)}d\omega. \quad (10)$$

Eliminating the complex functions in the integrals gives

$$F(t) = \frac{A}{\pi}\int_0^{\omega_1} \frac{\alpha}{\alpha^2 + \omega^2}[\text{Cos}\, \omega(t - t_0)$$
$$- e^{-\alpha T_P}\,\text{Cos}\, \omega\{(t - t_0) - T_P\}]d\omega$$
$$+ \frac{A}{\pi}\int_0^{\omega_1} \frac{\omega\, \text{Sin}\, \omega(t - t_0)}{\alpha^2 + \omega^2}d\omega$$
$$- \frac{A}{\pi}\int_0^{\omega_1} \frac{\omega e^{-\alpha T_P}}{\alpha^2 + \omega^2}\,\text{Sin}\, \omega\{(t - t_0) - T_P\}d\omega$$
$$+ \int_{-\omega_1}^{\omega_1} G(\omega)e^{j\omega(t-t_0)}d\omega. \quad (11)$$

The damping term $e^{-\alpha t}$ was originally introduced to provide the needed convergence for the time integral. For small values of α, the first few pulses of the original time function $f(t)$ are not appreciably affected. As α is decreased, the number of pulses unappreciably effected by the $e^{-\alpha t}$ term becomes larger. As the Lim $\alpha \to 0$, $f(t)$ approaches a square pulse wave beginning at $t = 0$ and continuing for an infinite time. In the remainder of the analysis, the limiting condition of α is used.

As Lim $\alpha \to 0$, the following conditions apply to the terms in Eq. (11)

$$\lim_{\alpha\to 0}\int_0^{\omega_1}\frac{\alpha}{\alpha^2+\omega^2}\left[\operatorname{Cos}\omega(t-t_0)-e^{-\alpha T_P}\operatorname{Cos}\omega\{(t-t_0)-T_P\}\right]d\omega=0$$

$$\lim_{\alpha\to 0}\int_0^{\omega_1}\frac{\omega}{\alpha^2+\omega^2}\operatorname{Sin}\omega(t-t_0)d\omega=\operatorname{Si}\omega_1(t-t_0) \tag{12}$$

$$\lim_{\alpha\to 0}\int_0^{\omega_1}\frac{\omega}{\alpha^2+\omega^2}\operatorname{Sin}\omega\{(t-t_0)-T_P\}d\omega=\operatorname{Si}\omega_1\{(t-t_0)-T_P\}.$$

Thus the response of the network becomes

$$f(t) = \frac{A}{\pi}\left[\operatorname{Si}\omega_1(t-t_0) - \operatorname{Si}\omega_1\{(t-t_0)-T_P\}\right] + \lim_{\alpha\to 0}\int_{-\omega_1}^{+\omega_1}G(\omega)e^{j\omega(t-t_0)}d\omega. \tag{13}$$

The functions,[2] $1/\pi \operatorname{Si}\omega_1(t-t_0)$, $1/\pi \operatorname{Si}\omega_1\{(t-t_0)-T_P\}$, and the first two pulses represented by Eq. (13) are shown plotted in Fig. 10B. For values of $|\omega_1(t-t_0)| > 18$, the value of $1/\pi \operatorname{Si}\omega_1(t-t_0) \simeq \pm \frac{1}{2}$. In repeated pulse waves where the time of separation between pulses is such that $\omega_1(T_R - T_P) > 18$, each individual pulse may be described with negligible interference from the terms for other pulses by a term similar in form to $A/\pi[\operatorname{Si}\omega_1(t-t_0) - \operatorname{Si}\omega_1\{(t-t_0)-T_P\}]$. Under these conditions, the first term in Eq. (13) describes the first pulse in $F(t)$. The remainder of the pulses in $F(t)$ are similar in form but delayed in time from the first and are contained in the integral term of the equation. Thus the accuracy of the approximation obtained by limiting the bandwidth to $-\omega_1 < \omega < \omega_1$ may be determined from the behavior of the first term in Eq. (13). For bandwidths such that the shape of the pulse is approximately trapezoidal, the slope of the tangent to the pulse at $t - t_0 = 0$ is about average for the rising edge of the pulse, and has the equation

$$\left.\frac{d}{dt}F(t)\right|_{t-t_0=0} = \frac{A}{\pi}\frac{dt}{d}\left[\operatorname{Si}\omega_1(t-t_0)-\operatorname{Si}\omega_1\{(t-t_0)-T_P\}\right]\Big|_{t-t_0=0} = \frac{A\omega_1}{\pi}. \tag{14}$$

The time of rise, t_{rise}, of the pulse is calculated from the slope of the tangent to the pulse at $t - t_0 = 0$ as follows:

$$t_{\text{rise}} = \left.\frac{A}{\frac{d}{dt}F(t)}\right|_{t-t_0=0} = \frac{\pi}{\omega_1}. \tag{15}$$

However, the accuracy of a given approximation to a pulse should be expressed in terms of the ratio K of the rise time to the duration of the pulse.

$$K = \frac{t_{\text{rise}}}{T_P} = \frac{\pi}{\omega_1 T_P} = \frac{1}{2F_1 T_P}. \tag{16}$$

In order to obtain a convenient expression for the bandwidth needed to give a desired approximation to a pulse, F_1 is replaced nF_P, where n is the harmonic of the pulse-width frequency F_P. With this substitution, Eq. (16) reduces to

$$K = \frac{1}{n}.$$

Therefore the ratio of the time of rise to the pulse width for a given repeated pulse wave is inversely proportional to the number of harmonics of the pulse-width frequency that are included in the pass band, provided that the separation between pulses is such that $\omega_1(T_R - T_P) > 18$.

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1. GUILLEMIN, E. A., "Communication Networks," John Wiley & Sons, New York City, 1935, Volume II, page 461–487.
2. JAHNKE, E., AND EMDE, F., "Tables of Functions," G. Teubner, Leipzig and Berlin, 1933, page 83.

LIST OF SYMBOLS.

a_k Amplitude coefficient of the kth harmonic of F_R.
A Amplitude of the pulse in the repeated pulse wave.
C_0 Oscillograph input capacitance.
$f(t)$ The repeated pulse wave, defined differently in body and Appendix.
F_1 Frequency corresponding to ω_1.
F_P Pulse-width frequency. Has period equal to $2T_P$.
F_R Basic pulse repetition frequency. Has period equal to T_R.
$F(t)$ The response function of the network when the bandwidth is limited to ω_1.
$g(\omega)$ The frequency function corresponding to the $f(t)$ used in the transforms.
$G(\omega)$ A portion of $g(\omega)$ in the Appendix.
k Number of the harmonic of F_R.
K Ratio of the rise time to the pulse width.
m Series of positive integers which are involved in envelope of spectrum.
n Number of harmonics of pulse-width frequency included in pass band.
r The numbei of pulses that has occurred counted from the first.
R_1, R_2, L, C The constant in the measuring network.
t Time.
t_0 Time delay for a pulse.
T_P Duration of the pulse.
T_R Interval of time between successive pulses in the pulse wave.
ω Angular frequency.
ω_1 Limit of the pass band for the measuring networks.

Frozen Juice Like Fresh.—One of the very best of the orange juice products is a semi-concentrated juice stored at zero temperature and made ready for drinking by diluting with three parts of water to one of juice.

This is the opinion of workers at the U.S. Department of Agriculture's Winter Haven, Florida, Laboratory, who haveh ad opportunity to taste the products of many systems of processing. It was developed at the Laboratory in cooperation with the Florida Citrus Commission.

"In this method," says M. K. Veldhuis of the Bureau of Agricultural and Industrial Chemistry, "orange juice is vacuum-concentrated to about 5-fold. Then fresh juice is added to reduce the mixture to a 4-fold concentration.

"The addition of fresh juices," says Veldhuis, "replaces much of the flavor lost in the vacuum concentration process."

This 4-fold concentrate is then quick-frozen and marketed through frozen food channels. "At the usual zero storage temperature, the product is little harder than a slush and can be mixed readily with three volumes of tap water to quickly prepare a cold drink hardly distinguishable from fresh juice but with a decided gain from reducing costs for transportation and storage."

R. H. O.

X-Raying Oil Fields. (*Electrical Engineering*, Vol. 66, No. 7.)—A method of X-raying oil fields by remote operation from its research laboratories has been developed by Gulf Oil Corporation The new development helps solve the riddle of how oil flows through the sand and rocks under any particular field, and how it is affected by underground water and gas.

Since X-raying entire oil fields at their sites would be impractical, the fields were brought to the laboratory. Small 3-inch by 1-inch samples, or cores, from the drill holes are made to reproduce in miniature the layer of rock or sand from which they were taken. Reactions studied within the samples give a picture of flow conditions in a stratum perhaps hundreds of feet thick and miles in extent.

To make such studies, the core is subjected to artificial pressures and saturations of oil, gas, and water. The progress of these elements in the core is gauged by an X-ray device for determining permeability-saturation. The X-ray beams follow the reaction by means of an opaque tracer mixed with the liquid or gas.

First step of the researcher is to establish a basis for comparison by studies of the core at 100 per cent and at zero saturation. Then an extensive series of flow experiments, reproducing conditions which might be created by various recovery methods, must be made.

Effects of gravity, capillary attraction, and the amounts of oil, gas, and water already in the sand are charted. These procedures must be repeated sometimes on as many as 10 or 11 cores taken from the different layers through which the well extends.

By correlating such data, the laboratory can determine how natural pressure, or artificial gas injection, or water flooding will move oil through various strata of the field being studied toward well shafts. Operating methods can be planned accordingly to assure the greatest yield, and the extent of the yield forecast.

Permeability-saturation determination by X-rays represents an extension of the electrolytic model oil field technique developed for predicting generalized oil field behavior and guiding placement of wells.

R. H. O.

RIGOROUS AND APPROXIMATE TREATMENT OF LONG LINE TRANSMISSION PROBLEMS BY HYPERBOLIC FUNCTIONS.*

BY

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I. INTRODUCTION.

It is generally agreed that in rigorous treatment of long transmission lines hyperbolic functions must be employed. In order to facilitate the application of these functions, charts and tables have been prepared; the last chart, due to Woodruff, covers the values of $\theta^2 = ZY$ between 0.05 and 0.36, corresponding to the lengths of aerial lines between approximately 100 and 300 miles at 60 c/s, because θ/L is practically constant and equal to 0.0018–0.0022 per mile at 60 c/s.

Almost all authors state that when using the expansions in series of hyperbolic functions it is possible to obtain any precision desired by taking a sufficient number of terms. They consider however the mathematical precision only; many of them (for example, Nesbitt, Wagner and Evans, etc.) give in their books examples of computation of successive terms in order to show that these terms diminish very rapidly, so that quite soon they do not affect the first, say four, significant figures.

A reference—rather vague—to the physical problem is found in G. D. McCann's excellent chapter No. 4 in the "Westinghouse Transmission and Distribution Reference Book" (1944). McCann fixes at 0.5 per cent. the mathematical precision of the developments, because "the resistance, inductance and capacitance of a line can rarely be known within 3 or 4 per cent. and probably never within 1 per cent. This is due to conductor sag, its variation with differents spans and the variation that exists in conductor spacing, together with the effect of temperature upon conductor resistivity and sag."

In this paper we shall treat the physical aspect of the problem more comprehensively. We must consider two points:

(I) There is no physical significance in adding a new term of a development unless this term is large enough as compared to the maximum error committed in the value of the sum of preceding terms. We shall admit that it is so if a consecutive term is larger than 0.25 of the maximum error in the sum of the preceding teims Of course, a value other than 0.25 may be chosen without affecting the generality of the treatment;

* Lecture given at the Graduate School of Engineering, Harvard University, on April 10th, 1947.

(II) Even if it makes sense to add a new term in compliance with the consideration (I), its addition may not be worthwhile in view of the limited accuracy within which a given magnitude has to be known.

As a corollary of (II) we shall examine the problem under which conditions a line may be considered short, that is of negligible capacitance. The textbooks agree that an overhead line may be considered short when it is 20 or 30 miles long; some say that the effects of capacitance may be neglected whatever the length of line, if the voltage is lower than 40 kV. We shall see that the problem cannot be stated in this simple manner.

It is easy to see, applying to the term in θ^6 the same treatment that we shall use in respect to the terms in θ^2 and θ^4, that the term in θ^6 may be omitted for present usual lengths of lines. Consequently we shall limit all developments to the terms in θ^4 and below.

II. ERRORS ON THE CONSTANTS OF A LINE.

Let, per mile,

r—the resistance, in ohms,
$l\omega$—the inductive reactance, in ohms,
$c\omega$—the capacitive susceptance, in mhos.

We shall neglect the conductance g, ordinarily very small. We have

$$\bar{Z} = Z\underline{/\gamma} = \sqrt{r^2 + l^2\omega^2}L\underline{/\arctan l\omega/r},$$

$$\bar{Y} = Y\underline{/\pi/2} = c\omega L\underline{/90^\circ},$$

where L is the length of the line in miles.

The maximum errors on Z, γ and Y are

$$\frac{\Delta Z}{Z} = \frac{\Delta r}{r}\cos^2\gamma + \frac{\Delta(l\omega)}{l\omega}\sin^2\gamma + \frac{\Delta L}{L}, \tag{1}$$

$$\Delta\gamma = \left(\frac{\Delta(l\omega)}{l\omega} + \frac{\Delta r}{r}\right)\sin\gamma\cos\gamma, \tag{2}$$

$$\frac{\Delta Y}{Y} = \frac{\Delta(c\omega)}{c\omega} + \frac{\Delta L}{L}, \tag{3}$$

and are due to the variable atmospheric conditions, the variation of frequency and of the load, and the approximation of the formulas used for the calculation of the constants, or the errors in the measurements of these constants.

III. THE EQUIVALENT π CIRCUIT.

We have

$$\bar{Z}_e = \bar{Z}\frac{\sinh\bar{\theta}}{\bar{\theta}} = \bar{Z}\left(1 + \frac{\bar{\theta}^2}{6} + \frac{\bar{\theta}^4}{120} + \frac{\bar{\theta}^6}{5040} + \cdots\right), \tag{4}$$

$$\bar{Y}_e = \frac{\bar{Y}}{2}\frac{\tanh(\bar{\theta}/2)}{(\bar{\theta}/2)} = \frac{\bar{Y}}{2}\left(1 - \frac{\bar{\theta}^2}{12} + \frac{\bar{\theta}^4}{120} - \frac{17\bar{\theta}^6}{20140} + \cdots\right). \tag{5}$$

A. The Development of $\bar{Z}_e$.

We have, limiting the development to the term in $\bar{\theta}^4$,

$$\bar{Z}_e = \bar{Z}\left(1 + \frac{\bar{\theta}^2}{6} + \frac{\bar{\theta}^4}{120}\right) \tag{6}$$

and the projections of $\bar{Z}_e$, M on $\bar{Z}$ and N on the normal to $\bar{Z}$, are (Fig. 1):

$$M = Z\left(1 - \frac{\theta^2}{6}\sin\gamma - \frac{\theta^4}{120}\cos 2\gamma\right), \tag{7}$$

$$N = Z\left(\frac{\theta^2}{6}\cos\gamma - \frac{\theta^4}{120}\sin 2\gamma\right). \tag{8}$$

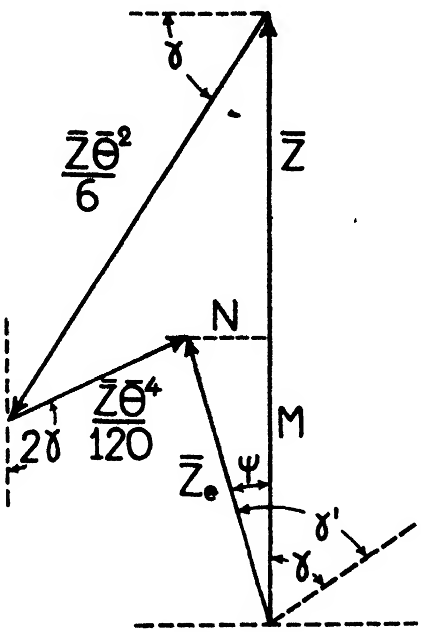

FIG. 1.

Hence

$$\tan\psi = N/M \cong \frac{v}{6}\cos\gamma + \frac{v}{180}\sin 2\gamma \cong \psi.$$

The modulus and argument of $\bar{Z}_e$ are

$$Z_e = \frac{M}{\cos\psi} \cong M(1 + \tfrac{1}{2}\tan^2\psi)$$
$$\cong Z\left[1 - \frac{\theta^2}{6}\sin\gamma - \frac{\theta^4}{120}(\cos 2\gamma - \tfrac{5}{3}\cos^2\gamma)\right], \tag{9}$$

$$\gamma' = \gamma + \psi \cong \gamma + \frac{\theta^2}{6}\cos\gamma + \frac{\theta^4}{180}\sin 2\gamma. \tag{10}$$

If the use of the second term, $(Z\theta^2 \sin \gamma)/6$, in (9) has to havé any physical meaning, we must have, according to the consideration (I):

$$\tfrac{1}{4}\Delta Z \leq (Z\theta^2 \sin \gamma)/6; \quad \text{hence} \quad \theta^2 \geq \frac{3}{2 \sin \gamma} \frac{\Delta Z}{Z}. \tag{11}$$

In examining the use of the third term, we should compare it with the maximum error in $Z - (Z\theta^2 \sin \gamma)/6$, but in an analysis of errors it will be sufficiently correct to compare it with the maximum error in Z only, $(Z\theta^2 \sin \gamma)/6$ being relatively small with respect to Z. Consequently, the use of the term in θ^4 in (9) has no physical meaning unless

$$\theta^4 \geq \frac{30}{|\cos 2\gamma - \frac{5}{8} \cos^2 \gamma|} \frac{\Delta Z}{Z}. \tag{12}$$

Let us consider now the argument γ'. The use of the second term in (10) has a meaning only if

$$\theta^2 \geq \frac{3}{2 \cos \gamma} \Delta\gamma \tag{13}$$

and the use of the term in θ^4 in (10) has a meaning if

$$\theta^4 \geq \frac{45}{\sin 2\gamma} \Delta\gamma. \tag{14}$$

To see the actual significance of the relations (11)–(14), and of the similar ones which we are going to establish for other magnitudes, we have to assume numerical values for errors. We shall admit

$$\frac{\Delta(c\omega)}{c\omega} = \frac{\Delta(l\omega)}{l\omega} = \frac{\Delta L}{L} = 0.01$$

and $\Delta r/r = 0.06$, what corresponds to a 15° C. variation in temperature. The relations (11 and 13) become.

$$\theta^2 \geq \frac{3}{2} \frac{0.06 \cos^2 \gamma + 0.01 \sin^2 \gamma + 0.01}{\sin \gamma}, \tag{11a}$$

$$\theta^2 \geq 0.105 \sin \gamma. \tag{13a}$$

The relations (11a) and (13a) show that the lengths L' (modulus) and L'' (argument) for which the terms in θ^2 have to be considered in (9) and (10), respectivély, depend on γ. Making $\theta = 2.10^{-3}L$, we find

modulus (11a): $L' =$ 85 miles for $\gamma \cong 90°$, and 155 miles for $\gamma = 45°$;
argument (13a): $L'' =$ 160 miles for $\gamma \cong 90°$, and 135 miles for $\gamma = 45°$.

Similarly, the relation (12) gives

$L' =$ 450 miles for $\gamma \cong 90°$, and 600 miles for $\gamma = 45°$.

The relation (14) gives $L'' =$ 550 miles independently of γ.

Consequently for present lines the term in θ^4 should not be taken into account. It not only means that it is sufficient to make $\bar{Z}_e = \bar{Z}\left(1 + \frac{\bar{\theta}^2}{6}\right)$, but also that, when calculating Z_e, we do not have to correct for $1/\cos\psi$, and ought to write $Z_e = M$.

B. The Development of $\bar{Y}_e$.

We find, in similar manner as for $\bar{Z}_e$ (Fig. 2):

$$Y_e = \frac{Y}{2}\left[1 + \frac{\theta^2}{12}\sin\gamma - \frac{\theta^4}{120}(\cos 2\gamma - \tfrac{5}{12}\cos^2\gamma)\right], \tag{15}$$

$$\tan\psi = \frac{\theta^2\cos\gamma}{12}\left(1 + \frac{7\theta^2}{60}\sin\gamma\right) \cong \frac{rLY}{12} \cong \psi. \tag{16}$$

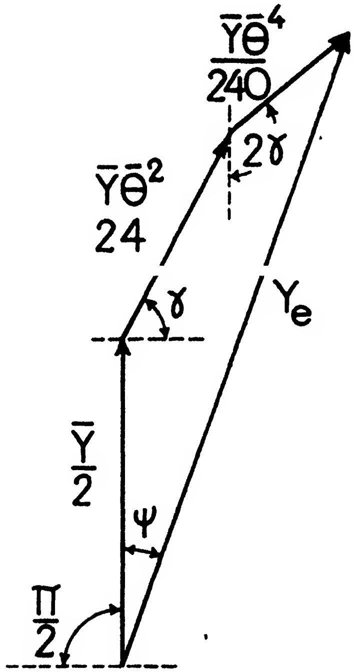

FIG. 2.

The second term in (15) will be used if

$$\theta^2 \geq \frac{3}{\sin\gamma}\frac{\Delta Y}{Y} \tag{17}$$

and the third one if

$$\theta^4 \geq \frac{30}{|\cos 2\gamma - \frac{5}{12}\cos^2\gamma|}\frac{\Delta Y}{Y}. \tag{18}$$

Let us consider now the argument of $\bar{Y}_e$, $(90° - \psi)$. In view that we assumed $g = 0$, the term in θ^2 in (16) must always be considered, as far as only the consideration (I) is concerned.

The second term in (16) has to be considered only if

$$\theta^2 \geq \frac{15}{7 \sin \gamma}\left(\frac{\Delta r}{r} + \frac{\Delta L}{L} + \frac{\Delta Y}{Y}\right) \tag{19}$$

c.

Until now we have considered $\bar{Z}_e$ and $\bar{Y}_e$ alone, but generally it is necessary to consider them in combination with themselves and with other magnitudes. The conclusions obtained before may not be valid, because of possibility of compensation and of the presence of new terms.

We are interested in obtaining correct values of the terms in θ^2 and θ^4 in the developments corresponding to various magnitudes. Note

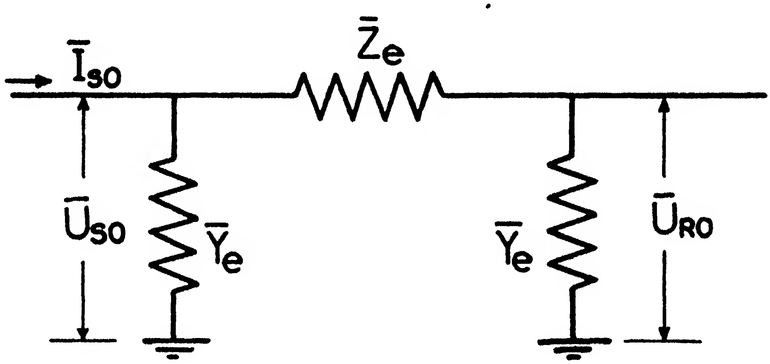

FIG. 3.

that for expression of a form $\bar{Z}_e^a \bar{Y}_e^b$, where a and b are positive integers or zero, it will be sufficient to make $\bar{Z}_e = \bar{Z}\left(1 + \frac{\bar{\theta}^2}{6}\right)$ and $\bar{Y}_e = \frac{\bar{Y}}{2}\left(1 - \frac{\bar{\theta}^2}{12}\right)$; the coefficient of the term in $\bar{\theta}^2$ will be always exact, and the one of the term in θ^4 will be exact if $a > 0$ and $b > 0$; if $a = 0$ or $b = 0$, this coefficient will be too small in $(a + b)/120$.

If we make $\bar{Z}_e = \bar{Z}$ and $\bar{Y}_e = \bar{Y}$, the coefficient of $\bar{\theta}^2$ will be exact only if $a > 0$ and $b > 0$.

IV. LINE AT NO LOAD RELATION OF THE VOLTAGES.

We have (Fig. 3):

$$\frac{\bar{U}_{SO}}{\bar{U}_{RO}} = 1 + \bar{Z}_e\bar{Y}_e = \bar{A} = A/\underline{\delta}, \tag{20}$$

where $\bar{A}$ is one of the constants $\bar{A}$, $\bar{B}$, $\bar{C}$ and $\bar{D} = \bar{A}$ of the equivalent four terminal network.

Let us make in (20) $\bar{Z}_e = \bar{Z}\left(1 + \frac{\bar{\theta}^2}{6}\right)$ and $\bar{Y}_e = \frac{\bar{Y}}{2}\left(1 - \frac{\bar{\theta}^2}{12}\right)$, we obtain

$$\bar{A} = 1 + \frac{\bar{\theta}^2}{2} + \frac{\bar{\theta}^4}{24}, \tag{21}$$

where the coefficient of $\bar{\theta}^4$ is exact, according to what we said above.

The modulus is

$$A = \frac{U_{SO}}{U_{RO}} = 1 - \frac{\theta^2}{2}\sin\gamma - \frac{\theta^4}{24}(\cos 2\gamma - 3\cos^2\gamma) \tag{22}$$

and there is a meaning in using the term in θ^4 if this term is larger than 0.25 of the error in the term in θ^2, that is, if

$$\theta^2 \geq \frac{3\sin\gamma}{1 + \cos^2\gamma}\left[\frac{\Delta(l\omega)}{l\omega} + \frac{\Delta L}{L} + \frac{\Delta Y}{Y}\right]. \tag{23}$$

If $\frac{\Delta(l\omega)}{l\omega} + \frac{\Delta L}{L} + \frac{\Delta Y}{Y} = 0.04$, for $\gamma \cong 90°$ the length must be greater than 175 miles, and for $\gamma = 45°$ greater than 120 miles.

The value of A is obtained with great accuracy by means of (22); for example, if $L = 120$ miles, we may neglect the term in θ^4 (which means here to make $\bar{Z}_e = \bar{Z}$, $\bar{Y}_e = \bar{Y}/2$, and $\cos\delta = 1$) and the maximum error on A will be between ϵ and $1.25\,\epsilon$, with

$$\epsilon = \frac{\theta^2 \sin\gamma}{2}\left[\frac{\Delta(l\omega)}{l\omega} + \frac{\Delta L}{L} + \frac{\Delta Y}{Y}\right]. \tag{24}$$

For $L = 120$ miles, and $\frac{\Delta(l\omega)}{l\omega} + \frac{\Delta L}{L} + \frac{\Delta Y}{Y} = 0.04$, $\epsilon \cong 0.001$. But normally it is sufficient that A have a much less accuracy. Indeed, if we determine A by measuring U_{SO} and U_{RO} on the respective switchboard voltmeters, it is impossible to expect to obtain A within a maximum error less than 2 per cent. In such a case it is useless to take into account the term $(\theta^2 \sin\gamma)/2$ if it is smaller than 0.02, that is, if $\sin\gamma \cong 1$, for lines of less than 100 miles. Then, under the conditions just exposed a line up to 100 miles has to be considered as "short."

V. LINE AT NO LOAD. CURRENT AND POWER AT THE SENDING END.

A. Current at the Sending End

We have (Fig. 3)

$$\frac{\bar{I}_{SO}}{\bar{U}_{SO}} = \bar{Y}_e\frac{2 + \bar{Z}_e\bar{Y}_e}{1 + \bar{Z}_e\bar{Y}_e} = \frac{\bar{C}}{\bar{A}}. \tag{25}$$

Let us make $\bar{Z}_e = \bar{Z}\left(1 + \frac{\bar{\theta}^2}{6}\right)$ and $\bar{Y}_e = \frac{\bar{Y}}{2}\left(1 - \frac{\bar{\theta}^2}{12}\right)$; we find

$$\frac{\bar{I}_{SO}}{\bar{U}_{SO}} = \bar{Y}\left(1 - \frac{\bar{\theta}^2}{3} + \frac{\bar{\theta}^4}{8}\right) \tag{26}$$

with an error in the coefficient of $\bar{\theta}^4$ equal to 1/120; the exact value of this coefficient is 2/15.

The modulus of $\bar{I}_{SO}/\bar{U}_{SO}$ is

$$\frac{I_{SO}}{U_{SO}} = Y\left[1 + \frac{\theta^2}{3}\sin\gamma - \tfrac{2}{15}\theta^4(\cos 2\gamma - \tfrac{5}{12}\cos^2\gamma)\right] \tag{27}$$

and there only is a meaning in using the term in θ^4 if

$$\theta^4 \geq \frac{15}{8}\,\frac{1}{|\cos 2\gamma - \frac{5}{12}\cos^2\gamma|}\,\frac{\Delta Y}{Y}, \tag{28}$$

that is, if $\Delta Y/Y = 0.02$, 220 miles for $\gamma \cong 90°$ and 320 miles for $\gamma = 45°$. For any length shorter than defined by equation (28) we may write

$$\frac{I_{SO}}{U_{SO}} = Y\left(1 + \frac{\theta^2}{3}\sin\gamma\right) \tag{29}$$

with a maximum error between $\Delta Y/Y$ and $1.25\,\Delta Y/Y$.

To neglect the term in θ^4 is not equivalent here to make $\bar{Z}_e = \bar{Z}$ and $\bar{Y}_e = \bar{Y}/2$. If we limit the developments of $\bar{Z}_e$ and $\bar{Y}_e$ to their first terms, we find

$$\frac{I_{SO}}{U_{SO}} = Y\left(1 + \frac{\theta^2}{4}\sin\gamma\right) \tag{30}$$

with a supplementary negative error $(\theta^2 \sin\gamma)/12$.

If it is sufficient to obtain I_{SO}/U_{SO} with an even less accuracy, we may write, neglecting $\bar{Z}_e\bar{Y}_e$ compared to 1,

$$\frac{I_{SO}}{U_{SO}} = 2Y_e = Y\left(1 + \frac{\theta^2}{12}\sin\gamma\right) \tag{31}$$

with a supplementary negative error $(\theta^2 \sin\gamma)/4$, or

$$\frac{I_{SO}}{U_{SO}} = Y \tag{32}$$

with a supplementary negative error of $(\theta^2 \sin\gamma)/3$. We have now to investigate when a line may be considered as "short" from the point of view of the no-load current. In order to give a physical significance to this problem we must refer I_{SO} to the load current I_n. Assume that the line is transmitting its "natural" power, magnitude introduced by Rüdenberg to characterize a load for which the line operates at nearly

its maximum efficiency. Under these conditions

$$\sqrt{Z \sin \gamma} \tag{33}$$

and hence

$$I_n = U_R \cos \varphi_R / \sqrt{\frac{Z \sin \gamma}{Y}}, \tag{34}$$

where $\cos \varphi_R$ is the power factor of the receiver.

From (32) and (34)

$$\frac{I_{SO}}{I_n} = \frac{U_{SO}}{U_R} \frac{\sqrt{\sin \gamma}}{\cos \varphi_R} \theta \cong \frac{U_{SO}}{U_R} \frac{\sqrt{\sin \gamma}}{\cos \varphi_R} 2.10^{-3} L \tag{35}$$

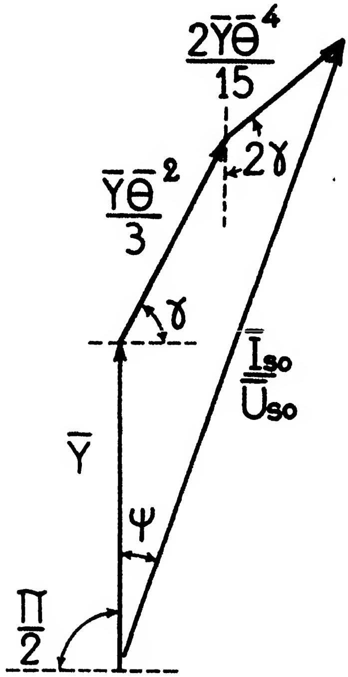

FIG. 4.

with L in miles. $U_{SO}\sqrt{\sin \gamma}/U_R \cos \varphi_R$ is nearly equal to 1; consequently we find that to call a line of a length less than 20 miles "short" means to neglect a no-load current equal approximately to 5 per cent of the full-load current.

B. Power at the sending end.

We have, calling ψ the angle between $\bar{I}_{SO}/\bar{U}_{SO}$ and $\bar{Y}$ (Fig. 4):

$$\frac{P_{SO}}{U_{SO}^2} = \frac{I_{SO}}{U_{SO}} \sin \psi = Y\left(\frac{\theta^2}{3} \cos \gamma + \frac{2\theta^4}{15} \sin 2\gamma\right) \tag{36}$$

and the use of the term in θ^4 has a meaning if

$$\theta^2 \geq \frac{5}{16 \sin \gamma}\left(\frac{\Delta r}{r} + \frac{\Delta L}{L} + \frac{2\Delta Y}{Y}\right), \tag{37}$$

that is, if $\Delta r/r + \Delta L/L + 2\Delta Y/Y = 0.11$ and $\sin \gamma \cong 1$, for lengths greater than 90 miles.

VI. LINE IN SHORT-CIRCUIT. CURRENT AT THE SENDING END.

We have

$$\frac{\bar{I}_{SS}}{\bar{U}_{SS}} = \frac{1 + \bar{Z}_e\bar{Y}_e}{\bar{Z}_e} = \frac{\bar{A}}{\bar{B}}. \tag{38}$$

Let us make $\bar{Z}_e = \bar{Z}\left(1 + \frac{\bar{\theta}^2}{6}\right)$ and $\bar{Y}_e = \frac{\bar{Y}}{2}\left(1 - \frac{\bar{\theta}^2}{12}\right)$; we obtain

$$\frac{\bar{I}_{SS}}{\bar{U}_{SS}} = \frac{1}{\bar{Z}}\left(1 + \frac{\bar{\theta}^2}{3} - \frac{\bar{\theta}^4}{72}\right) \tag{39}$$

with an error of $\bar{\theta}^4/120$; the exact value of the coefficient of $\bar{\theta}^4$ is $-1/45$. The modulus is

$$\frac{I_{SS}}{U_{SS}} = \frac{1}{Z}\left[1 - \frac{\theta^2}{3}\sin\gamma + \frac{\theta^4}{45}(\cos 2\gamma + \tfrac{5}{2}\cos^2\gamma)\right] \tag{40}$$

and the use of the term in θ^4 only has a meaning if

$$\theta^4 \geq \frac{45}{4}\,\frac{1}{1 + \frac{5}{2}\cos^2\gamma}\,\frac{\Delta Z}{Z}, \tag{41}$$

that is, if $\Delta r/r = 0.06$, $\Delta(l\omega)/l\omega = 0.01$ and $\Delta L/L = 0.01$, if the length is greater than 350 miles for $\gamma \cong 90°$ and 400 miles for $\gamma \cong 45°$. We may then write, with an accuracy between $\Delta Z/Z$ and $1.25\,\Delta Z/Z$,

$$\frac{I_{SS}}{U_{SS}} = \frac{1}{Z}\left(1 - \frac{\theta^2}{3}\sin\gamma\right). \tag{42}$$

If we may tolerate a larger error, we can make $\bar{Z}_e = \bar{Z}$ and $\bar{Y}_e = \bar{Y}/2$, obtaining

$$\frac{I_{SS}}{U_{SS}} = \frac{1}{Z}\left(1 - \frac{\theta^2}{2}\sin\gamma\right), \tag{43}$$

the supplementary error being negative and equal to $(\theta^2 \sin\gamma)/6$.

We may also simplify (38) neglecting $\bar{Z}_e\bar{Y}_e$ compared to 1; we find

$$\frac{I_{SS}}{U_{SS}} = \frac{1}{Z_e} = \frac{1}{Z}\left(1 + \frac{\theta^2}{6}\sin\gamma\right) \tag{44}$$

with a supplementary positive error $(\theta^2 \sin\gamma)/2$.

Finally, let us observe that the simplest expression for I_{SS}/U_{SS},

$$\frac{I_{SS}}{U_{SS}} = \frac{1}{Z}, \tag{45}$$

is equivalent to the consideration of the line as "short." It leads to a supplementary positive error of $(\theta^2 \sin\gamma)/3$. If we want to find I_{SS}/U_{SS} within a total accuracy of 10 per cent., with $\Delta r/r = 0.06$, $\Delta(l\omega)/l\omega = 0.01$, $\Delta L/L = 0.01$ and $\sin\gamma \cong 1$, the length should be less than about 240 miles. For 300 miles, using (45), the total error would be = 14 per cent. if $\gamma \cong 90°$, 13 per cent. of $\gamma = 45°$.

VII. LINE AT LOAD.

A. Relation of the Voltages.

We have

$$\frac{\bar{U}_S}{\bar{U}_R} = 1 + \bar{Z}_e\bar{Y}_e + \bar{Z}_e\bar{Y}_c = \bar{A} + \bar{B}\bar{Y}_c, \tag{46}$$

where $\bar{Y}_c$ is the admittance of the load.

Let us make $\bar{Z}_e = \bar{Z}\left(1 + \frac{\bar{\theta}^2}{6}\right)$ and $\bar{Y}_e = \frac{\bar{Y}}{2}\left(1 - \frac{\bar{\theta}^2}{12}\right)$; we find

$$\frac{\bar{U}_S}{\bar{U}_R} = 1 + \bar{\theta}^2\left(\frac{1}{2} + \frac{\bar{Y}_c}{\bar{Y}}\right) + \frac{\bar{\theta}^4}{6}\left(\frac{1}{4} + \frac{\bar{Y}_c}{\bar{Y}}\right). \tag{47}$$

If the power transmitted is the natural power of the line, we have

$$Y_c = \sqrt{\frac{Y}{Z\sin\gamma}}\cos\varphi_R, \qquad \text{and} \qquad \frac{Y_c}{Y} = \frac{\cos\varphi_R}{\sqrt{\sin\gamma}}\frac{1}{\theta}. \tag{48}$$

For $\cos\varphi_R \cong 1 \cong \sin\gamma$, and a length of 300 miles, $Y_c/Y = 1.6$, so that we may admit, only for the purpose of discussion of errors,

$$\frac{1}{2} + \frac{\bar{Y}_c}{\bar{Y}} \cong \frac{1}{4} + \frac{\bar{Y}_c}{\bar{Y}} \cong \frac{\bar{Y}_c}{\bar{Y}} = \frac{Y_c}{Y}\underline{/-\varphi_R - \pi/2}.$$

For the same purpose we may neglect the correction of the cosine of the angle between 1 and $\bar{U}_S/\bar{U}_R$, and find the following approximate expression for the modulus of $\bar{U}_S/\bar{U}_R$:

$$\frac{U_S}{U_R} = 1 + ZY_c\cos(\gamma - \varphi_R) - \frac{\theta^2 ZY_c}{6}\sin(2\gamma - \varphi_R), \tag{49}$$

and the use of the third term in (49) only has a meaning if

$$\theta^2 \geq \frac{3}{2\sin(2\gamma - \varphi_R)}\left[\frac{\Delta L}{L}\cos(\gamma - \varphi_R) + \frac{\Delta r}{r}\cos\gamma\cos\varphi_R + \frac{\Delta(l\omega)}{l\omega}\sin\gamma\sin\varphi_R\right], \tag{50}$$

for example, if $\gamma \cong 70°$, $\varphi_R \cong 25°$, $\Delta r/r = 0.06$, $\Delta(l\omega)/l\omega = 0.01 = \Delta L/L$, for $L \geq 110$ miles. For lengths less than the one defined by (50), we may make $\bar{Z}_e = \bar{Z}$ and $\bar{Y}_e = \bar{Y}/2$ in (46), which is equivalent to neglecting the third term in (47). For greater lengths all depends on the accuracy within which we want to know the relation U_S/U_R. For a line of 300 miles, to neglect the term in θ^4 in (49) is equivalent to a positive error equal to 0.02 Y_c/Y, that is, about 3 per cent. if the natural power is transmitted. It must be observed that this error is proportional to the fourth power of the length, L^4.

B. Relation of the Currents.

We have

$$\frac{\bar{I}_S}{\bar{I}_R} = 1 + \bar{Z}_e\bar{Y}_e + \bar{Z}_c\bar{Y}_e(2 + \bar{Z}_e\bar{Y}_e) = \bar{A} + \bar{C}\bar{Z}_c, \tag{51}$$

where $\bar{Z}_c$ is the impedance of the load. Let us make $\bar{Z}_e = \bar{Z}\left(1 + \frac{\bar{\theta}^2}{6}\right)$ and $\bar{Y}_e = \frac{\bar{Y}}{2}\left(1 - \frac{\bar{\theta}^2}{12}\right)$; we find

$$\frac{\bar{I}_S}{\bar{I}_R} = 1 + \bar{\theta}^2\left(\frac{1}{2} + \frac{\bar{Z}_c}{\bar{Z}}\right) + \frac{\bar{\theta}^4}{6}\left(\frac{1}{4} + \frac{\bar{Z}_c}{\bar{Z}}\right). \tag{52}$$

If the line transmits the natural power

$$\frac{Z_c}{Z} = \frac{\sqrt{\sin\gamma}}{\cos\varphi_R}\frac{1}{\theta}, \tag{53}$$

that is, for a line of 300 miles, $\sin\gamma \cong 1$ and $\cos\varphi_R \cong 1$, $\bar{Z}_c/\bar{Z} = 1, 6\underline{/\varphi_R - \gamma}$, so that we may still make for the purpose of the discussion of errors

$$\frac{1}{2} + \frac{\bar{Z}_c}{\bar{Z}} \cong \frac{1}{4} + \frac{\bar{Z}_c}{\bar{Z}} \cong \frac{\bar{Z}_c}{\bar{Z}}.$$

Furthermore we may neglect for the same purpose the correction of the cosine of the angle between 1 and $\bar{I}_S/\bar{I}_R$, and find the following approximate expression for the modulus of $\bar{I}_S/\bar{I}_R$:

$$\frac{I_S}{I_R} = 1 - Z_cY\sin\varphi_R - \frac{Z_cY\theta^2}{6}\cos(\gamma + \varphi_R). \tag{54}$$

Here the third term has a meaning if

$$\theta^2 \geq \frac{3}{2}\frac{\Delta Y}{Y}\frac{\sin\varphi_R}{|\cos(\gamma + \varphi_R)|}, \tag{55}$$

that is, if $\Delta Y/Y = 0.02$, $\gamma \cong 70°$ and $\varphi_R \cong 25°$, for lengths greater than 200 miles. Furthermore it may often be neglected because we only need to know I_S/I_R within a limited accuracy. For a line of 300 miles,

and for $\cos(\gamma + \varphi_R)$ as high as 0.5 the error in neglecting the third term, thus making $\bar{Z}_e = \bar{Z}$ and $\bar{Y}_e = \bar{Y}/2$ in (51), is positive and lower than 2 per cent.

VIII. LINE AT LOAD. POWER CIRCLES AT SENDING AND RECEIVING ENDS.

We have

$$\frac{\bar{P}_S}{U_S^2} = \frac{1 + \bar{Z}_e\bar{Y}_e}{\bar{Z}_e} - \frac{\bar{U}_R}{\bar{U}_S}\frac{1}{\bar{Z}_e} = \frac{\bar{A}}{\bar{B}} - \frac{1}{\bar{B}}\frac{\bar{U}_R}{\bar{U}_S} \tag{56}$$

and

$$\frac{\bar{P}_R}{U_R^2} = \frac{\bar{U}_S}{\bar{U}_R}\frac{1}{\bar{Z}_e} - \frac{1 + \bar{Z}_e\bar{Y}_e}{\bar{Z}_e} = \frac{\bar{U}_S}{\bar{U}_R}\frac{1}{\bar{B}} - \frac{\bar{A}}{\bar{B}}. \tag{57}$$

The modulus of the relation $\bar{A}/\bar{B}$ has been studied in VI. We found that in practically all cases it is sufficient to take

$$\frac{A}{B} = \frac{1}{Z}\left(1 - \frac{\theta^2}{3}\sin\gamma\right) = \frac{1}{Z}\left(1 - \frac{l\omega LY}{3}\right). \tag{58}$$

The argument of $\bar{A}/\bar{B}$ is

$$\gamma'' = -\gamma + \frac{\theta^2}{3}\cos\gamma(1 + \tfrac{7}{15}\theta^2\sin\gamma), \tag{59}$$

where the use of the term in θ^4 only has a meaning if

$$\theta^4 \geq \frac{45}{28}\left[\frac{\Delta(l\omega)}{l\omega} + \frac{\Delta r}{r}\right], \tag{60}$$

that is, for $\Delta(l\omega)/l\omega + \Delta r/r = 0.07$, for $L > 280$ miles.

We may then write for all usual cases

$$\gamma'' = -\gamma + \frac{\theta^2}{3}\cos\gamma = -\gamma + \frac{rLY}{3}. \tag{61}$$

For the radii, $\frac{U_S}{U_R}\frac{1}{Z_e}$ and $\frac{U_R}{U_S}\frac{1}{Z_e}$, according to the study done in III, is practically always sufficient to write

$$Z_e = Z\left(1 - \frac{l\omega LY}{6}\right). \tag{62}$$

The zero position of the radii, for which $\bar{U}_S$ and $\bar{U}_R$ are in phase, is defined by

$$-\gamma' = -\gamma - \frac{\theta^2}{6}\cos\gamma = -\gamma - \frac{rLY}{6} \tag{63}$$

The relations (58), (61), (62) and (63) determine the centers and radii of the circles of power with all the accuracy which has physical meaning for lines up to 300 miles, and without any necessity of using the hyperbolic functions.

CONCLUSIONS

We have shown that because of the errors in the constants of the line, generally there is no physical meaning in using more than two terms in the developments of $\bar{Z}_e$ and $\bar{Y}_e$. Furthermore, because it is of no use to try to determine the magnitudes beyond a certain accuracy, in many cases it is perfectly correct to limit the expansions of $\bar{Z}_e$ and $\bar{Y}_e$ to the first terms, that is $\bar{Z}$ and $\bar{Y}/2$, respectively.

The definition of a line as a "short" (of negligible capacitance) if the length is less than a given value, is not rational. It is necessary to specify the criterion employed. We have seen that a line is "short" if

$L \leq$ 20 miles by the condition of neglecting $\bar{I}_{SO} \lesssim 0.05\ I_n$, (V)

$L \leq$ 100 miles by the condition $U_{SO}/U_{RO} \geq 0.98$, (IV)

$L \leq$ 240 miles by the condition I_{SS}/U_{SS} exact within 10 per cent., (VI)

for specified values of γ, $\Delta r/r$, $\Delta(l\omega)/l\omega$, $\Delta L/L$.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

DETECTION OF ISOTOPES BY TRACER MICROGRAPHY.

A new method for the more effective tracing of radioactive isotopes in materials in which they have been intentionally introduced has been developed by L. Marton of the National Bureau of Standards with the cooperation of P. H. Abelson of the Department of Terrestrial Magnetism, Carnegie Institution of Washington. In this procedure, by means of a magnetic focusing arrangement, the radiation given off by a radio-isotope within a sample material is made to form an image of the emitting surface upon a photographic plate. This image may then be used in studying the distribution and concentration of the radioactive element present in the sample.

In many chemical, biological, biochemical, and other fields of research, there is growing application of the method of tracers, in which the isotope of a given element is used as an indicator to tag or label certain groups of atoms so that they may be distinguished from other atoms of the same kind. Identification of tracer elements is at present greatly facilitated through the use of radioactive isotopes, which, because of recent developments in atomic energy, are now available in large quantities and are relatively easy to detect through their radiations.

In the well-known method of radio autography a radio-isotope is introduced in a biological or other system, and the distribution of that particular element within the system is determined by bringing the sample in close contact with a photographic emulsion. This method lacks resolving power because, even in case of perfect contact of the sample with the emulsion, the circle of confusion from every point of emission is so great that details less than a tenth of a millimeter are very difficult or impossible to distinguish.

In order to improve the resolution of this tracer method, it was decided to use electron optical image formation for determination of the distribution of a radioactive element within a given sample. This process, which may be called "tracer micrography," is based on the emission of high speed electrons (beta rays) by many tracer elements and the use of magnetic lens elements to form an image on a suitable recording surface.

In the absence of any means for correction of the chromatic aberration of electron optical lenses, the first micrographs were limited to those elements which emit electrons of uniform speed. After some at-

* Communicated by the Director.

tempts with Columbium^{93}, Yttrium^{87}, Strontium^{85}, Strontium^{87}, and Protactinium^{233}, Gallium^{67} was selected for the initial tests. Gallium chloride, prepared by chemical separation from zinc, was bombarded by heavy hydrogen nuclei in the cyclotron at the Carnegie Institution, and the solution was evaporated drop after drop on a ¼-inch tantalum disc. Radiation emitted from the surface of the disc, upon passing through a magnetic lens consisting of a small ironclad coil with Armco iron pole pieces, was brought to a focus upon a photographic film at a distance of about 3½ inches. An image of the tantalum disc was thus obtained showing radioactive areas. The conditions were selected so that a linear magnification of 2 was obtained.

For calibration of the instrument, the photographic film was replaced by a Geiger counter, and the lens current necessary to produce a maximum number of counts in unit time was determined for radiations of varying velocities. This establishes the focusing current for a given type of radiation.

In preliminary experiments with samples of different concentration and thickness of the radioactive layer, exposure times ranged from 2 to 12 hours according to the age and concentration of the sample and the numerical aperture of the lens. It was found that micrographs with good definition were obtained consistently when the layer was sufficiently thin to avoid considerable self-absorption The best resolving power attained so far has been about 30 microns

The simplicity of this method, both in apparatus and technic, is one of its more important features Vacuum requirements are very moderate, since the mean free path of the electrons is large in comparison with the apparatus dimensions, even at forepump pressure.

Further improvements in tracer micrography are expected through after-acceleration of the beta particles by means of an homogeneous electrostatic field. Such after-acceleration may well result in reduced exposure time and in better resolution due to a reduction in spherical aberration. A further reason for after-acceleration is that chromatic aberration, which is always present, even in sources emitting particles of uniform speed, can be markedly decreased if the accelerating potential is at least comparable in magnitude to the energy of the primary emission.

Location of radioactive tracer elements is facilitated by a new electron optical method, "tracer micrography."

TEMPERATURE DISTRIBUTION IN A TEST BUNGALOW.

Uniformity of temperature throughout houses is a tacitly accepted American ideal of heating. It is probably never attained in practice, and departures from it depend upon the design and construction of the house and upon the characteristics of the heating system or device used.

Distribution of warmth in houses has usually been judged only qualitatively by individual engineers on the basis of personal experience or observation. Quantitative data have been limited almost entirely to laboratory tests on parts of the system. In order to obtain data on the temperature distribution in an entire house, a series of heat distribution tests of several types of heating systems were conducted at the National Bureau of Standards by Richard S. Dill and Paul R. Achenbach.

A full scale house in which complete heating systems can be installed was constructed at the Bureau. Designated as a test bungalow, it is similar in plan to house B described in F.H.A.'s Technical Bulletin No. 4, "Principles of Planning Small House." It has four rooms and bath, with a central hallway. The walls are conventional in construction consisting of ½-inch gypsum board on the inside and 2- by 4-inch studding with sheathing and lap siding on the outside, separated by a layer of building paper. All the windows are double hung with the exception of that in the bathroom, and one of the two in the kitchen. The double floor of 1-inch pine includes building paper between the subfloor and finish floor. For the heat distribution tests the walls were not insulated but a 2-inch blanket of wood-fiber insulating material was laid over the ceiling. Weather-stripping is not provided for windows or doors.

During the tests, data on temperature conditions inside the bungalow were recorded by various types of instruments. Heat-transfer coefficients for floors, side walls and ceilings were measured by the Nicholls type of heat-flow meters fastened to the surfaces. Thermocouples located at various levels of the rooms, on top and bottom of the floor, on the inside and outside of the walls, and in the basement and attic air, gave a complete record of temperature conditions. Observations were made while heat was being supplied in turn by an experimental electric heater, an oil-burning warm-air furnace, a jacketed gas-firing space heater, a jacketed oil-fired space heater, a single gas-burning gravity floor furnace, two gas-burning gravity floor furnaces, a gas-burning floor furnace with forced circulation, an oil-burning gravity floor furnace, and a conventional gravity hot-water heating system.

From the standpoint of comfort, the temperatures from the floor to 5 feet above the floor are more significant than the temperatures at higher levels. The tests showed that the average temperature differences produced in the test bungalow between the 2-inch level and the 60-inch level, with continuous forced circulation of air through a plenum chamber, with a forced-circulation gas floor furnace, and with a gravity hot-water heating system, were less than 10° F. when the outside temperature was 32° F. The temperature differences produced in this same zone by all the other devices tested ranged from 14 to 18° F. at comparable outside temperatures.

The average horizontal temperature differences between rooms at

all levels of measurement ranged from 2° to 4° F. for the several floor furnaces and for the gravity hot-water system. For the electric warm-air furnace and the oil-burning furnace, when attached to a plenum chamber, these differences were nearly as small for the 2-, 30-, and 60-inch levels, but were considerably greater in the upper levels of the house. The temperature difference between rooms was greater for the space heaters in the living room than for the other heaters tested. This effect can probably be attributed to the fact that the rooms other than the one containing the heater are warmed by the overflow of heat from the living room through the doorways. Except for the space heaters, the average horizontal temperature differences from 2 inches to 60 inches above the floor did not increase with lower outside temperatures.

Certain types of heaters may be suitable for houses of one construction and not suitable for other constructions. For example, a high temperature near the ceiling would not cause much additional heat loss if the ceiling were well insulated, or an insulated floor might permit the use of heaters that do not deliver warmed air at or near the floor level. Furthermore, some types of heaters that provide comfort in areas where extremely low temperatures do not occur may not provide comfort in colder climates.

Although standards of performance in the field of vertical or horizontal temperature differences in residences have not been established, the Bureau tests indicated that the air temperatures in the living zone of all rooms should be in the range from 65° to 80° F, for comfort when heated by the conventional types of heating systems. A floor temperature of 60° F., when continued for 1 hour or more, was found to be too low for foot comfort; when a floor temperature of 65° F, was continued for the same period, discomfort was no longer apparent. It was further observed that air temperatures of approximately 85° F. around the level of the head became oppressive, especially if the air had an appreciable velocity.

The results obtained with the devices used in the test bungalow show need for further improvement in the design of house construction and heating systems in order to provide comfort in all rooms of basementless houses. A more uniform temperature can probably be obtained either by the use of more insulation in the house elements, by improving the heat-distribution systems, or by a combination of both.

Entire heating systems were installed in this test bungalow, constructed by the Bureau for studies of heat transfer phenomena, in order to determine the performance of various heating devices designed for use in low-cost housing. The bungalow is now enclosed with an insulated shell so that any desired temperature can be maintained outside of the house.

NOTE: The complete technical report on this project is contained in the National Bureau of Standards Building Materials and Structures Report BMS108, "Temperature Distribution in a Test Bungalow with Various Heating Devices."

MERCURY-BALANCE METHOD FOR THE MEASUREMENT OF GEL STRENGTH.

An improved method for more precise measurements of the tensile strength of gels [1] has been developed by W. J. Hamer of the Bureau's electrochemistry laboratory. This procedure, which gives the strength or "yield value" and resistance to deformation of the material under study, essentially involves the determination of the shearing force necessary to fracture the gel as given by the weight of mercury required to pull a standard disc from the gel in which it is embedded.

In many applications of gelatin, glue, starch, agar, and various gums, it is important to know the tensile strength of the gel or jelly. This property was originally measured by comparison with a set of arbitrary standards on the basis of the resistance offered by the gel to finger pressure. Although this procedure was undoubtedly satisfactory for many commercial purposes, there was need for a more objective method. Such a method is very important, for example, in studies of gelatin coatings in photographic operations, particularly with the increasing use of flexible film supports instead of rigid glass bases.

The new Bureau development represents objective method for gel-strength measurement. While this procedure was designed specifically for the study of the paste walls of dry cells, it may be applied to all types of gels and used in the study of food starches, in following small changes in starch pastes for use in adhesives, and in many other applications.

The method involves the balancing of the cohesive forces of the gel against a gradually increasing quantity of mercury until the yield point is reached and the gel is broken. A brass disc is suspended in a beaker containing the gel by means of a wire attached to one beam of an analytical balance. An adjustable platform over the balance pan supports the beaker. The gel is covered with liquid petrolatum to prevent "skin" formation. Mercury is added from a burette at a constant rate to a container on the other pan. Different flow rates are made possible by use of a removable capillary tip held to the burette by a rubber tube. The capillary tip is calibrated before use by weighing the mercury it delivers in a given time. As the mercury is added, the height of the platform is changed manually, so that the pointer of the balance is always kept at zero. A vertical scale indicating the height of the adjustable platform is read at frequent regular intervals until the gel is broken.

The weight of mercury required to produce a fracture in the gel divided by the area of the brass disc gives gel strength in grams per square centimeter. The rate of change in height of the adjustable platform gives an indication of the resistance the gel offers to deformation. A fast rate means that the gel is easily deformed, whereas a

[1] Details of this work are described in Bureau Research Paper RP1810, an improved method for measurement of gel strength and data on starch gels. W. J. Hamer, *J. Research*, NBS **39** (1947).

slow rate indicates that the gel resists deformation and is probably quite rigid. Although rigid gels usually offer high resistance to deformation, this is not always true.

This method has a precision greater than 0.5 per cent. Its limitations lie more in the reproducibility with which gels can be prepared than in errors inherent in the method. It may be applied to all types of gels, whether they are of the reversible (gelatin) or irreversible (starch) type.

Gel-strength measurements are used in the control of several industrial processes, such as the manufacture of adhesives, pectin jellies, starch sponges, and dextrins; in the preparation of foods; and to a more limited extent in the paper and textile industries. They are also frequently used in studies of the granular structure of starches and in determinations of the effects of acids, heat, aging, alcohols, polyhydrols, and other physical or chemical factors on the elasticity of starch and gelatin gels.

The paste wall of most dry cells consists of a mixture of starch and flour in the form of an aqueous gel. Recently the Bureau has made studies of the paste-wall separator of dry cells of the Leclanche type. Gel-strength measurements were used to differentiate walls of different starches, to find out whether the paste reacted with the other constituents of the cell, and to determine the effects of chemical and physical modifications on the strength of the paste wall. The results were correlated with the shelf life of dry cells made with the various types of paste walls.

The characteristics of the paste walls are often described in terms of gelling temperature of the paste or the variety and amount of starch and flour used in its preparation. Frequently the viscosity of the paste is also used for this purpose. For quantitative results, however, a more sensitive method is needed, preferably one that may be used with the gels themselves rather than the pastes from which they are formed. Numerous procedures have been proposed for objective measurements of gel strengths. Several of these are limited in application, because they depend on the breaking or compression of the surface of the gel and are subject to errors owing to skin formations of variable thickness. Furthermore, many methods give a measure of different characteristics of a gel depending upon the point of view of the experimenter. For example, they may indicate rigidity, elasticity, plastic viscosity, or resistance to some force such as cutting action or torsion. The new method developed at the Bureau gives consistent results, well adapted to use in the Bureau's studies of the paste walls of dry cells.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING, WEDNESDAY, DECEMBER 17, 1947.

The stated monthly meeting of The Franklin Institute was held on Wednesday, December 17, 1947, at 8:15 P.M. The President, Mr. Richard T. Nalle, presided over the meeting. The President announced that the minutes of the stated monthly meeting for October were printed in full in the November issue of the JOURNAL OF THE FRANKLIN INSTITUTE, and if there were no corrections or additions, the minutes would stand approved as read. The minutes were approved.

The President then called on the Assistant Secretary, Dr. Frazer, for his report and announcements. Dr. Frazer announced that the following members had been elected during the month of November:

Sustaining	1
Active	30
Associate	19
Student	8
Total membership as of November 30	5417

Dr. Frazer then made the announcement of our Christmas Week Lectures, which are given for young people through the James Mapes Dodge Foundation. It was announced that these lectures would be held on December 30th and 31st at 4:30 P.M., and that the speaker this year would be Mr. Vincent J. Schaefer, who would speak on "Scientific Fun in the Fields, the Mountains, and the Sky."

Dr. Frazer further announced that according to the By-Laws, Article IV, Section 5, nominations for the Board of Managers were to be made at this December meeting. The following members of the Board of Managers, to serve for a period of three years, were nominated:

HIRAM S. LUKENS
ORUS J. MATTHEWS
MORTON GIBBONS-NEFF
CHARLES S. REDDING
GEORGE WHARTON PEPPER
M. M. PRICE
JAMES S. ROGERS
CLARENCE TOLAN, JR.

The President then called for any nominations from the floor. Since no nominations were made, the President announced that the nominations for members of the Board of Managers were closed.

The President then presented the speaker of the evening, Dr. Donald H. Andrews, Professor of Chemistry at the Johns Hopkins University, in Baltimore, Maryland. Dr. Andrews spoke on "Radio and Infra-red Detection by Superconductivity." Dr. Andrews demonstrated a new type of bolometer, which was developed during the recent war at the Johns Hopkins University. The bolometer was used for detecting infra-red rays. It consists of a small ribbon of columbium nitride cemented to the top of a copper block 1 cm. square and 1 cm. high. When cooled to −258.5° C., the ribbon becomes partially superconducting and detects both infra-red and radio waves with a sensitivity in many ways superior to any other known device. Recordings of various kinds of signals received by the bolometer were demonstrated both with an oscilloscope and a loud speaker.

After Dr. Andrews' very edifying lecture, the President adjourned the meeting with a rising vote of thanks to the speaker.

JOHN FRAZER, *Assistant Secretary.*

NEW MEMBERS OF THE FRANKLIN INSTITUTE ELECTED JUNE 1, 1947 TO DECEMBER 1, 1947.

SUSTAINING MEMBERS.

Mr. Sherman Mills Fairchild
Mr. Eugene J. Houdry

ACTIVE MEMBERS WITH FAMILY PRIVILEGES.

Mr. Pierre Aureli
Mr. N. Newlin Baily
Mr. Robert W. Barker
Mr. Mayer I. Blum
Mr. Elmer J. Boetefuer
Mr. Edward G. Budd, Jr.
Mr. Emil Cohn, Jr.
Mr. John M. Diem
Mr. Moses Ehrlich
Mr. Joseph H. Gillies
Mr. Charles H. Godschall
Mr. Harry C. Hambridge
Mr. Harry R. Hirshorn
Mr. David H. Hopkins
Mr. Laurence T. Howell
Mr. Edward Kohlhepp
Mr. George Lipsky
Mr. John S. Malick
Mr. John Cochrane Martin
Mr. John F. McNelis
G. Herbert Moffses, M.D.
Mr. Harry E. Mousley
Mr. Robert Nagel
Mr. Gordon Null
Mr. Marvin H. Pond
Mr. Howard C. Pruyn
Mr. Harry Rubin
Mr. Theodore A. Seraphin
Mr. Samuel A. Weston
O. Wilson Winters, D.D.S.
Mr. Carl A. Wolf

ACTIVE MEMBERS.

Mr. William Loughridge Aiken
Mr. Edward J. Bailey
Major General Milton G. Baker
Mr. Henry S. Bamford
Mr. Naaman F. Barr
Mr. William L. Baumner, Jr.
Mr. Charles W. Berg
Mr. Sanford Bliwise
Mr. Lou Block
Mr. M. A. Bordman
Mr. Harry S. Boud
Mr. John E. Bower
Mr. William H. Brearley, Jr.
Mr. W. Ronald Briggs
Mr. Harold W. Brown
Mr. James M. Castle
Mr. George A. Clickenger
Mr. Frank G. Conrad, Jr.
Mr. Edward F. Cooper
Mr. Edward P. Cornely
Mr. George J. Crits
Mr. Joseph Croskey
Mr. E. B. Curdts
Mr. Edwin A. Dages
Mr. Joseph De Frenes
Mr. William J. Devinney
Mr. Felix Di Arenzo
Mr. Ralph Earle
Mr. A. J. Ellis
Mr. Jay Emanuel
Paul J. Ernst, Ph.D.
Mr. Woodfin R. Fagge
Mr. William F. Famille
Mr. Mort F. Farr
Mr. Myer Feinstein
Mrs. Jeanne Ferber
Mr. Alex C. Fergusson
Mr. Harry B. Fertik
Thomas Fitz-Hugh, Jr., M.D.
Mr. Lawrence S. Ford
Mr. John J. Fox
Mr. Benjamin H. Freeman
Mr. Charles H. Fryburg
Mr. Edward E. Gallob
Mr. Arthur S. Gleason
Mr. Richard A. Griefen
Mr. H. K. Groff
Mr. William P. Groves, Jr.
Mr. Amil S. Gumula
Mr. Fred C. Haab
Mr. Joseph J. Hagan
Mr. Wilson C. Hanline
Mr. Paul V. Hannas
Mr. Abdullah Feyzi Hamdi
Mr. W. J. Heggie
Mr. Robert F. Herr
Gerald E. Herrnstadt, Ph.D.
Mr. Emil Hoffman
Mr. William P. Hood
Mr. William Hopkin
Mr. Wayne L. Hopkins
Mr. John J. Howard
Mr. A. D. Howry
Mr. Robert V. Hudson
Mr. Joseph Gray Jackson
Mr. Bernard Jacobs
Mr. C. Brooke Jones
Mr. Morris Katz
Mr. John Keefe, Jr.
Mr. John Kerbeck
Mr. Ellis Knowles
Mr. William Kolb
Mr. Alfred Theodore Kornfield
Mr. Ellis M. Landis
Mr. Henri Lauer
Miss Dorothy K. Lawrence
Mr. Hurley Lazarus
Mr. Alfred Leith
Mr. Adam Leva
Norton M. Levin, D.O.
Mr. Willard F. Levy
Mr. Joseph F. Lewis
Mr. John Franklin Long
Mr. Thomas C. Longstreth
Mr. James Luckman
Mr. Lionel P. Maggenti
Mr. Charles W. Maloney
Mr. John Mallo
Mr. Thomas I. Manly
Mr. Samuel A. Mann
Mr. Vernon L. Marquart
Mr. Emil A. Mathias
Mr. T. E. McBride
Mr. Laurence T. McCurdy
Mr. Sterling R. Mensch

Mr. C. B. Miller
Mr. Raymond Millman
Mr. Harry W. Morgan
Mr. Robert V. Morris
Mr. Richard J. Mott
Mr. Arthur Niessen
Mr. Charles H. Nonamaker
Mr. M. John O'Donoghue
Mr. Gordon Palmer, Jr.
Mr. Daniel C. Parr
Mr. Edward Paul
Mr. Richard D. Pomerantz
Mr. Clarence H. Porter
Mr. F. B. Powers
Mr. Harold C. Rahn
Mr. Leonard Rodenhausen
Mr. Frank B. Russell
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Harold G. Scheie, M.D.
Mr. Morris Schiff
Mr. Frederick J. Schneider
Mr. Howard I. Scherr
Mr. Gerry W. Schulz
Mr. Richard Schweder
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Mr. John B. Shields
Mr. Chao Tze Shih
Dr. Alfred R. Smith
Mr. Cortland Gray Smith
Mr. Edward W. Smith, Jr.
Mr. Perry C. Smith
Mr. Albert Soffa
Mr. Robert Solts, Jr.
Mr. Ulrich Sontheimer
Mr. James C. Stambaugh
Mr. Albert D. Stave
Mr. Saul H. Steel, Jr.
Dr. John A. Stevenson
Mr. Charles M. G. Stewart
Mr. E. Lee Strock
Mr. Lisle J. Sykes
Mr. John J. Townsend
Mr. John W. Temple
Mr. John M. Tinker
Mr. Irving S. Towsley
Mr. Wilmer S. Trinkle
Mr. William A. Tucker
Mr. Russel R. Tull
Mr. Edward S. Turner
Mr. Ralph Vezin
Mr. Alfred M. Whitney
Mr. Edward J. Williams
Mr. R. M. Wilson
Mr. Samuel S. Wolfman
Mr. R. E. Worden
Carroll S. Wright, M.D.
Mr. Joseph A. Wurster, Jr.
Mr. Samuel M. Zollers

ACTIVE NON-RESIDENT MEMBERS.

Mr. George W. Alcock
Mr. Carleton Craig
Mr. John R. Heizmann
Mr. Arthur T. Hinckley
Mr. Mortimer Francis Nahon
Mr. Guy F. Rolland
Mr. P. A. Shields
Mr. George H. Woodard

NECROLOGY.

Mr. Charles S. Beach '01
Miss Elizabeth C. Biddle '09
Mr. C. H. Bierbaum '97
Mr. George Imlay Bodine, Jr. '44
Mr. Julius Boekel '84
Mr. Franklin S. Chambers '41
Mr. Robert Cherry, Jr. '37
Mr. Morris L. Clothier '26
Mr. Walter T. Dwyer '46
Mr. Charles J. Eisenlohr '36
Mr. William M. Elkins '42
Mr. John J. Foulkrod, Jr. '36
Mr. William H. Frey '43
Mr. Anthony H. Geuting '36
Mr. John Elgin Gilmer '43
Mr. Joseph Handler '43
Mr. Frederick A. Healy '36
Boyden W. Kowalski, M.D. '44
Mr. John M. Krupp '43
Mr. James Barton Longacre '38
Mr. Sylvester A. Mahan '46
Mr. James T. J. Mellon '42
Samuel Nicholas, M.D. '44
Max Planck, Ph.D. '27
Mr. J. Benton Porter '41
Mr. Paul C. Richter '46
Mr. Isaac B. Ritter '42
Mr. Charles E. Schmidt '43
J. E. Shrader, Ph.D. '22
Mr. J. Stogdell Stokes '36
Mr. S. M. Swaab '46
Mr. Henry F. Wageman '46
Miss Hepsey Norris Wells '34
Mr. William H. B. Whitall '36
Thomas L. Wilcox, D.D.S., '45
Major Richard R. Wright, Sr. '46

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, November 12, 1947.)

HALL OF THE COMMITTEE,
PHILADELPHIA, NOVEMBER 12, 1947.

MR. WALTER C. WAGNER in the Chair.

The following report was presented for final action:

No. 3186: Henderson Medal.

This report recommended the award of a George R. Henderson Medal to Charles Duncanson Young, of Whitford, Pennsylvania, "In consideration of his contribution to the

scientific advancement of the steam locomotive which has resulted in improving the reliability and efficiency and reducing the cost of steam locomotives thereby producing a more effective transportation unit."

(*Abstract of Proceedings of Stated Meeting held Wednesday, December 10, 1947.*)

HALL OF THE COMMITTEE,
PHILADELPHIA, DECEMBER 10, 1947.

MR. WALTER C. WAGNER in the Chair.

The following report was presented for final action:

No. 3185: Electrical Gun Director.

This report recommended the award of a Howard N. Potts Medal each to David Bigelow Parkinson, of Maplewood, New Jersey, and Clarence Anding Lovell, of Summit, New Jersey, "In consideration of their combined contributions, both to the theoretical and practical design of the Electrical Gun Director, one of the outstanding pieces of equipment developed during the war period, which development has further contributed largely to the theory and practical application of servo mechanisms and smoothing filters in general."

JOHN FRAZER,
Secretary to Commmittee.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M., Wednesdays and Thursdays from 2 P.M. until 10 P.M.

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NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Enzymatic Hydrolysis of Desoxyribonucleic Acid Prepared with the Use of Strong Sodium Hydroxide.—CHARLES A. ZITTLE. It was previously reported (1) that desoxyribonucleic acid (DNA) prepared under mild conditions (method of Hammarsten) (2) was readily hydrolyzed by specific nuclease so that it became soluble in HCl, whereas DNA prepared with the use of hot 1.25 N NaOH (method of Levene) (3) was not hydrolyzed by the nuclease. Further study has shown that under the proper conditions the alkali-treated DNA can be slowly hydrolyzed by the nuclease at a rate about 20 per cent. of that of DNA (Hammarsten). The previously reported erroneus conclusion that DNA (Levene) could not be hydrolyzed was in consequence of this low rate of hydrolysis and the instability of the enzyme under the conditions used.

Experimental.

Preparation of the enzymes and substrates and the manometric and other procedures have been described (4). In initial experiments with DNA (Hammarsten) this nucleic acid was contemplated as a substrate for the phosphoesterase from calf intestinal mucosa (5), and for this purpose it was treated with specific nuclease since hydrolysis of DNA by the phosphoesterase is only rapid and complete after it has been acted on by specific nuclease (4). For preparing the nuclease-treated DNA the nuclease was obtained in the form of a dried ammonium sulfate precipitate and weighed amounts of this material were added directly to the nucleic acid soultions. The above observations about hydrolysis were made under these conditions. In subsequent attempts to use dilute solutions of the nuclease in manometric experiments it was found to be very unstable, but could be stabilized by the addition of neopeptone following observations of McCarty (6).

A stable nuclease solution was prepared as follows: 300 mg. were dissolved in 20 cc. of 0.0005 N H_2SO_4 and dialyzed against the same to remove the ammonium sulfate; 6.0 cc. of this solution were mixed with 4.0 cc. of neutralized neopeptone, 1.3 cc. of 0.5 M $NaHCO_3$ and 0.7 cc. of 0.3 M magnesium chloride. This solution, prepared from the dialyzed enzyme for each experiment, was used in the experiments to be described.

The initial rates of hydrolysis of the two nucleic acids were determined manometrically. In each experiment 2.0 cc. of 0.6 per cent. nucleic acid together with 0.2 cc. of 0.5 M $NaHCO_3$, variable amounts of enzyme in the side arm of Warburg flasks, and water to make the

final volume 3.5 cc. were used. In these experiments the initial rates of hydrolysis were: for DNA (Hammarsten), 91.2 cmm./10 min./1.0 cc. of nuclease; for DNA (Levene), 17.0.

Precipitates obtained by acidifying solutions of DNA (Levene) with HCl turn a light brick red on standing for some time; initially the precipitates are white. This observation and the slow rate of hydrolysis of this nucleic acid suggested that an inhibitory substance might be present from the reagents (colloidal iron, for example) used in its preparation. Hydrolysis of the two preparations of nucleic acid in the same flask (followed manometrically: 6.0 mg. of each nucleic acid, 0.5 cc. of enzyme), however, was equal to the sum of the two taken alone, indicating that the alkali-treated nucleic acid preparation was not inhibitory to hydrolysis of the other. With the same thought in mind the alkali-treated nucleic acid was dialyzed for 24 hours, precipitated with HCl (final concentration, 0.24 N; pH 1.2) and dried with acetone. Fifty-eight per cent. of the starting material were recovered. The enzymatic hydrolysis of this material (followed manometrically: 3.0 to 9.0 mg. of nucleic acid; 0.5 cc. enzyme), however, was the same as that of the original.

For complete hydrolysis of the alkali-treated nucleic acid at 23°, measured by solubility in 0.25 N HCl, the following procedure was adequate. Nine parts of 0.6 per cent. solution of the nucleic acid were treated with successive portions of the enzyme solution: with two parts initially, with two parts at the end of 4 hours, and with two parts at the end of 7 hours. The last portion of enzyme was permitted to act for 18 hours with the solution saturated with chloroform. Although precipitation with HCl was now negative, precipitation with the uranium reagent had remained unchanged.

Phosphoesterase was permitted, as was done previously (1), to act on the above digest without and with silver nitrate present, the silver to inhibit the adenosine deaminase in these preparations. With this complete digest the results are comparable to those obtained with digest of DNA (Hammarsten); with silver present more CO_2 is released due to the inhibition of the deaminase and consequent production of ammonia. This is in contrast to previous results with DNA (Levene). From the previous results it was difficult to decide whether the DNA had been deaminated by the strong alkali used in its preparation or that the phosphoesterase had not liberated adenosine. From the present data it appears that the latter explanation was the correct one.

Discussion.

The previous observation (1) that DNA (Levene) was not hydrolyzed was made because of the instability of the enzyme under the conditions used and also because of the low rate of hydrolysis of this prepation of nucleic acid. The DNA (Hammarsten) was rapidly hydrolyzed

and hydrolysis was complete before the activity of the enzyme was lost, but, the DNA (Levene) showed little or no hydrolysis during the same period of time. The results of the present study indicate that adenosine deaminase can act on both of these hydrolysates and there is no reason for believing that the adenine radical is involved in the low rate of hydrolysis.

Summary.

In contradiction to results previously reported, it has been found that DNA prepared by a method which utilizes strong NaOH is acted on by the specific nuclease, although at a rate only 20 per cent. of that of DNA prepared by a milder method. The nature of the changes responsible for the low rate of hydrolysis of the alkali-treated DNA are not known.

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BOOK REVIEWS.

AIR CONDITIONING AND ELEMENTS OF REFRIGERATION, by Samuel P. Brown. First Edition. 644 pages, drawings and illustrations, 15 × 23 cms. New York and London, McGraw-Hill Book Co., Inc. 1947. Price $6.00.

"The objective of this book is to provide a complete, practical, working textbook and handbook suitable for the study by any interested person and to those working in design, installation, operation of heating, ventilating, and other mechanical equipment of buildings." The selection of equipment, automatic controls, and design of air ducts and liquid piping systems are covered, as well as refrigeration theory.

The tables necessary for elementary design work are included, although complex theory and higher mathematics have been eliminated.

Throughout the text, illustrative practical problems are presented, which are then answered in the appendix. There are over 230 graphs and illustrations throughout the book.

HENRY E. MICHAEL.

FUNDAMENTALS OF ELECTRICITY AND MAGNETISM, by Leonard B. Loeb. Third Edition. 669 pages, drawings, 15 × 24 cms. New York, John Wiley & Sons, Inc., 1947. Price $6.00.

This textbook, originally written to fill a specific need at the University of California, presents the subject, in its third edition, as a consistent and related whole, rather than isolated and dogmatically stated topics.

The historical survey of previous editions is replaced by a brief historical outline and the space thus gained is devoted to the enlargement of the section on atomic structure.

The book begins with elementary magnetism and direct current electricity followed by the study of static electricity. The c.g.s. is utilized, since it is felt by the author that electricity, basically so closely associated with electronic and atomic phenomena, profits more from its use rather than the m.k.s. system, which has its advantages in application to gross mechanics or engineering physics.

HENRY E. MICHAEL.

TABLES OF INTEGRALS AND OTHER MATHEMATICAL DATA, by Herbert Bristol Dwight. Revised edition. 250 pages, tables, 14 × 21 cms. New York, The Macmillan Co., 1947. Price $2.50.

This book contains a variety of mathematical data in formula and tables including algebraic, trigonometric, inverse trigonometric, exponential, logarithmic, hyperbolic, inverse hyperbolic, elliptic, and Bessel functions. There are also probability integrals, surface zonal harmonics, definite integrals and differential equations. An appendix gives tables of numerical values and a list of 62 references referred to in the text where the derivation of the results is given and where further results may be found. At the end of the book there is also a subject index giving more details on material location so that for instance gamma function or Bernoulli's numbers may be found quickly. This second addition of this work is a convenient tool for those who use mathematics to any extent.

R. H. OPPERMANN.

THE CHEMISTRY OF COMMERCIAL PLASTICS, by Reginald L. Wakeman. 836 pages, tables and illustrations, 15 × 24 cms. New York, Reinhold Publishing Corporation, 1947. Price $10.00.

Twenty years ago it was not uncommon to hear it said that all pioneering in organic chemistry was over—organic chemistry was no longer a virile science for no discoveries of fundamental importance had been made by its devotes since the brilliant éclat of the aniline dye

industry. But the foundations were being laid for a new glorious period for organic chemistry, the Age of Plastics. Plastics, as we know them today, cover a large group of organic substances, either wholly or in part synthetic, which can be molded into coherent solid articles that retain their shape indefinitely at room temperature. This book is an analysis of the fund of information on the subject which correlates the scientific data with actual present industrial practice.

After an introductory chapter and a reference to the history of the plastics industry the work examines in a general way some of the relationships which exist between the various raw materials and the plastics made from them. This leads to treatments on general chemical principles of resinification and ways in which synthetic resins and plastics are manipulated in making diverse products. Thereafter consideration is given separately to specific subjects such as phenolic, urea and melamine plastics, aniline-formaldehyde resins, polyamides-nylon, coumarone-indene resins, polyvinylidene chloride, polystyrene, rubber derivatives, natural and synthetic elastomers, cellulose, lignin and protein plastics, polymeric organosilicon-oxygen compounds. These represent the most important plastics of commerce. There have been developed many other synthetics of more or less resinous character and are marketed for special purposes in relatively small volumes. Some of these are taken up in the last chapter together with a few which have been studied extensively in industrial laboratories but which still have not appeared on the market. In the back of the book there is an author and a large well arranged subject index.

This book is the result of an exhaustive survey of the literature and an intimate knowledge of the industry. The many illustrations, tables, curves, diagrams coupled with skill in presentation and a progression to present practice pointing to the future make this book a valued contribution to the literature as a text and reference work.

R. H. OPPERMAN.

TEXTILE FIBERS by J. M. Matthews and H. R. Mauersberger. Fifth Edition. 1133 pages, tables and illustrations, 15 × 24 cms. New York, John Wiley & Sons, Inc., 1947. Price $12.50.

The history of fiber development has been a strange pattern of trial and error ever since fibers were first used for making apparel. Climate had much to do with it since wool, hair and fur fiber had great and continual development in cold climates and in hot climates development was first found in vegetable fibers. With the invention of machinery, fiber development was more rapid. Some fibers were delayed in development because of man's inability to dye them satisfactorily and to degum with sufficient thoroughness. Then came the need, particularly of the totalitarian nations, of producing ersatz materials to take the place of natural fibers. New methods, new products, new machines, and new techniques continually displace the old. This volume on fibers depicts the progress that has been made in textile fibers, their economic utilization, and technical development.

Now in its fifth edition, the book has 1095 pages of text, written for the student in colleges and technical school and as a reference for those who deal with the sale, production, processing, and conversion into fabric of all the different fibers now used in the textile and clothing industries. A suitable introduction leads into a general detail of the essential physical, microscopic, and chemical properties which make fiber a valuable textile material. Later, these properties are given considerable space with regard to cotton, the bast fibers, structural or hard vegetable fibers, wool, specialty hair fibers, silk, rayon, new synthetic textile fibers, and material or inorganic fibers and filaments. A complete coverage is given including the economics, production, cultivation, statistics, prices, dyeing, spinning, etc. A considerable section is devoted to fiber identification methods, the quantitative determination of fibers present in a product, and fiber testing methods. There is a bibliography at the end of each of the 24 chapters and in the back of the book there is a large comprehensive subject index.

This monumental work is the sum of the work of forty-six contributors, collaborators and reviewers whose work was supervised in selection, revision and arrangement by an editor with much experience in the industry. It is of practical value today and points to fields of research and promise for the future.

R. H. OPPERMANN.

SERVOMECHANISM FUNDAMENTALS, by Henri Laurer, Robert Lesnick, and Leslie E. Matson. First Edition. 277 pages, drawings, 16 × 23 cms. New York and London, McGraw-Hill Book Co., Inc., 1947. Price $3.50.

The term"servomechanism" is generally used to denote a particular class of automatic control system where the controlling signals of the system change fast and frequently. This type of system is different from the "regulator" in that in the case of the regulator the controlling signals do not change or change slowly and the primary problem is to prevent extraneous causes from affecting the function controlled by the system. Servomechanisms are used to steer ships, control airplanes, automatically tune radio receivers, position guns, etc. Regulators control temperatures, maintain liquid levels etc. This newer servomechanism principle has the outstanding features that the controlled and controlling elements of the system may be widely separated, that the work performed may be many times greater than required for control operation and the often complicated processes can be made to take place according to predetermined standards.

This book is designed for practicing engineers as an introduction to the principles underlying the theory of servomechanisms. As such it starts with the elements and proceeds to the more complex with sympathy to the needs of those who require refreshing or review coverage on many forgotten subjdcts. In detail, the work begins with defining the concepts of control of open and closed control systems and the means employed for accomplishing control operations. Simple practical examples are described, enough to give an idea of duties performed which could hardly be accomplished by other means. The servo system follow up link; a differential device which produces an indication of the magnitude and direction of the error between the input and output members of the system, is next taken up.

Because servo-control systems involve a knowledge of both mechanical and electrical engineering principles, the authors have inserted at this point a chapter on fundamentals of mechanics and electricity as a review covering only the most elementary concepts. Then proceeding onward the book treats on principles of the transient analysis method of elementary servomechanisms. This is divided as follows: analysis of servomechanisms with viscous output damping, with error rate damping, with combined viscous output damping and error-rate damping, with integral control, and an enlargement in greater detail devoted to practical means for obtaining error rate damping, and a discussion of error rate stabilization networks. There is an introduction to the transfer function analysis of servomechanisms which is a frequency analysis and the relation of this to the transient analysis is indicated. The last chapter of the book is devoted to typical design calculations and general consideration. At the end there is a subject index.

The practicing engineer who has a need of familiarity with servomechanisms will find in this book a useful tool.

R. H. OPPERMANN.

PETROLEUM PRODUCTION, by Park J. Jones. Volume III. 271 pages, 16 × 24 cms., tables and drawings. New York, Reinhold Publishing Corporation, 1947. Price $5.00.

This is Volume III of a series of books and is limited to oil production by water. This excludes primary gas caps and oil production by gas. The book is divided into two parts, the first dealing with foundations. Essentially in the form of a text, formulas, curves, and charts are given with discussions on reservoir space in barrels, the fluid factor and its effects, oil production by expansion, well producing capacity and the factors effecting it, injection of water, the optima for reservoirs involving economical as well as physical factors, and the migration of oil. Part two of the book is on applications and its coverage includes discussions on radial reservoirs, two pay intervals, impermeable wedges, saturated oil, linear reservoirs, elongated reservoirs, and bottom water. A subject index is in the back.

The presentation shows a practical application of facts and figures relating to this type of petroleum production.

R. H. OPPERMANN.

PUBLICATIONS RECEIVED.

The Chemistry and Technology of Waxes, by Albin H. Warth. 519 pages, drawings, tables and illustrations, 15 × 23 cms. New York, Reinhold Publishing Corporation, 1947. Price $10.00.

Architectural Construction, by Theodore Crane. 414 pages, drawings and illustrations, 16 × 25 cms. New York, John Wiley & Sons, Inc., 1947. Price $6.00.

Field Practice, by Elwyn E. Seelye. A Data Book for Civil Engineers. 306 pages, tables and drawings and illustrations, 13 × 21 cms. New York, John Wiley & Sons, Inc., 1947. Price $4.50.

Saga in Steel and Concrete, by Kenneth Bjork. 504 pages, 16 × 24 cms. Northfield, Norwegian-American Historical Association, 1947. Price $4.00.

Industrial Health Engineering, by Allen D. Brandt. 395 pages, drawings, tables and illustrations, 15 × 24 cms. New York, John Wiley & Sons, Inc., 1947. Price $6.00.

Flight Engineering and Cruise Control, by Harris G. Moe. 209 pages, drawings and illustrations, 15 × 23 cms. New York, John Wiley & Sons, Inc., 1947. Price $4.00.

Very High-Frequency Techniques, compiled by the staff of the Radio Research Laboratory, Harvard University. Volumes I and II. First Edition. 554 pages, and 1057 pages, drawings and illustrations, 16 × 24 cms. New York, McGraw-Hill Book Co., 1947. Price $14.00, two volumes.

The American Annual of Photography 1948, edited by Frank R. Fraprie and Franklin I Jordan. 216 pages, illustrations, 18 × 25 cms. Boston, American Photographic Publishing Co., 1947. Price $2.00.

Chemical Engineering Fundamentals, by Chalmer G. Kirkkride. 419 pages, tables and drawings, 15 × 24 cms. New York, McGraw-Hill Book Company, Inc., 1947. Price $5.00.

Detecting Impurities in Uranium. (*Electrical Engineering*, Vol. 66, No. 7.)—Thirty-three impurity elements, some in concentrations as low as one part in 20 million, have been detected and estimated through a modified spectrographic method developed at the National Bureau of Standards and used since 1942 in the Manhattan Project for analysis of uranium and its compounds.

Impurities differ in their effects on the nuclear chain reaction. Some elements such as boron and cadmium may interfere if present in concentrations as low as a few tenths of a part per million, and many other elements should not exceed a few parts per million. Rapid, sensitive, and accurate methods therefore are required for the determination of at least 60 chemical elements in a variety of uranium-base materials. Established spectrographic methods of analysis are unsuccessful because of interference by the complex uranium spectrum in which more than 20,000 lines have been observed.

This difficulty was overcome by separating the impurities from the uranium in the carrier-distillation method by converting the uranium sample to a refractory compound having low volatility (the black oxide of uranium U_3O_8) and distilling the impurities from this compound in a d-c electric arc.

In order to sweep out the minute quantities of impurity vapors from the sample without volatilizing the uranium, a small amount of a volatile material, termed a "carrier," is added to the sample. Gallium oxide was found most useful as a carrier and is added at a concentration of two per cent in the uranium oxide. When the mixture is heated by a d-c arc in a carbon electrode of special design, the carrier material and impurities are volatilized into the arc. The uranium, remaining as a residue, can be recovered readily from the electrode—an important consideration, particularly with some active forms of uranium. The light of the arc, examined with a spectrograph, provides a spectrum consisting of the simple spectrum of gallium plus the spectral lines characteristic of volatile impurities in the uranium.

For quantitative determinations, carefully prepared standards of known composition are submitted to the same treatment as the samples. Amounts of impurities are then estimated by photometric measurement, or by visual comparison of the spectrograms of samples and standards. Determinations are made to an accuracy of plus or minus 10 per cent of the amount of the element present. For example, boron and cadmium can be observed easily at a concentration of 1 part in 10 million, and the concentration can be determined to within one tenth of this amount. The greatest sensitivity, observed for the detection of silver, was 1 part in 20 million.

R. H. O.

Stand Built to Test Governors Under Service Conditions. (*Compressed Air Magazine*, Vol. 52, No. 6.)—Governors for diesel engines must be built with accuracy to perform the important function for which they are designed. These units control the amount of fuel injected into the engine cylinders by

the fuel pumps and come into action as soon as the engine overspeeds or underspeeds. In many cases where diesels drive equipment such as electric generators, governors must be extremely sensitive to slight changes in engine speed, and it is for this reason that they have to undergo rigid testing and minute adjustment under simulated running conditions before they leave the factory. This applies not to one unit picked at random from a lot but to each one, and has led the Woodward Governor Company to design and construct a test stand that does this work not only in routine fashion but also with precision.

The stand consists of a cast-iron surface plate and pedestal and is equipped with an air motor of the Multi-Vane Type that has a speed range from 150 r.p.m. to 4200 r.p.m. This unit drives the governor which, in turn, controls the speed of the motor by throttling the air supply. Air at from 80 to 100 p.s.i. is used, and the pressure must be kept constant close to the stand for satisfactory testing.

Different types of governors can be coupled to the motor by means of a mounting plate and drive-shaft adapters, of which three are standard equipment. Depending upon whether it is keyed or serrated, the adapter is either attached to the governor drive shaft or dropped into a flywheel recess in the table top. The shaft is then inserted in the recess until the governor rests squarely upon the flat surface plate, something that is necessary for smooth running. It may also involve moving the motor up or down in order to get it in proper position to drive the shaft.

After the governor has been filled with oil to a predetermined level and been linked with an adjustable fulcrum on the stand, the air motor is started. This is done by flexing the cable that connects the fulcrum and the air-control lever until the governor takes over. Testing involves dispelling all air bubbles by allowing the governor to surge for about five minutes, warming the oil by letting the unit run from 15 to 20 minutes, adjusting the compensating needle valve, and checking the response of the governor under its specified high and low speeds and under other service conditions. Excessive heating, vibration, oil leakage, and defective parts can be detected by the stand operator, and different measuring and indicating instruments enable him to make the corrections and adjustments necessary to insure a governor that will do the exacting work for which it is designed.

R. H. O.

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GUIDED BOMBS IN WORLD WAR II.[1]

BY

L. O. GRONDAHL, Ph.D., Sc.D.(Hon.).*

This paper is based in whole or in part on work done for the Office of Scientific Research and Development under Contracts OEMsr-240 and OEMsr-1013 with Massachusetts Institute of Technology, under Contract NDCrc-183 with the Gulf Research and Development Corporation, and under Contracts OEMsr-1081, OEMsr-1285, and OEMsr-1415 with the Union Switch and Signal Company.

My purpose is to tell you the story of a project of the war years which was completed from the initial research stages through development, engineering, and production to effective combat service in 3½ years. The achievement can probably be matched by many others of that time and is therefore only a sample of what can be done when it is appropriate to follow long shots and to make all the false starts—and the good ones—simultaneously. The story is interesting as an illustration both of effective cooperation of a great number of organizations and of quick results. Such a development in industry usually takes 10 to 20 years to complete. The story has to do with the development of a dirigible high angle bomb. Three varieties were completed, but only one went into actual combat service. The research work was conducted at colleges and universities, at aeronautical laboratories, air fields, industrial laboratories, and Army establishments.

The laboratories and the air fields were distributed over the country from Boston to Los Angeles, and at many points between these extremes. We shall mention some of them as we proceed with the story. The men who had to do with the promotion of the use of the bombs in

[1] This paper consists of the essential portions of a talk before the Franklin Institute on October 15, 1947. Some of this material was published in *Ordnance*, May–June, 1947, p. 503.

* Consultant in Research, Westinghouse Air Brake Company and Union Switch and Signal Company. Formerly a member of Section D-3 and Division 5, and Chief of Section 5.2, National Defense Research Committee.

the combat zones flew to London, France, Germany, North Africa, Sicily, and Italy to the east, and to Hawaii, China, and Burma to the west.

The principal contributions to the project were made by the National Defense Research Committee, which provided the funds and the administration: Massachusetts Institute of Technology at which a small group of physicists made the early experiments, and at which much development of the fundamental apparatus, wind tunnel tests, and analysis of trajectories were made; the Army Air Forces which furnished the personnel and the airplanes, and other facilities necessary for the field tests; the Gulf Research and Development Corporation which completed the development that had been started at M.I.T. and did a vast amount of analytical work and field testing; and the Union Switch and Signal Company which did the engineering for production and the final manufacture. In addition to these there were more than 20 other organizations which contributed to the development and the manufacture of component parts.

The high angle bomb is the bomb that you have seen many times in motion pictures, and which is dropped out of a bomb bay at an instant determined by the bombardier by the use of a bomb sight into which he feeds the necessary information. The most important parts of this information are the speed of the plane and the altitude. The plane flies in a straight line towards the target and the bomb is dropped at a predetermined instant, such that if everything functions exactly correctly it falls on the target. As the bomb falls away from the plane its vertical velocity increases very rapidly in response to the action of gravity. Its horizontal velocity is approximately constant and equal to that of the plane, so that if the plane is traveling 200 m.p.h. the bomb's velocity parallel with the ground will continue to be nearly 200 m.p.h. during the whole drop. Its vertical velocity may increase to 700 or 800 m.p.h. depending upon the altitude from which it is dropped. If a bomb is dropped from 15,000 ft. or higher, the last portion of its trajectory is nearly vertical.

When a bomb is dropped in this way, small errors in manipulation of the plane or of the bomb sight, or errors in information in regard to the exact altitude of the target and the exact speed of the plane relative to the ground result in quite large errors in the strikes. The purpose of our study was to find a method of eliminating these errors, for, of course, every bomb that is dropped in the wrong spot is not only a waste in itself but is a waste of the time, personnel, and equipment used in making the bombing mission.

The problem may be visualized if one imagines that one looks down on a bomb that is falling from an airplane. Before the bomb is dropped the plane has been flying in a straight line toward the target for some time. This portion of its flight is called the bombing run. If the flight

of the plane is continued in the same direction as the bombing run after the bomb has been dropped, the bomb seems to remain approximately vertically under the plane and stays that way until it hits the ground. If one draws an imaginary line through the bomb in the apparent direction of its flight, it is easy to see whether it is going to the right or to the left of the target. One can then imagine at once a possibility of doing some correcting of any bombardier's error by steering the bomb to the right or left. It is not possible by looking at such a bomb to tell whether it is going to hit the ground too soon or too late to land on the target. For this purpose it is necessary to use a special maneuver or special apparatus. The special maneuver was unsatisfactory and the special apparatus which was developed at the Franklin Institute was used.

The analysis of bombing experience in combat reveals that to correct bombardiers' errors completely it is necessary not only to correct the direction, or, as we often say, the azimuth of the flight of the bomb, but also to correct its range. We can, therefore, consider the problem as consisting of two parts, the right and left deflection and the change in range. The former is controlled by means of rudders and the latter may be controlled by means of elevators.

To control the flight of a bomb, it is necessary to keep it from rotating about its axis. Otherwise rudders would sometimes be elevators and vice-versa, and control would be impossible. The first method that suggests itself for eliminating this rotation or roll is to provide stabilization by means of a gyro. The simplest way to apply these controls is by means of ailerons, a name which we borrow from aeronautics. These control surfaces are so arranged that when they are operated by the gyroscopes they produce a rotation of the bomb about its axis but no yaw or pitch.

If a bomb is to be controlled from an airplane, it is necessary also to provide in some way a means of communication between the airplane and the bomb to transmit the controls. Radio is the obvious solution and is the one that was used in our development.

To give an idea of the course the development necessarily had to take, a few of the early questions may be enumerated.

(1) Is it possible to control the high angle bomb sufficiently to correct the errors that are likely to be encountered? The errors from 15,000 ft. were of the order of a few feet to several hundred, or even occasionally a thousand feet.

(2) What kind of aerodynamic surfaces must be used in order to get this control?

(3) What physical shape should the controlled bomb take?

(4) What is the behavior of bombs in their flight to the ground? Do they all rotate or do they very seldom rotate?

Many control systems were proposed and during the first two or three years of the development all possibilities were followed. One of these was the use of television equipment in the bomb. This was arranged to transmit to the operator a picture of what could be seen from the nose of the bomb itself. If this picture had the target in the center it was an indication that the course would carry it to the target. If the target was not in the center, application of the control from the plane would correct the course of the bomb.

Another promising type of apparatus made use of target seeking equipment and was operated by radiations, either of light or heat. The action was automatic in that the equipment was arranged so that the heat sensitive or the light sensitive equipment would initiate controls to keep the target in the center of the bomb's field of view.

Both of these methods of attack had to be abandoned because the apparatus was too bulky, and too complicated to be perfected in a short time. There remained the direct sight method, and since that was the one finally used, it will be discussed more in detail.

To establish the possibility of getting effective controls, the earliest experiments were made with small models in the wind tunnels at M.I.T. The wind tunnel experiments indicated that the controls would be adequate. In order to get verification of the results obtained in the wind tunnel and of the very large extrapolation that was necessary, 100 lb. practice bombs were taken to Aberdeen Proving Grounds, and tested to determine their behavior in the air. For these experiments small rudders were built into the tail structure of the practice bomb, with the idea of determining how large were the effects of the rudder surfaces. In these experiments it was soon noticed that the bombs rotated so much that it was difficult to get satisfactory estimates of the effects of the surfaces. Occasionally, however, the rotation was small enough to give promising indications. It was necessary to make observation from the ground because the men in the plane were not able to see the the bombs throughout the entire flight. In spite of all the difficulties and shortcomings in the early experimental methods, something was learned about the size of rudders that were needed and everyone was convinced that gyro stabilization was absolutely necessary. It was learned also that if we were to think of using direct sight in our control system, it would be necessary to provide the bombs with flares.

Television and direct sight bombs were now under construction at Massachusetts Institute of Technology. All essential apparatus including gyro equipment, radio, and the servo mechanism had been initiated and some of it was under construction.

At about this stage of the development, the Gulf Research and Development Corporation came into the project under a contract with the National Defense Research Committee. It was realized that a better method of experimentation in the field was necessary. Motion

picture cameras were mounted in the nose of the bomb and recorded what the bomb saw during its fall. Clockwork mechanisms were provided and were pre-set to apply controls for predetermined periods during the drop. The bomb was "exploded" near the end of its flight so that the various parts of apparatus were carried to earth on parachutes.

As soon as an adequate flare was made available, another experimental method was used very extensively, namely to take motion pictures of the bomb as seen from the plane. The motion pictures taken from the plane and those taken from the nose of the bomb could be combined to provide material for a very satisfactory analysis of the actual trajectory. These analyses showed how the bomb responds to the control surfaces, how large the control surfaces needed to be to get a certain over-all result, what angle of deflection was necessary, how the bomb responded to aileron control, how much yaw or pitch was involved in the application of controls, the period of oscillation of the bomb both in yaw or pitch and in rotation, the rate at which the bomb responds to control and many other details that were necessary for the solution of the problem.

Early tests with television and with direct vision controls in azimuth and range (Razon), and also in azimuth only (Azon) were made at Eglin Field, Florida. By this time the essential elements of the bomb control equipment had all been included and some study of the aerodynamics had been made by the workers at M.I.T. The completion of the development of practical experimental units, both Azon and Razon, was done by the Gulf Research and Development Corporation, as was also the extensive and elaborate field tests, wind tunnel tests, analyses and calculations of trajectories that followed and that were necessary to determine aerodynamic and other characteristics.

A primary component part of the bomb equipment in direct sight missiles is the radio receiver, which in the experimental units consisted of a super-regenerative radio amplifier arranged to select functions by means of a code.

The gyro equipment consisted of two gyroscopes, one called a free or directional gyro and the other a rate gyro. The free gyro was used to return the bomb from any position at which the rudders were not vertical and the rate gyro was used to keep the speed of rotation of the bomb below a certain value. It was found that if the directional gyro were used alone, the bomb would return to its normal position with such high angular velocity that it would over-shoot and go into violent oscillations. With the rate gyro cooperating it was possible to bring the the bomb back to its normal position at a reduced speed so that it would not over-shoot by more than about three degrees and immediately settle down into its normal position. This gyroscope assembly was developed in its original form by M.I.T. and was very much improved

by the Gulf Laboratories. In its experimental form the gyroscopes were driven by air, being connected to the vacuum system of the plane.

At this stage it was realized that it would not be possible in a reasonable length of time to develop all the bombs that had been started. The Chief of Division 5, N.D.R.C., therefore called a meeting in February 1943 to discuss this question and to decide which method of attack should be used. The meeting was attended by many representatives of the Army Air Forces and all the members of Division 5 and Section 5.2 N.D.R.C. The Chief of Section 5.2 recommended that for the present we concentrate our attention on the direct vision method of control and apply it to azimuth only. He was alone in his argument

FIG. 1 1000 pound Azon (VB-1)

for this program and the Air Force representatives and the aerodynamicists present were pessimistic about the possibility of getting adequate control by the method that was suggested. After a long discussion it was decided that the work on the direct vision bomb should be discontinued and the work on the homing bombs should be pushed. The reason for this was that it was thought humanly impossible to guide a missile that travels as fast as a high angle bomb does during the last part of its flight. On the other hand it was believed that if the apparatus could be made to report automatically to the control equipment, all of which is in the bomb itself, the results would be more easily attained.

Following this meeting and while preparations were being made for a more complete test of target seeking bombs, the Azon tests were continued in order to get additional aerodynamic data. Two months later, after having tested 13 Azons, there had been obtained some very satis-

factory movies taken from the airplane showing the performance of an Azon in flight, and a strike on a road target 12 ft. wide. Bomb No. 13 in this series was controlled all the way to the ground, and the motion pictures showed that by watching the flare it was very easy to tell when the bomb was to the right or to the left of the road, and the effects of application of control were very evident It had been found that from 15,000 ft. the bomb could be deflected from its normal path by 2,500 to 3,000 ft The motion picture was taken to Washington and shown to

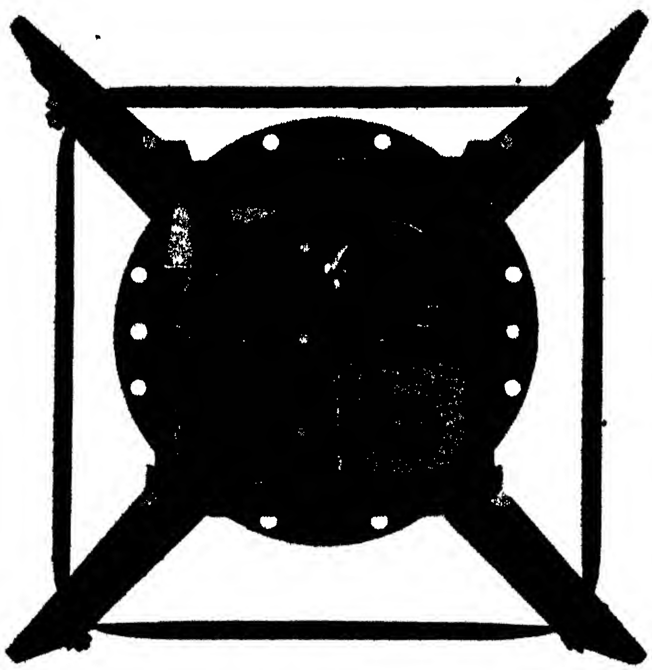

FIG. 2 Leading end of tail structure showing radio receiver gyro and battery

a number of interested Army officers in the Pentagon building. After seeing this performance, there was a complete reversal of opinion and a demand for a crash program to get Azon into production. Azon had been recommended for use on roads and bridges, and it was obvious from the experiment that it would be a very effective weapon against such targets.

The engineering and production was assigned to the Union Switch and Signal Company, and the changes from the experimental unit that were necessary for production and for use in the field may be briefly stated as follows:

(1) The body of the fin structure was changed from a round section to a square section and to welded construction (Fig. 1).

(2) The gyroscope unit was changed from air-drive vacuum-operated to electric-drive (Fig. 2).

(3) The battery was changed from 12 volts to a smaller 24-volt battery using the NT-6 battery which had been developed by the Willard Battery Company for the Navy for another purpose.

(4) Solenoids for operating the ailerons were changed to eliminate much mechanical linkage and to provide individual adjustment (Fig. 3).

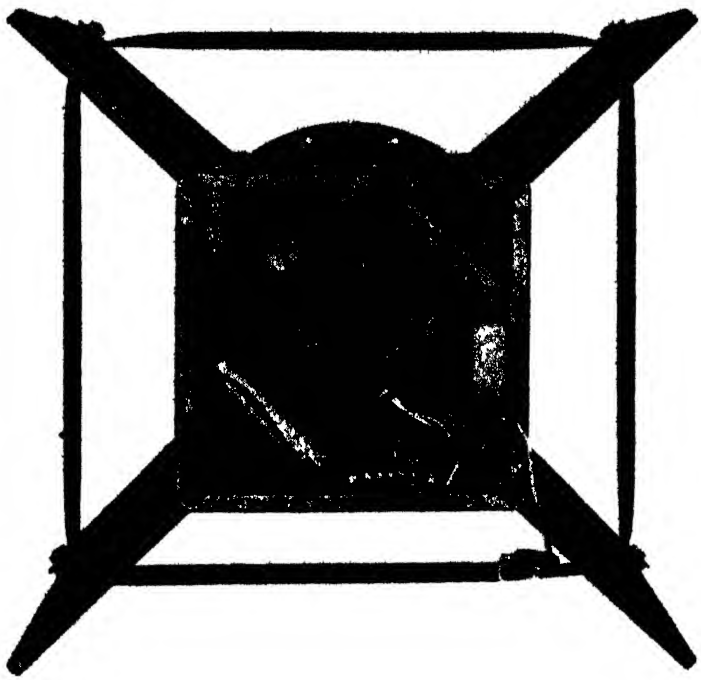

FIG. 3. Trailing end of tail structure showing solenoids and servo motors.

(5) Radio. In the production units the selection of functions was made by means of audio frequency modulations instead of by means of the long and short pulses. The receiver was later changed to a superheterodyne.

(6) The struts which were installed to stiffen the fins were used also as antennas. They were streamlined and insulated and were brought inside the circumscribed square.

(7) The servo motors were wound for 24 volts.

(8) The circuits were modified by adding a kick-out switch through which the equipment could be warmed up and through which the battery could be charged during flight, and by means of which all the circuits could be transferred to the bomb's battery when the bomb was

released. Flare ignition was added to the electrical circuit and the ignition was made by means of a squib instead of with a match. A protective relay was added which made it impossible to ignite the flare until the bombardier had thrown a switch, which was done any time after take-off. An electric gyro release was added to the circuits.

(9) The flare was perfected by the Army Ordnance Department.

(10) The disposition of the various components in the tail structure was changed so that they were all accessible for removal.

One hundred and fifty units of this type were built on a pilot production order. The first of these were delivered November 25, 1943 and were immediately tested at Eglin Field. Many of the units from the pilot production were used in combat.

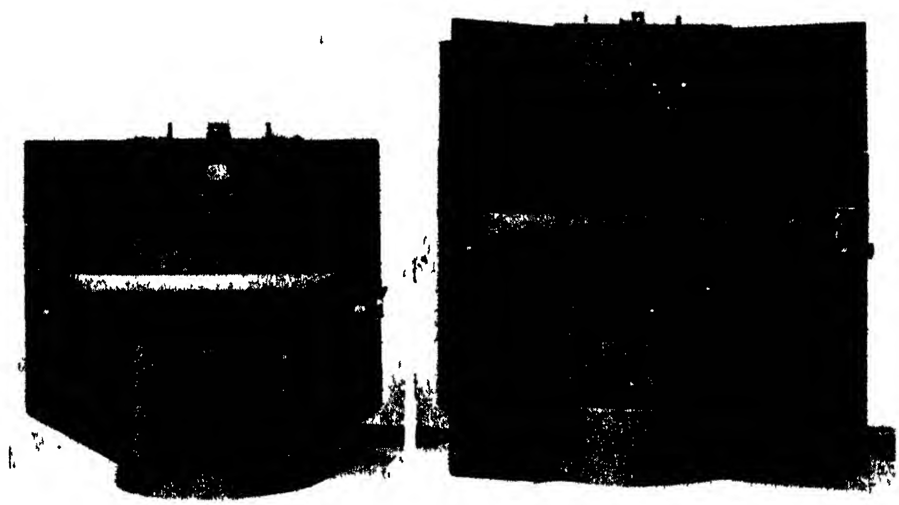

FIG. 4. Relative sizes of a 1000 pound and a 2000 pound (VB-2) Azon tail structure.

During the work on the 1,000 lb. Azon, VB-1, the development had also been completed of a 2,000 lb. Azon, VB-2 (Fig. 4). This unit was given a thorough test by Division 5 and the Air Forces, but was not used in combat.

During and following the work on VB-1, the development of the Razon was completed by the Gulf Research Laboratories. It had been started at M.I.T.

In the case of the Razon it was thought wise to go to a different type of tail construction because of the possibility that the cruciform fin structure had a tendency to produce rotation if the bombs were subjected simultaneously to yaw and pitch, and because of the necessity of getting something that was definitely practical at the earliest possible time. The cruciform structure was not finally abandoned, but was set

aside for the moment on account of the possibility that it might require a more extensive development. The structure that was finally adopted for the Razon, known to the Army as VB-3, had aerodynamic surfaces in the form of two shrouds (Fig. 5). One shroud was mounted at the edge of the structure next to the bomb. The second shroud was mounted with its trailing edge flush with the rear edge of the box part

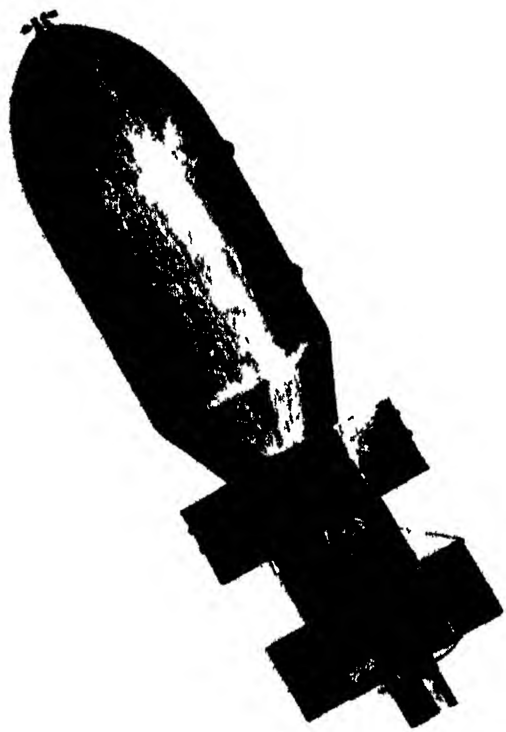

FIG 5. Artist's drawing of 1000 pound Razon (VB-3)

of the fin structure. The former shroud may be called a lift shroud and the latter a control shroud.

The control surfaces were built as flaps on the trailing edges of the long sides of the control shroud. The ailerons were built into the struts supporting the control shroud and were four in number. Except for the changes necessary to control in two directions, the apparatus contained in the apparatus compartment was the same as in the Azon (Fig. 6).

The development of the Razon was completed as were also the field tests. After providing certain corrections that were necessary in the

use of the sighting equipment, the errors in drops from an altitude of 15,000 ft. were reduced to approximately 45 ft. in range and 10 to 15 ft. in azimuth. The Razon was not used in combat. At the end of the war it was in production at the plant of the Union Switch and Signal Company, and 3000 units were made for experimental use by the Army Air Forces.

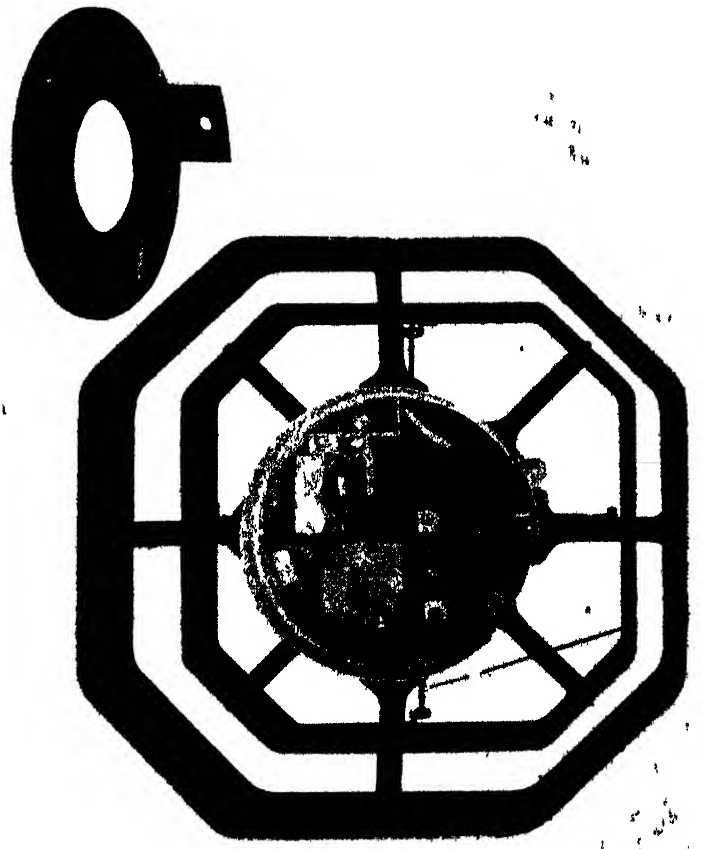

FIG. 6 Production model of VB-3 apparatus showing shrouds, radio, gyro, and battery

Since the components that had been developed for the Azon (Known as VB-1 and VB-2) and their method of mounting were also used in the experimental Razon (known as VB-3), very little extra engineering was required for production.

(In the oral presentation there were shown moving pictures of tests and combat performance of the bombs, and a demonstration of the Azon apparatus.)

The total construction of high angle guided missiles by the Union Switch and Signal Company was as follows:

VB-1	15,271
VB-2	132
VB-3	3,150
Total	18,553

At the end of the war there were on order an additional 10,000 VB-1's and 20,000 VB-3's. These orders were cancelled.

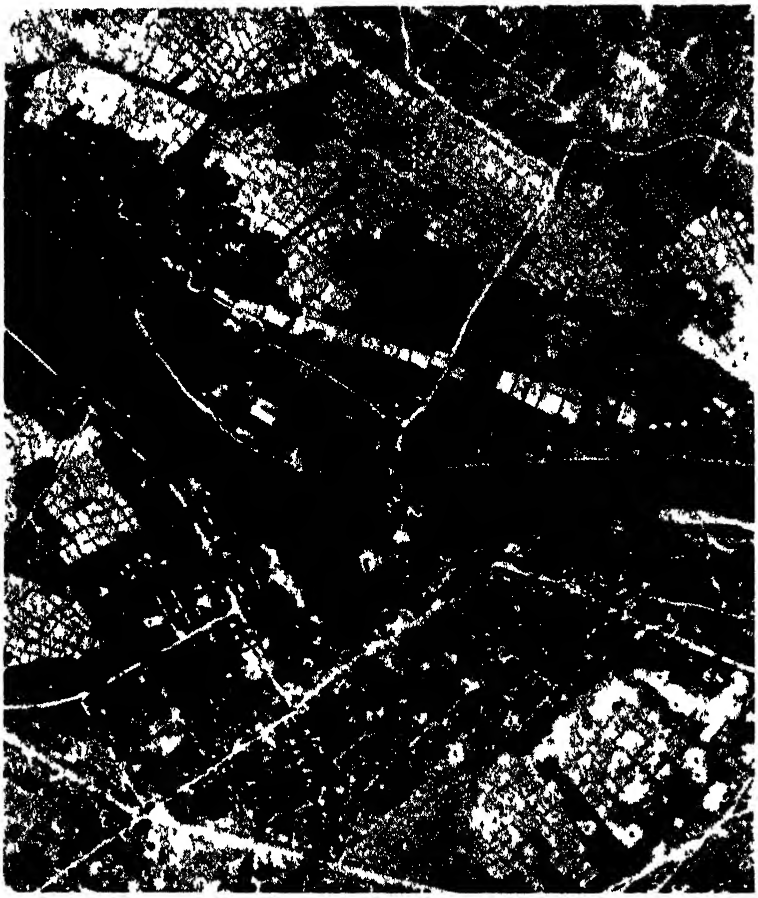

FIG. 7. Azon hits on a bridge in Burma theatre.

The first delivery of regular production models was made March 27, 1944.

A squadron of B-17's took the Azon into combat in February, 1944, in Italy, three and one-half years after the initiation of the research work. These were pilot production units and the 15th Air Force made

the first applications. During the Italian campaign some bridges were destroyed, and at one time the viaduct leading to the Brenner Pass was put out of service.

The next use of the Azon was made in France by the 8th Air Force operating from England. Here also a number of bridges were destroyed, specifically bridges on the Loire and the Seine, and the members of the group that were using the Azon were enthusiastic, but in the latter half of June, 1944, conditions at the front changed so that there was no further need for that type of bombing, and the unit that had been using the Azon was assigned to another combat duty.

The really effective use of the Azon was made in Burma by the 10th Air Force. Here there were not only adequate equipment and trained crews, but also Army representatives and civilian representatives who were well trained in the use of Azons and who knew their capabilities. Results in Burma were very gratifying. Approximately 150 bridges were destroyed. Sometimes a considerable number were destroyed in one mission. The result was a complete disruption of the Japanese system of communications in this theatre.

The increase in effectiveness of high angle bombing as applied to long targets that results from the use of Azons instead of standard bombs is illustrated by the fact that the number of bombs required to destroy a bridge was reduced from 300 or 400 of the standard bombs to six or seven Azons. Illustration of the effectiveness is shown in Fig. 7. In this case four planes dropped one Azon and one standard bomb apiece, each bombardier steering his own Azon, and it will be seen that of the four Azons three hit the bridge. The fourth landed near the end of the bridge. The standard bombs may be seen at various considerable distances from the target.

To have had a part in such a development was a great privilege, the opportunity to tell you about it has been a great pleasure and honor, and the interest that you have shown has been very gratifying and much appreciated.

Thank you.

Engineering at the University of Pennsylvania during the summer of 1943. Ultrasonic delays, particularly for use in radar, were subsequently employed in this country, England and Germany.

Radar systems, designed to employ pulse-to-pulse cancellation, had need for a means of faithfully delaying microsecond pulses for times of the order of 10^{-3} sec. In England, H. Grayson at TRE developed a water line satisfactory for this purpose. At Radiation Laboratory, work was initiated around the end of 1943 to investigate the possibilities of a mercury line for a similar use. In the course of time, as various projects presented their own specifications, each conditioned by their particular problems, the work broke up into separate programs and several small groups of individuals went ahead on more or less parallel lines of development.

The original research was carried out by G. D. Forbes and H. Shapiro. Mercury delay lines, built according to their design, gave very faithful pulse reproduction at the required delay items of the order 500 to 1,000 microseconds. These units were incorporated into an experimental radar system where their successful performance gave a great impetus to the whole program. Forbes and Shapiro investigated other possibilities such as solid rods, narrow tubes, etc. Shapiro continued with research on mercury tanks and on transmission through wires while Forbes later collaborated with P.R. Bell in designing a mercury delay line for a radar system.

Following the successful experimental system several different radar systems were developed which incorporated the ultrasonic mercury delay line as an essential component. The units had to be engineered to withstand a wide variety of field conditions and were installed in trucks, planes and ships. The length of the temporal delay varied from one application to another from roughly 500 to 3,500 microseconds. The length of delay had in turn a bearing on the choice of carrier frequency. All this required considerable development and some research. As an example of the latter, it became important to measure the velocity of mercury as a function of temperature to a high degree of accuracy. These measurements were performed by R. I. Jacobson.[2]

In general a well-designed delay line should possess the following characteristics: (a) rugged mechanical construction, (b) accurate preservation of pulse shape, which requires adequate bandwidth, (c) no accessory signals, (d) moderate attenuation, and, for some applications, (e) accurately controlled delay.

B. DEVELOPMENTS AND TECHNIQUES.

1. Capillaries and Wires.

Much of the early research was directed toward using capillaries or wires to transmit the supersonic pulses but these efforts met with

[2] Radiation Laboratory Report 745: "Measurement of Supersonic Velocity in Mercury at 15 Megacycles per Second as a Function of Temperature."

very little success. As the saving of space was always desirable and particularly so in the case of the longer delays, the possibility of using a coiled delay line appeared an inviting prospect. It is known from work at lower frequencies that it is possible to send an acoustic beam around a moderately curved tube if the wave length is comparable with tube diameter. For this reason, it was natural to try mercury in a curved capillary. It was found that the pulse could traverse several bends in such a capillary with little distortion, but that the attenuation was excessive.

Properties of thin rods and wires when used to transmit supersonics will be only briefly mentioned here. Transmission through solid rods involves many difficulties. One of the most troublesome effects is the appearance of accessory signals caused by the excitation of transverse and surface modes at the surface of the rod. This effect can be reduced by roughening the sides of the rod or increasing the diameter. If, on the other hand, one decreases the diameter with a view to approaching the condition of the coiled wire, attenuation increases and pulse shape becomes distorted.

2. Mechanical Construction and Assembly Technique for the Laboratory Delay Line.

The delay lines built for the original experimental system were successful in providing broad bandwidth, faithful waveform reproduction, accurate alignment and freedom from accessory signals for delays of the order of 500 microseconds at a carrier frequency of 15 Mc. In addition, no deterioration of their performance was noticed with time.

To achieve the required bandwidth mercury was used for the transmitting medium and cells filled with mercury were used as electrodes in back of the crystals. The high acoustic impedance of the mercury broadens the crystal's response, as has already been shown (I, Sec. B). Many of the original lines were constructed from textolite, since it was easily machined. An example is depicted in Fig. 1, which shows clearly the mercury cells backing the crystals.

The purpose of these cells was to eliminate accessory signals which would otherwise arise from multiple reflections at the crystal surfaces. Since the crystals were usually used at resonance, they acted as half wave sections of nearly lossless transmission line. Consequently, for the same material on both sides of the crystal there was practically no impedance mismatch and the supersonic beam was transmitted into the end cell. There the beam struck the end glass plate and was completely dispersed by multiple reflections inside the end cell so that no fraction could emerge in phase to give an accessory signal at a later time.

Because of its superior mechanical and electrical properties, cold rolled steel tubing was used in constructing later models. The impedance mismatch between the quartz and the lacquer layer employed to

make it adhere to the steel appeared to be sufficient so that no perceptible accessory signal was transmitted through the steel. The alignment of the crystals at the end of the main tube was facilitated if the ends were machined parallel to each other. This could be done with a parallelism of better than several hundredths of a degree in a good lathe by placing arbors in the ends of the line and mounting the tube accurately on the lathe centers. With the cutting tool one end of the tube was faced off and then, without removing the tube from the lathe, or readjusting its position, the tool was moved to the other end of the tube to face that end also.

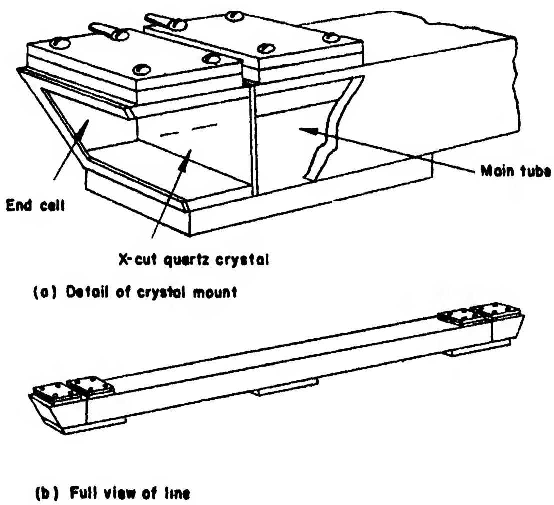

FIG. 1. Liquid delay line for experimental system.

Cleaning is an important preliminary to the assemply of mercury lines as scum caused by dirt or grease in the mercury will adversely affect performance by causing excessive attenuation. Material delivered from the shop was carefully cleaned by immersion in or filling with some organic grease solvent. The opertion was to be repeated several times, the final rinse being done with unused solvent.

The crystals used in this work are usually thinner than 0.3 mm. in thickness and some care is required in their handling. They are generally waxed to plate glass by their manufacturer before delivery. A gentle heating will suffice to melt the wax and the crystal can then be slid off. Before mounting it should be cleaned carefully, again with grease solvent, to remove all wax and dirt.

To assemble a steel line (see Fig. 2), one crystal was lacquered in place across the end of the tube and the lacquer allowed to harden. The second crystal was then applied to the other end and, while the lacquer was still hardening, small adjustments could be made to improve the alignment. The accuracy of the alignment could be accurately

checked by observing how closely a point object lined up with its optical images from both crystals. Next the end cells were similarly lacquered at the end and applied in place to the crystals. Four bakelite straps held the end cell to the main tube. To maintain the rigidity of the

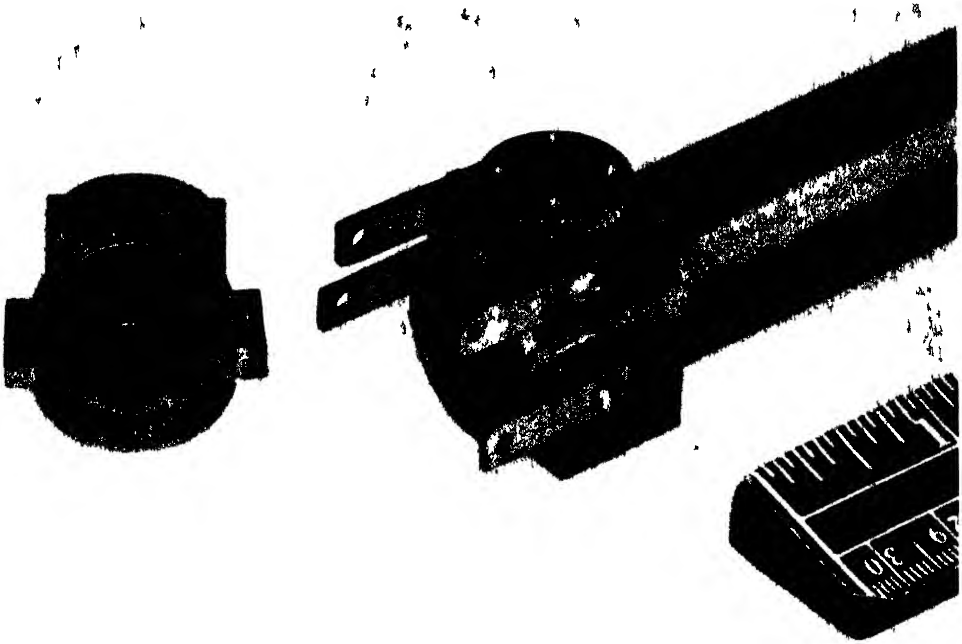

FIG 2 Delay line before assembly

line, it was mounted on a flat length of steel channel which had previously been strain-relieved by heat treatment.

3 The Elimination of Accessory Signals

Though the mercury end cell described in the preceding subsection proved to be satisfactory for a laboratory delay line or use in an experimental system, the quartz crystal supported by mercury on both sides was liable to fracture under shock. When the crystal mounting with absorbing backing was put into field use, some provision had to be made to prevent transient differentials in pressure from breaking the crystal. If the fluid on both sides of the crystal completely filled its container so that there was no air space into which it could be driven by sudden shock, equality of pressure would be maintained under mechanical test but not under thermal variation. The problem was to a large extent solved by inserting a coiled, steel capillary (Fig. 3), which acted as a connecting link between the completely filled chamber in the line and a partially filled chamber in the loading plug. The short time transients

were damped by the viscous forces in the capillary walls but slow thermal variations altered the mercury level in the cap chamber and produced no net pressure change.

Later some modifications in absorbing backings for crystals were introduced. A steel backing piece was used with a deeply cut, saw-

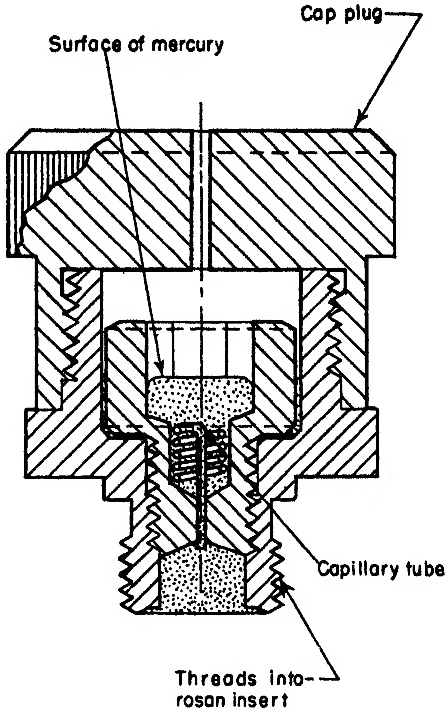

FIG. 3. Mercury inlet with capillary loading.

toothed surface (Fig. 4). The tops of the sawteeth supported the crystal's back surface and the space between the teeth was filled with mercury. The supporting steel area should be too small to give appreciable reflection and still support the crystal adequately under variations of static pressure as well as sudden shock.

There is another promising possibility for an absorbing backing which has not yet been incorporated in any system application, namely, to use a crystal soldered to a solid material which matches well the transmitting medium. For mercury a good acoustic match can be obtained with a backing of hard lead (6 per cent antimony). Preliminary measurements have shown that the reflected signal is reduced by some 18 db under these conditions as compared to the case when the crystal is supported by a dry electrode. The material has the advantages that it is machinable and readily attenuates the transmitted energy. For soldering a metal to quartz, a metallic film must first be

deposited by evaporation on the quartz. Nickel is used for the first few layers nearest the glass, usually followed by metals of higher vapor pressure. Once the crystal has been soldered to the lead electrode, the latter can be cemented into a dielectric support and incorporated into the end assembly. Some precaution should be taken to prevent the mercury from coming in contact with the solder or the lead. The final step would be to machine the aligning surfaces so that when the end assembly is bolted to the tube, the crystal will be perpendicular to the

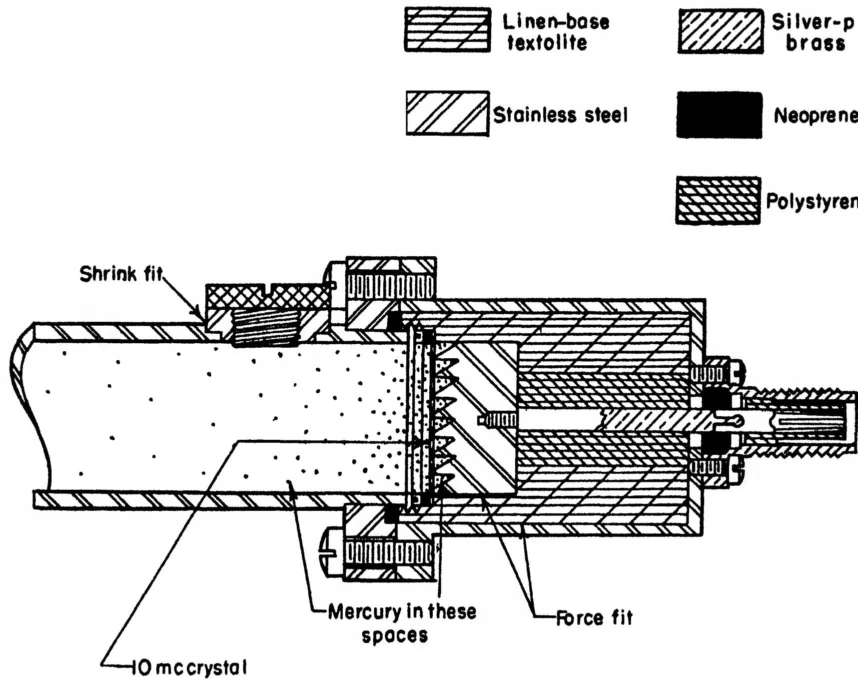

FIG. 4. End assembly with sawtooth backing.

tube axis. For those projects which required longer delays, 1,000 microseconds more, there was little need for the crystal mounting to absorb the incident energy. The attenuation in transit was sufficient in these cases to obliterate any accessory signals that might have been caused by multiple reflections. This meant that the crystal could be supported by dry metal electrodes which give rise to nearly total reflection. The acoustic impedance of air is so small that even the thin layer that exists between the metal plate and the quartz is sufficient to supply nearly complete mismatch. Since it was considerably simpler to support the crystal with mountings of this sort, it became general practice to make use of them wherever possible. To a certain extent, it was possible to control the attenuation loss by transit through the medium by appropriate choices of the carrier frequency and of the tube

size. It has been pointed out in I, Sec. D, that the free-space attenuation in a liquid medium varies as the square of frequency of propagation. There are also losses from the diffraction spreading of the beam and, in a tube, losses possibly arise from the viscous forces at the walls. Such losses depend on the size of the crystals and the inner cross section of the tube.

4. Folded Lines.

Though the longer lines simplify the problem of eliminating the accessory signals, their very length if unbroken make them unwieldly and impractical. It is accordingly a general practice for lines of roughly over 500 microseconds to "fold" the beam on itself one or more times so that the longest dimension is reduced to a convenient size. The folding has been accomplished in many different ways, of which all, nevertheless, have these two common features. First, the reflection of the beam in every case is achieved by two plane reflectors set at 45° with the path of the beam and 90° with respect to each other so that they form a two dimensional corner reflector. Secondly, the various segments of the folded path run through parallel tubes or channels which are interconnected by the reflectors. Figure 8 shows how the folding is accomplished for a line which is discussed in detail in the last section of this part.

With steel reflectors in mercury 45° is outside the critical angle for transmission of supersonic energy into the steel. Nevertheless, losses at the reflector plates are often observed. This matter has been investigated by Mr. H. J. McSkimin of Bell Telephone Laboratories at Murray Hill, New Jersey. He finds that there is negligible loss on reflection from a very smooth surface such as polished plate glass and also a roughened flat surface such as steel ground with 160 mesh carborundum gives no noticeable loss. For surfaces of intermediate roughness, however, a loss can occur. McSkimin explains this effect as the results of phase incoherence in the reflected beam. Since mercury does not wet surfaces with which it does not form an amalgam, small bubbles of air will lodge between the mercury and the steel at any irregularities in the steel surface. That portion of the compressional beam which is reflected from the air surfaces will be reflected with 180° change of phase with respect to the incident beam. The phase change of the part of the beam reflected from the steel had been calculated to be about 80° with respect to the incident beam. If the parts of the beam are nearly equal, then one might expect a loss due to phase incoherence as high as 3 db per reflection. For the very rough surface the reflection is practically all from the air and consequently the phase incoherence is small. McSkimin recommends this surface for actual reflectors as dirt and scum has less effect on this surface than on the polished smooth surface where the reflection is primarily from the solid material of the reflector.

5. Tanks.

The search for a method to package the supersonic beam even more compactly than is done in the folded lines led, naturally, to the investigation of liquid tanks in which the beam could be multiply reflected from the tank walls. The design on paper of such a device proves to be rather an amusing pastime which appears to call for some ingenuity on the part of the designer. Actually with any scheme, one is fundamentally limited by spreading of the beam, since it is important to avoid striking the receiving crystal with even a part of the beam at any time other than at the end of the desired delay time. Scaling up a particular two-dimensional pattern would give a delay that is proportional to the linear scaling factor. However, a larger tank permits a larger crystal which then reduces the diffraction spreading of the beam. This permits a more complex pattern involving a greater number or reflections. It turns out that the volume needed to contain a given delay path is roughly proportional to the length of path. (This assumes the depth of the tank is fixed.)

In regard to crystal mounting it is possible to use absorbing, mercury-backed cells similar to those used in the experimental lines, and to have them immersed in the transmitting fluid. The advantage of the latter construction is that it can be rotated to obtain exact alignment. Its disadvantages are that it occupies valuable space and is difficult to make leak-proof. If the required mechanical tolerances can be met, it is preferable to use external crystal assemblies bolted to the walls of the tank. Actually the tanks do not offer very promising possibilities and work on them at Radiation Laboratory never passed the experimental stage. In the first place, their performance was not predictable as the mechanism causing loss at reflection was not well understood. (The work on tanks preceded McSkimin's investigation of reflections in mercury.) Secondly, the mechanical problems in alignment and leak-proof construction appeared formidable. Lastly, the net gain in space and weight did not appear very considerable. It seems that by increasing the number of corner reflector and parallel channels, one could use the folded line technique to get within a factor or two of the space required by a tank. With most liquids it is true there would be a considerable saving in weight, but with mercury the tank would probably be heavier than the folded line.

If one should, nevertheless, decide to construct a mercury delay tank on the basis of what is now known something like the design in Fig. 5 might serve as a basis. The tank would be rectangular with inner dimensions na by $(n + 1)a$. The total delay path would be $\sqrt{2}\,a$ $[n(n + 1) - 1]$ and the final beam spreading should be considerably less than $a/\sqrt{2}$. All reflections would be at 45° and should give no serious loss if the walls are flat and roughened. The depth of the tank

in most cases should not greatly exceed the diameter of the crystals, as this would help decrease the loss from beam spreading.

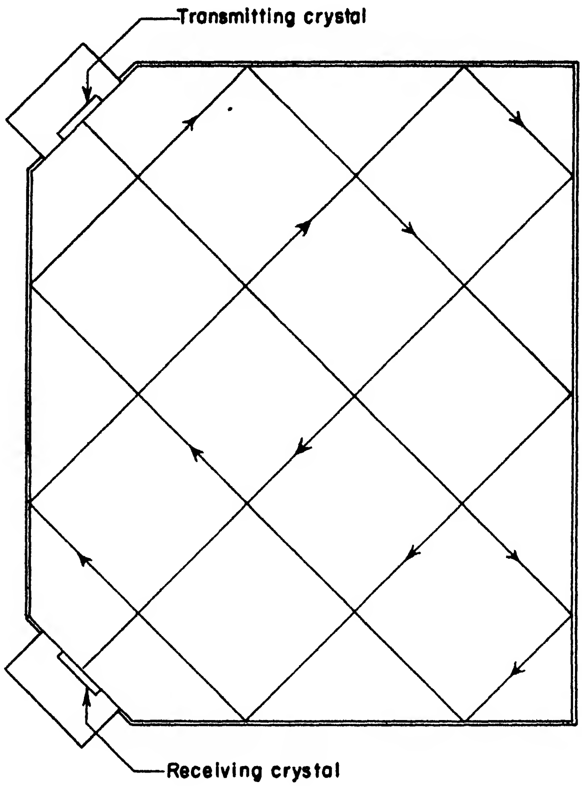

FIG. 5. Design for liquid delay tank.

C. DESIGN CONSIDERATIONS.

If one is asked to construct a line which is to delay pulses of a given duration and rise-time for a given time, one has the following design parameters at his disposal: transmitting medium, carrier frequency, crystal fundamental frequency, crystal and tube size, number of channels and reflectors, and required mechanical tolerance. An attempt will be made to discuss the factors which influence the choice of the various parameters and to point out the interrelations involved.

In choosing the transmitting medium one is limited to those with moderately low attenuation. Highly viscous liquids are immediately ruled out.

The next consideration is the elimination of beam distortion arising from thermal gradients in the medium, since the acoustic velocity is generally a function of temperature. Little trouble is experienced with mercury since its heat conductivity is high and its thermal coeffi-

cient of velocity rather low, about 3 parts in 10^{-4}. With water and most other liquids, the effect is very serious, unless the medium is held at a temperature for which the thermal coefficient of velocity vanishes. Measurements [3] at the Bell Telephone Laboratories have shown that the acoustic velocity in water goes through a maximum for 74° C. and that many water mixtures have similar maxima at lower temperatures. Such media held at their temperature of maximum velocity will give constant delay. In addition, the attenuation of water decreases with rising temperature.[4]

Finally, the acoustic impedance of the transmitting medium has an important influence on the pass band of the transducer (see I, Sec. B). For applications requiring broad bandwidth the large acoustic impedance of mercury gives an immediate solution. It is possible, however, to obtain comparable bandwidth with a medium of low acoustic impedance if the crystal is backed by a material of high acoustic impedance, such as mercury or a soldered metal electrode. One obtains in this case a broad, double-humped passband (see Fig. 7, Part I) which is not likely to cause much distortion in the passband of the transmitter. However, the voltage ratio is decreased through mismatch in such a scheme by about a factor of Z_1/Z_2 (see eq. (15) of I, Sec. B) as against the case where the material of high acoustic impedance is used for the transmitting medium. For the case of a water line with mercury-backed crystals, $Z_1 = 1.5 \times 10^5$ and $Z_2 = 19.8 \times 10^5$ and the added loss amounts to about 22 db.

With the medium chosen, the range of the carrier frequency is clearly indicated. It must be chosen so that the transit attenuation will not be excessive but, if reflecting backings for the crystals, the attenuation must be large enough to absorb the accessory signals. Usually about 20 db one way is sufficient. One must bear in mind that a high carrier frequency means thinner crystals, more critical alignment and more severe mechanical tolerances. Lower carrier frequencies involve greater spreading of the beam and increased dispersion in velocity between the different frequency components of the signal pulse (I, Sec. C). In addition, where one wishes the pulse *envelope* to be faithfully reproduced regardless of phase, one requires a certain number of cycles per pulse. This involves a dependence of carrier frequency on the duration of the pulses to be delayed.

It has already been pointed out in I, Sec. B that to drive a crystal at an odd harmonic of its fundamental frequency generally increases the voltage loss and this procedure should only be employed for those frequencies at which a crystal operating on the fundamental would be too thin for manufacture or convenient handling.

[3] G. W. Willard, *J. Acous. Soc. Am.*, **19**, 1 (1947).
[4] C. E. Teete, Jr., *J. Acous. Soc. Am.*, **18**, 488 (1946).

On the other hand, the tolerance on crystal thickness can be very loose if the crystal is operating into a medium of high acoustic impedance such as mercury. Crystals whose fundamental frequency lay outside the passband of the transmitter have been used satisfactorily in the mercury delay lines.

The size of the crystal diameter is primarily tied up with the problem of minimizing voltage loss through mismatch. It has been shown in I, Sec. B, that, for low impedance driving and receiving circuits, the minimum voltage loss for constant bandwidth occurs when the live capacity across the excited area of the crystal is of the same order as the stray capacity at the crystal when the latter has been reduced to a minimum. At present there is no known reason to have the inner cross section of the tube differ from that of the active area of the crystal. With smaller crystals and tubes, the beam spread is greater and the tube boundary conditions become more important. The effect of tubular attenuation is increased and also whatever pulse distortion may exist from velocity dispersion (I, Sec. C).

The number of channels and corner reflectors depends on the need for reducing the long dimension of the delay line. It is primarily a problem in packaging and utilization of space. The introduction of reflectors and additional channels complicates the manufacture and increases the need for accurate machining of individual pieces. It decreases, however, any difficulty that may be experienced with bending or sagging of the line.

The over-all mechanical tolerances are in effect determined when the medium and carrier frequency have been chosen. It is the ratio of wave length in the transmitting medium to crystal diameter which determines allowable tolerances. If $\sin\theta = 1.22\,\lambda/d$, then something like $0.2\,\theta$ is a permissible over-all tolerance. Where the final alignment of the crystal depends on the net result of several individual tolerances, such as is the case with many corner reflectors, the respective individual tolerances have to be proportionately more severe. For a corner reflector, for which the reflecting surfaces are accurately perpendicular, it is only necessary to align the reflector in the angle at right angles to the plane of the reflection.

D. AN EXAMPLE OF DELAY LINE DESIGN.

A line which was designed for 3,300 microseconds delay has been chosen as an example to illustrate some of the points discussed in the preceding section. The crystal end assemblies are shown in Figs. 6 and 7. Fig. 8 shows the corner reflector.

The line is variable in length with a variation of roughly 1 in. possible (about 17 microseconds at ordinary temperatures for mercury). This variation is made in the end assembly shown in Fig. 6, which was largely designed by Mr. P. C. Bettler. In this end assembly, the long

shaft holding the crystal, which protrudes into the end block, can be made to travel by means of a screw having 16 threads per inch. The mercury is retained by a packing nut and packing gland, consisting of thread or linen cloth impregnated with ceresine wax, a kind of paraffin.

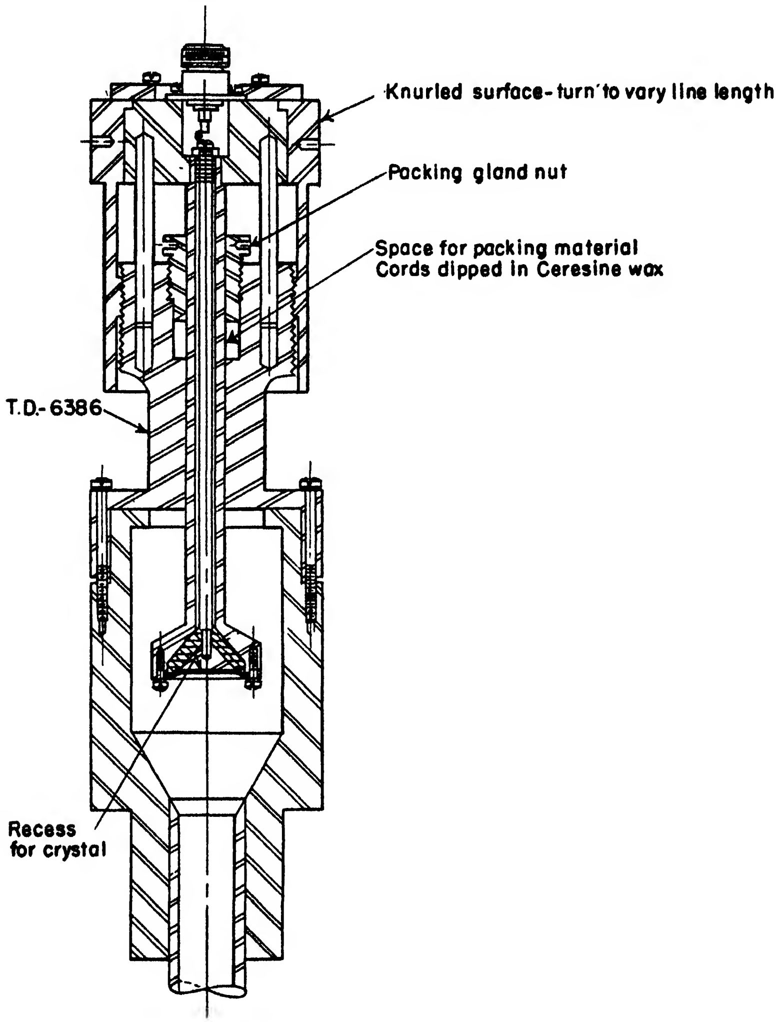

FIG. 6. Variable end assembly.

Ultrasonic 10 megacycle waves leaving the crystal of the adjustable crystal holder assembly travel to the reflector block, where they are twice reflected through 90°, and then continue back to the fixed crystal holder (Fig. 7), where they are picked up by a crystal similar to the transmitting crystal. These crystals are backed with a conical piece of stainless steel insulated by Bakelite from the rest of the crystal mount assembly.

The two crystal mount gaskets are made of polythene, which keeps the mercury from leaking. The long tubes are shrunk into the end blocks and reflector block. It is found that no mercury leaked when the

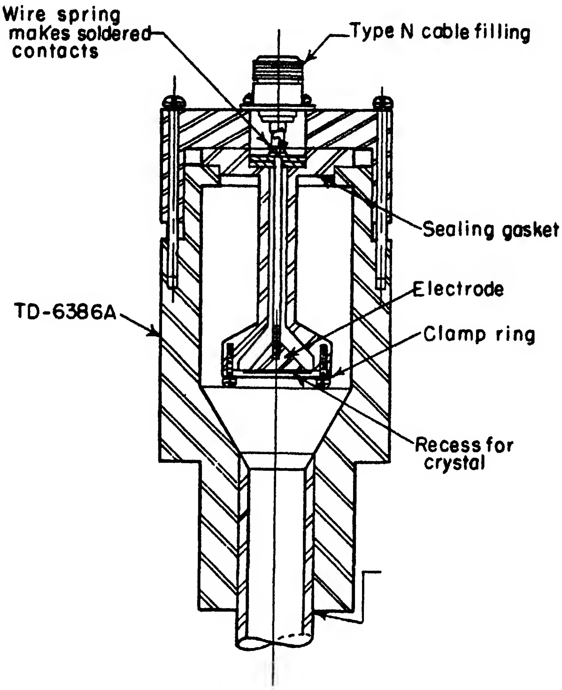

FIG. 7. End assembly with clamp ring.

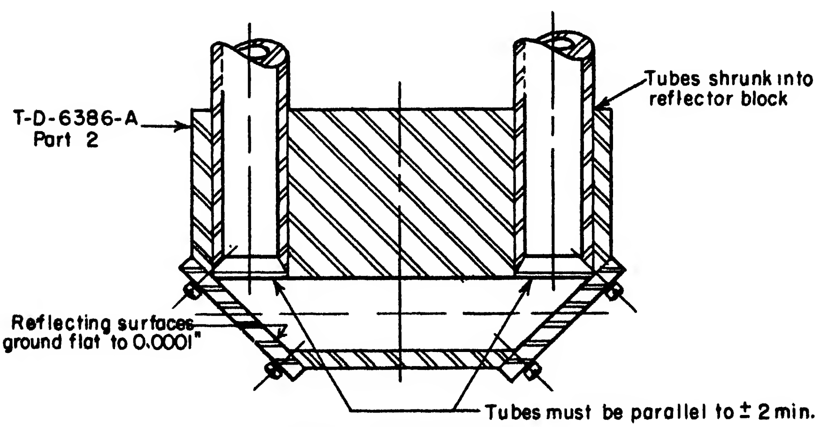

FIG. 8. Reflector block.

line is in a horizontal position, as it always is when operating. However, during filling, when the line was on end, small leaks were sometimes found at the shrunk fits mentioned above, since the mercury pressure is as high as 3 atmospheres at the reflector block (Fig. 8).

The end blocks are not completely filled with mercury; a volume of air of 3 cu. in. is left in each to allow for the expansion of the mercury should the temperature increase greatly in the field.

The constants for this delay line are as follows:

Capacity of adjustable crystal holder assembly	53 $\mu\mu$f
Capacity of (fixed) crystal holder	37 $\mu\mu$f
Overall attenuation (using an output resistor of 1,000 ohms)	60–70 db

The attenuation varies from line to line, depending mostly on the alignment, which is critical. The bandwidth of the line was determined by values of the capacities quoted above and their associated resistors. The bandwidth of the quartz crystal-mercury-quartz crystal system taken by itself is more than 6 megacycles broad.

In the actual field application four such lines are packed into parallel compartments of a special containing box. One of these lines is used to establish the repetition rate for the repeating pulse pattern. Two of the other lines were adjusted to give delays synchronizing with the repetition interval. The fourth line was used as a spare. Because all lines were in the same thermal environment, it was seldom necessary to make adjustments to maintain synchronization.

Three-Dimensional Inspection of Castings.—Inspection and layout of castings by a three-dimensional method of projecting accurate layout drawings upon a rough casting has been developed by G-E engineers at the Pittsfield Works and has seen several years of successful application to thousands of intricate ferrous and nonferrous castings. This method has effected considerable savings in time and material. Moreover, an unskilled operator with but a few days training can employ the method, whereas the usual bench layout requires inspectors familiar with machining practice.

Originally designed to inspect and lay out parts for subsequent machining, this method has also been used for a rapid inspection of finished parts. Projection may also be employed during an actual machining process, whether the part to be machined is stationary or revolving. The equipment assures layout within fifteen thousandths of an inch.

The installed apparatus consists of a layout image projector containing a photographic glass slide of the finished casting layout. The layout is projected by a lens directly upon the surface of the casting. To establish the plane of true projection and correct dimensions, a second projector—the inspection plane projector or light wand—over the inspection position is used. The light wand emits a sheet of light which falls vertically upon the casting.

Both projectors operate in unison through a selsyn system. Thus, the light wand always designates the proper plane for the main projector's inage as well as its correct focus and size.

The inspection pedestal assists in positioning the rough casting by means of an electric drive. An adjustable surface plate allows for the final location of the casting coincidental to the sheet of light from the inspection plane projector. Automatically, the finished layout will be projected properly on the rough casting.

When machining a piece which revolves and is symmetrical about the axis of rotation, the finished outline may be projected to serve as a template.

This template-projection system may be used in fabricating duplicate metal parts for tanks, boilers, and other burned-out and welded pieces. Variations of this system can be used for laying out intricate pieces, locating parts to be welded, shearing, and general layout operations.

To make this three-dimensional method practical, drawings of finished castings, which are suitable for photographing, are prepared. The size and focus of the drawing image on the ground glass of the camera are checked by microscope for accuracy.

In comparison to this three-dimensional inspection system, bench layout procedure required a highly skilled layout man. A reference surface was selected. Processing across or around the casting, each machinable area was checked for location and size with suitable tools, such as height gage, steel rule and scriber. Only by laying out the entire casting could the layout man be sure that all the machinable surfaces were properly located.

R. H. O..

AN INTEGRAL-EQUATION APPROACH TO PROBLEMS OF VIBRATING BEAMS.*

BY

WALTER T. WHITE, Sc.D.†

PART II.

5. CONSTRUCTION OF GREEN'S FUNCTION.

The normal modes of a vibrating beam depend on its *mass* and *stiffness*. For one-dimensional continuous beams, the mass and stiffness are each functions of a single variable along the length of the beam. The mass function is calculated from the physical specification of the beam while the stiffness function is specified by a *Green's function*. The Green's function gives the deflection at any point of the beam caused by application of a unit-concentrated load at any other point. The superposition principle is used with the differential equation of the beam for the construction of Green's functions.

For the freely vibrating beam, the partial differential equation is

$$\frac{\partial^2}{\partial z^2}\left(EI\frac{\partial^2 y}{\partial z^2}\right) = -m\frac{\partial^2 y}{\partial t^2}. \tag{37}$$

In this equation the mass m per unit length and the moment of inertia I are either constants or functions of z. Equation 37 is reduced to an ordinary differential equation by the substitution

$$y(z, t) = y(z)\cdot\sin\omega t. \tag{38}$$

By this substitution Eq. 37 becomes

$$\frac{d^2}{dz^2}\left(EI\frac{d^2 y}{dz^2}\right) = m\omega^2 y, \tag{39}$$

where y is a function of z alone and ω is the natural frequency of the beam in radians per second.

The term on the right of Eq. 39 represents the inertial loading of the beam and is replaced by a distributed loading $f(z)$ per unit length of beam. This substitution gives

$$\frac{d^2}{dz^2}\left(EI\frac{d^2 y}{dz^2}\right) = f(z). \tag{40}$$

* This paper is from a thesis, "Integral-Equation and Approximate Solutions for Normal Modes of Vibration," submitted to the department of Electrical Engineering, M.I.T., 1941.

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For a beam with uniform flexural rigidity, Eq. 40 becomes

$$EI\frac{d^4y}{dz^4} = f(z). \tag{41}$$

The solution of Eq. 40 or 41 is the *static* deflection of the beam caused by the loading $f(z)$.

The Green's function for a uniform beam is constructed by the solution of Eq. 41, with the appropriate boundary conditions, provided $f(z)$ is a *unit*-concentrated load. In the notation of operational calculus, a unit-concentrated load is written as $u_1(z - v)$, where the load is applied at $z = v$. With this substitution Eq. 41 becomes

$$EI\frac{d^4y}{dz^4} = u_1(z - v). \tag{42}$$

The boundary conditions for the cantilever are that the deflection and slope at the fixed end are zero, and the bending moment and shear at the free end are zero. These conditions are expressed by the relations

$$\left.\begin{aligned} y(0) &= y'(0) = 0 \\ EIy''(L) &= EIy'''(L) = 0 \end{aligned}\right\}. \tag{43}$$

The Laplace transformation [7] of Eq. 42 is

$$s^4Y(s) - y(0)s^3 - y'(0)s^2 - y''(0)s - y'''(0) = e^{-v}/EI, \tag{44}$$

where

$$\text{Laplace, } [y(z)] \equiv Y(s). \tag{45}$$

Division of Eq. 44 by s^4 and the inverse Laplace transformation gives

$$\begin{aligned} y(z) = y(0) + y'(0)z + y''(0)z^2/2! + y'''(0)z^3/3! \\ + (z - v)^3u(z - v)/3!\,EI, \end{aligned} \tag{46}$$

where $u(z - v)$ is the *unit function*, defined as zero for $z < v$ and unity for $z > v$. By application of the boundary conditions of Eq. 43, the constants $y''(0)$ and $y'''(0)$ are calculated. Evaluation of the constants gives the Green's function $G(z, v)$ for the cantilever as

$$\begin{aligned} y(z) = G(z, v) &= z^2(3v - z)/6EI, \qquad z \leq v \\ &= v^2(3z - v)/6EI, \qquad v \leq z \end{aligned} \tag{47}$$

This relation is identical with Eq. 2 which is the static deflection of a uniform cantilever caused by a unit-concentrated load applied at $z = v$.

By application of the appropriate boundary conditions, Green's functions for uniform beams with other types of end conditions are constructed from Eq. 46.

The construction of Green's functions for nonuniform beams requires the solution of two second-order differential equations. The

first of these equations gives the bending moment $M(z)$ for a beam as

$$M(z) = EI(z)\frac{d^2y}{dz^2}. \tag{48}$$

Substitution of this relation in Eq. 40 gives the second equation as

$$\frac{d^2M}{dz^2} = f(z). \tag{49}$$

A bending-moment Green's function is constructed from Eq. 48 when $M(z)$ is a *unit*-concentrated bending-moment; that is,

$$M(z) = u_{1m}(z - w) \tag{50}$$

is a bending moment that is unity at $z = w$ and zero elsewhere. Substitution in Eq. 48 gives

$$EI(z)\frac{d^2y}{dz^2} = u_{1m}(z - w). \tag{51}$$

Since $u_{1m}(z - w)$ has a non-zero value only at $z = w$ and $I(z)$ does not vanish along the beam, $I(z)$ is replaced by $I(w)$ and Eq. 51 written as

$$\frac{d^2y}{dz^2} = u_{1m}(z - w)/EI(w). \tag{52}$$

Solution of Eq. 52 by Laplace transformation gives

$$y(z) = y(0) + y'(0)z + (z - w)u(z - w)/EI(w). \tag{53}$$

Substitution of the boundary conditions for the cantilever gives the bending-moment Green's function $H(z, w)$ as

$$\left.\begin{aligned} y(z) = H(z, w) &= (z - w)/EI(w), & w \le z \\ &= 0 \qquad , & z \le w \end{aligned}\right\}. \tag{54}$$

The solution of Eq. 49 when $f(z)$ is a unit-concentrated load applied at $z = v$ gives a bending-moment distribution $M(z, v)$ as

$$\left.\begin{aligned} M(z, v) &= (v - z), & z \le v \\ &= 0 \qquad , & v \le z \end{aligned}\right\}. \tag{55}$$

To construct the Green's function, consider the deflection of the beam caused by an element of bending moment $M(z, v)\Delta z$ applied at $z = w$. For small values of Δz this is a concentrated bending moment and the deflection is

$$\Delta y = H(z, w)M(w, v)\Delta w, \tag{56}$$

where $H(z, w)$ is given by Eq. 54. The deflection caused by the bending-moment distribution of Eq. 55 is the required Green's function and by

integration of Eq. 56 is given as

$$G(z, v) = \int_0^L H(z, w)M(w, v)dw. \tag{57}$$

By substitution from Eqs. 54 and 55 in Eq. 57, the Green's function for the non-uniform cantilever is

$$\left.\begin{aligned} G(z, v) &= \int_0^z \frac{(z - w)(v - w)dw}{EI(w)}, \qquad z \leq v \\ &= \int_0^v \frac{(z - w)(v - w)dw}{EI(w)}, \qquad v \leq z \end{aligned}\right\}. \tag{57a}$$

When the function $I(w)$ is known, these integrals can be evaluated by any of the usual methods.

By application of the superposition principle and the appropriate boundry conditions, the Green's functions for nonuniform beams with other types of end conditions are found by methods analogous to the foregoing.

The Green's functions for naturally twisted beams are constructed by consideration of the coupling between the axes of maximum and minimum flexural rigidity. In this discussion a naturally twisted beam in an unstrained condition has: (a) the ellipses of inertia of all sections so displaced that they remain parallel; and (b) the centers of gravity of all sections along a straight line.

The coordinate system of Fig. 2 is used for the naturally twisted beam. Coordinate z extends along the length of the beam. Coordinates x and y are respectively the axes of maximum and minimum inertiae at the section $z = 0$. Coordinates α and β are respectively the axes of maximum and minimum inertiae at sections along the beam. The angle θ is the rotation of the α and β axes. The positive directions for the bending-moment vectors M_x, M_y, M_α, and M_β are shown in Fig. 2b. The principal moments of inertia are I_α and I_β ($I_\alpha < I_\beta$).

The differential equations for a freely vibrating, naturally twisted beam as reduced from the partial differential equations are

$$\left.\begin{aligned} d^2M_x/dz^2 &= m\omega^2x(z) \\ d^2M_y/dz^2 &= m\omega^2y(z) \end{aligned}\right\}. \tag{58}$$

The terms on the right of Eqs. 58 result from the inertial loadings and are replaceable respectively by static-load functions $f_x(z)$ and $f_y(z)$.

The bending-moment distribution caused by unit-concentrated loads $u_{1x}(z - v)$ and $u_{1y}(z - v)$ applied respectively in the x and y directions at $z = v$ is obtained by solutions of the differential equations

$$\left.\begin{aligned} d^2M_x/dz^2 &= u_{1x}(z - v) \\ d^2M_y/dz^2 &= u_{1y}(z - v) \end{aligned}\right\}. \tag{59}$$

For the cantilever, the solutions of Eqs. 59 are

$$\left.\begin{aligned} M_x(z, v) &= (v - z)u(v - z) \\ M_y(z, v) &= (v - z)u(v - z) \end{aligned}\right\}. \tag{60}$$

The bending-moment Green's functions are obtained from the differential equations for the deflection of the beam in the α and β coordinates.

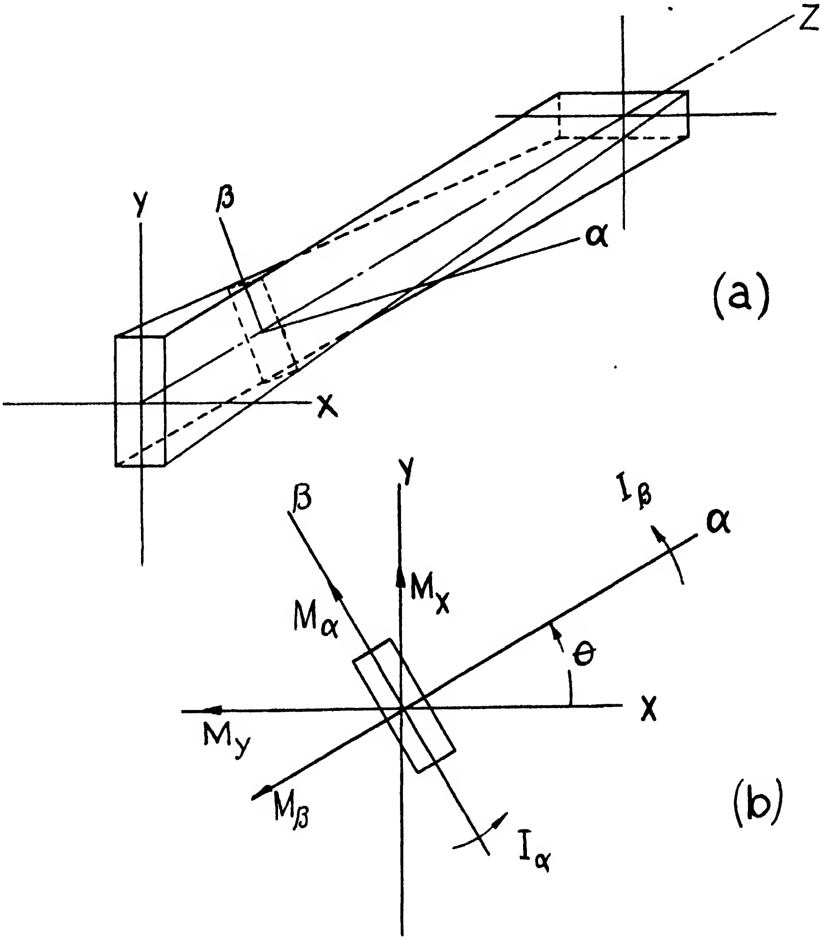

FIG. 2. Naturally twisted beam.

Since these equations are obtained by consideration of the curvature of a differential element, they are not affected by the twist of the beam. The equations for the deflections α and β are

$$\left.\begin{aligned} d^2\alpha/dz^2 &= M_\alpha/EI_\alpha \\ d^2\beta/dz^2 &= M_\beta/EI_\beta \end{aligned}\right\}. \tag{61}$$

These equations are solved for unit-concentrated bending-moments

$$\left.\begin{aligned} M_\alpha &= 1_\alpha u_{1m}(z - w) \\ M_\beta &= 1_\beta u_{1m}(z - w) \end{aligned}\right\}, \tag{62}$$

applied at $z = w$. The boundary conditions for a cantilever with the bending moments of Eqs. 62 are

$$\begin{aligned} \alpha'(0) &= \alpha'(z) = \alpha(0) = \alpha(z) = 0, \qquad (z < w) \\ \beta'(0) &= \beta'(z) = \beta(0) = \beta(z) = 0, \qquad (z < w) \end{aligned} \tag{63}$$

Solutions of Eqs. 61 and 62 for the cantilever are

$$\begin{aligned} \alpha_w(z) &= 1_\alpha(z - w)u(z - w)/EI_\alpha(w) \\ \beta_w(z) &= 1_\beta(z - w)u(z - w)/EI_\beta(w) \end{aligned} \tag{64}$$

The subscript w is attached to $\alpha_w(z)$ and $\beta_w(z)$ to indicate the deflections are in the directions respectively of α and β at $z = w$.

The corresponding x and y deflections are calculated from the geometry of Fig. 2 and are

$$\begin{aligned} x(z) &= \alpha_w(z) \cos \theta(w) - \beta_w(z) \sin \theta(w) \\ y(z) &= \alpha_w(z) \sin \theta(w) + \beta_w(z) \cos \theta(w) \end{aligned} \tag{65}$$

Similarly, the relationships between the bending moments at the point w are

$$\left.\begin{aligned} 1_\alpha &= 1_x \cos \theta(w) + 1_y \sin \theta(w) \\ 1_\beta &= -1_x \sin \theta(w) + 1_y \cos \theta(w) \end{aligned}\right\}. \tag{66}$$

From Eqs. 64, 65 and 66 the deflections caused by unit-concentrated bending-moments 1_x and 1_y applied at w are

$$\left.\begin{aligned} x(z) &= [1_x a_{xx}(w) + 1_y a_{xy}(w)](z - w)u(z - w)/EI_0 \\ y(z) &= [1_x a_{yx}(w) + 1_y a_{yy}(w)](z - w)u(z - w)/EI_0 \end{aligned}\right\}, \tag{67}$$

where

$$\left.\begin{aligned} a_{xx} &\equiv (I_0/I_\alpha) \cos^2 \theta + (I_0/I_\beta) \sin^2 \theta \\ a_{yy} &\equiv (I_0/I_\alpha) \sin^2 \theta + (I_0/I_\beta) \cos^2 \theta \\ a_{xy} &= a_{yx} \equiv (I_0/I_\alpha - I_0/I_\beta) \sin \theta \cos \theta \end{aligned}\right\}, \tag{68}$$

and

$$I_0 \equiv I_\alpha(0). \tag{69}$$

The deflections $x(z)$ and $y(z)$ in Eqs. 67 are the bending-moment Green's functions $II_x(z, w)$ and $II_y(z, w)$, since they are deflections caused by unit-concentrated bending-moments. Equations 67 are written in

terms of *self* and *mutual* bending-moment Green's functions to give

$$\left.\begin{aligned} H_x(z, w) &= 1_x H_{xx}(z, w) + 1_y H_{xy}(z, w) \\ H_y(z, w) &= 1_x H_{yx}(z, w) + 1_y H_{yy}(z, w) \end{aligned}\right\}, \tag{70}$$

where the *self* and *mutual* bending-moment Green's functions are

$$\left.\begin{aligned} H_{xx}(z, w) &\equiv a_{xx}(w)(z - w)u(z - w)/EI_0 \\ H_{yy}(z, w) &\equiv a_{yy}(w)(z - w)u(z - w)/EI_0 \\ H_{xy}(z, w) &= H_{yx}(z, w) \equiv a_{xy}(w)(z - w)u(z - w)/EI_0 \end{aligned}\right\}. \tag{71}$$

The Green's functions $G_x(z, v)$ and $G_y(z, v)$ are constructed from the bending-moment distribution of Eqs. 60 and the bending-moment Green's function of Eqs. 70; they are

$$\left.\begin{aligned} G_x(z, v) &= \int_0^L H_{xx}(z, w)M_x(w, v)dw + \int_0^L H_{xy}(z, w)\, M_y(w, v)dw \\ G_y(z, v) &= \int_0^L H_{yx}(z, w)M_x(w, v)dw + \int_0^L H_{yy}(z, w)M_y(w, v)dw \end{aligned}\right\}. \tag{72}$$

These Green's functions are in a more useful form when separated into *self* and *mutual* Green's functions, where

$$\left.\begin{aligned} G_{xx}(z, v) &\equiv \int_0^L H_{xx}(z, w)M_x(w, v)dw \\ G_{xy}(z, v) &\equiv \int_0^L H_{xy}(z, w)M_y(w, v)dw \\ G_{yx}(z, v) &\equiv \int_0^L H_{yx}(z, w)M_x(w, v)dw \\ G_{yy}(z, v) &\equiv \int_0^L H_{yy}(z, w)M_y(w, v)dw \end{aligned}\right. \tag{73}$$

In terms of self and mutual Green's functions, Eqs. 72 are

$$\left.\begin{aligned} G_x(z, v) &= G_{xx}(z, v) + G_{xy}(z, v) \\ G_y(z, v) &= G_{yx}(z, v) + G_{yy}(z, v) \end{aligned}\right\}. \tag{72a}$$

From Eqs. 60 it follows that

$$M_x(w, v) = M_y(w, v), \tag{60a}$$

and therefore in Eqs. 72a

$$G_{xy}(z, v) = G_{yx}(z, v). \tag{74}$$

The formulation of the integral equations follows from integration of the Green's functions and the inertial loadings of Eqs. 58. Accord-

ingly, the integral equations are

$$\left.\begin{aligned} x(z) &= \omega^2 \int_0^L G_{xx}(z, v)m(v)x(v)dv + \omega^2 \int_0^L G_{xy}(z, v)m(v)y(v)dv \\ y(z) &= \omega^2 \int_0^L G_{yx}(z, v)m(v)x(v)dv + \omega^2 \int_0^L G_{yy}(z, v)m(v)y(v)dv \end{aligned}\right\}. \tag{75}$$

The kernels of these integral equations are placed in a symmetric form by transformations similar to Eqs. 16 and 17. Hence, if

$$\left.\begin{aligned} X(z) &\equiv x(z)\sqrt{m(z)} \\ Y(z) &\equiv y(z)\sqrt{m(z)} \\ K_{xx}(z, v) &\equiv G_{xx}(z, v)\sqrt{m(z)m(v)} \\ K_{xy}(z, v) &\equiv G_{xy}(z, v)\sqrt{m(z)m(v)} \\ K_{yx}(z, v) &\equiv G_{yx}(z, v)\sqrt{m(z)m(v)} \\ K_{yy}(z, v) &\equiv G_{yy}(z, v)\sqrt{m(z)m(v)} \\ \lambda &\equiv \omega^2 \end{aligned}\right\}, \tag{76}$$

the integral equations are

$$\left.\begin{aligned} X(z) &= \lambda \int_0^L K_{xx}(z, v)X(v)dv + \lambda \int_0^L K_{xy}(z, v)Y(v)dv \\ Y(z) &= \lambda \int_0^L K_{yx}(z, v)X(v)dv + \lambda \int_0^L K_{yy}(z, v)Y(v)dv \end{aligned}\right\}. \tag{77}$$

From Eqs. 74 and 76 it follows that

$$K_{xy}(z, v) = K_{yx}(z, v). \tag{78}$$

The orthogonality condition for the pair of integral equations in Eqs. 77 is obtained by consideration of the separate eigenvalues λ_i and λ_j, and the corresponding eigenfunctions $X_i(z)$, $X_j(z)$, $Y_i(z)$ and $Y_j(z)$ of the symmetric kernels. The eigenfunctions are solutions of Eqs. 77; hence

$$\left.\begin{aligned} X_i(z) &= \lambda_i \int_0^L K_{xx}(z, v)X_i(v)dv + \lambda_i \int_0^L K_{xy}(z, v)Y_i(v)dv \\ Y_i(z) &= \lambda_i \int_0^L K_{yx}(z, v)X_i(v)dv + \lambda_i \int_0^L K_{yy}(z, v)Y_i(v)dv \end{aligned}\right\}, \tag{79}$$

and

$$\left.\begin{aligned} X_j(z) &= \lambda_j \int_0^L K_{xx}(z, v)X_j(v)dv + \lambda_j \int_0^L K_{xy}(z, v)Y_j(v)dv \\ Y_j(z) &= \lambda_j \int_0^L K_{yx}(z, v)X_j(v)dv + \lambda_j \int_0^L K_{yy}(z, v)Y_j(v)dv \end{aligned}\right\}. \tag{80}$$

Multiply Eqs. 79 and 80 respectively by $\lambda_j X_j(z)$, $\lambda_j Y_j(z)$, $\lambda_i X_i(z)$ and $\lambda_i Y_i(z)$ and by subtraction form the following equations:

$$\begin{aligned}(\lambda_j - \lambda_i)X_i(z)X_j(z) &= \lambda_i\lambda_j\Big[\int_0^L K_{xx}(z, v)X_j(z)X_i(v)dv \\ &\quad + \int_0^L K_{xy}(z, v)X_j(z)Y_i(v)dv \\ &\quad - \int_0^L K_{xx}(z, v)X_i(z)X_j(v)dv \\ &\quad - \int_0^L K_{xy}(z, v)X_i(z)Y_j(v)dv\Big] \\ (\lambda_j - \lambda_i)Y_i(z)Y_j(z) &= \lambda_i\lambda_j\Big[\int_0^L K_{yx}(z, v)Y_j(z)X_i(v)dv \\ &\quad + \int_0^L K_{yy}(z, v)Y_j(z)Y_i(v)dv \\ &\quad - \int_0^L K_{yx}(z, v)Y_i(z)X_j(v)dv \\ &\quad - \int_0^L K_{yy}(z, v)Y_i(z)Y_j(v)dv\Big]\end{aligned} \tag{81}$$

The orthogonality condition is given by integration of Eqs. 81 with respect to z, and the addition of the integrated Eqs. 81 after the appropriate interchange of z and v; the result is

$$(\lambda_j - \lambda_i)\int_0^L [X_i(z)X_j(z) + Y_i(z)Y_j(z)]dz = 0. \tag{82}$$

Since the eigenvalues λ_i and λ_j are postulated different, the orthogonality condition is obtained from Eq. 82 as

$$\int_0^L [X_i(z)X_j(z) + Y_i(z)Y_j(z)]dz = 0, \qquad i \neq j. \tag{83}$$

Substitution from Eqs. 76 gives the orthogonality condition in terms of $x(z)$ and $y(z)$ as

$$\int_0^L [x_i(z)x_j(z) + y_i(z)y_j(z)]m(z)dz = 0, \qquad i \neq j. \tag{84}$$

If $j = i$ the normalization constant N_i is given by the relation

$$N_i = \int_0^L [x^2{}_i(z) + y^2{}_i(z)]m(z)dz. \tag{85}$$

The bilinear expansions for the self and mutual kernels can be written as

$$
\begin{aligned}
K_{xx}(z, v) &= \sum_1^\infty \frac{X_n(z)X_n(v)}{\lambda_n N_n} \\
K_{yy}(z, v) &= \sum_1^\infty \frac{Y_n(z)Y_n(v)}{\lambda_n N_n} \\
K_{xy}(z, v) &= \sum_1^\infty \frac{X_n(z)Y_n(v)}{\lambda_n N_n} \\
K_{yx}(z, v) &= \sum_1^\infty \frac{Y_n(z)X_n(v)}{\lambda_n N_n}
\end{aligned}
\tag{86}
$$

The correctness of these relations can be demonstrated by substitution in Eqs. 77 for the nth eigenfunction. The uniform convergence is assured for practical problems.

Substitutions from Eqs. 76 in Eqs. 86 give the expansions for the self and mutual Green's functions as

$$
\begin{aligned}
G_{xx}(z, v) &= \sum_1^\infty \frac{x_n(z)x_n(v)}{\lambda_n N_n} \\
G_{yy}(z, v) &= \sum_1^\infty \frac{y_n(z)y_n(v)}{\lambda_n N_n} \\
G_{xy}(z, v) &= \sum_1^\infty \frac{x_n(z)y_n(v)}{\lambda_n N_n} \\
G_{yx}(z, v) &= \sum_1^\infty \frac{y_n(z)x_n(v)}{\lambda_n N_n}
\end{aligned}
\tag{87}
$$

6. NORMAL MODES OF A TWISTED TURBINE BLADE.

Application of the integral-equation method is made to the calculation of the normal modes of a twisted turbine blade. This example is restricted to a single stationary blade although the method can be extended to a single rotating blade or to a group of rotating blades with shroud bands.

It is convenient to non-dimensionalize the integral equation for the twisted beam by the substitutions in Eqs. 75 of

$$
\left.
\begin{aligned}
w &= w'L \\
v &= v'L \\
z &= z'L
\end{aligned}
\right\}.
\tag{88}
$$

The non-dimensional equations are written with primes omitted as

$$\left.\begin{aligned} x(z) &= \frac{\lambda}{m_0}\int_0^1 k_{xx}(z, v)m(v)x(v)dv + \frac{\lambda}{m_0}\int_0^1 k_{xy}(z, v)m(v)y(v)dv \\ y(z) &= \frac{\lambda}{m_0}\int_0^1 k_{xy}(z, v)m(v)x(v)dv + \frac{\lambda}{m_0}\int_0^1 k_{yy}(z, v)m(v)y(v)dv \end{aligned}\right\}, \tag{89}$$

where

$$\begin{aligned} \lambda &= m_0L^4\omega^2/EI_0 \\ k_{xx}(z, v) &= EI_0G_{xx}(z, v)/L^3 \\ k_{yy}(z, v) &= EI_0G_{yy}(z, v)/L^3 \\ k_{xy}(z, v) &= EI_0G_{xy}(z, v)/L^3 \end{aligned} \tag{90}$$

The kernels of Eqs. 89 are calculated from Eqs. 60, 71, 73 and 90 as

$$\begin{aligned} k_{xx}(z, v) &= \int_0^z a_{xx}(w)(z - w)(v - w)dw \\ k_{yy}(z, v) &= \int_0^z a_{yy}(w)(z - w)(v - w)dw \qquad z \leq v. \\ k_{xy}(z, v) &= \int_0^z a_{xy}(w)(z - w)(v - w)dw \end{aligned} \tag{91}$$

The kernels are symmetric in z and v, and therefore

$$k(z, v) = k(v, z). \tag{92}$$

TABLE III.
Data for Turbine Blade.

z	$\frac{m(z)}{m_0}$	$I_\alpha(z)$	$I_\beta(z)$	$\theta(z)$	$a_{xx}(z)$	$a_{yy}(z)$	$a_{xy}(z)$
0.0	1.000	0.143	0.818	19.0°	0.913	0.263	0.253
0.1	0.941	0.127	0.816	24.2	0.965	0.335	0.355
0.2	0.876	0.103	0.810	29.3	1.098	0.470	0.518
0.3	0.808	0.081	0.784	34.0	1.272	0.680	0.736
0.4	0.725	0.062	0.732	38.5	1.491	1.013	1.025
0.5	0.646	0.049	0.712	42.4	1.671	1.425	1.339
0.6	0.588	0.039	0.730	45.6	1.895	1.966	1.725
0.7	0.552	0.030	0.772	48.6	2.194	2.761	2.270
0.8	0.536	0.022	0.818	51.3	2.676	4.049	3.120
0.9	0.532	0.016	0.866	53.8	3.337	6.058	4.320
1.0	0.548	0.010	0.914	56.1°	4.338	9.429	6.220

L = 31.6 cm.; length of beam.
A_0 = 1.53 cm.^{2}; cross-sectional area at z = 0.
I_0 = 0.143 cm.^{4}; moment of inertia $I_\alpha(0)$.
m_0 = 1.227 $\times$ 10^{-6} kg.-sec.^{2}/cm.^{2}; mass per unit length at z = 0.
E = 2.05 $\times$ 10^6 kg./cm.^{2}; Young's modulus.
p = 0.00785 kg./cm.^{3}; density of material.
$\sqrt{\frac{EI_0}{m_0L^4}} = \frac{\omega}{\sqrt{\lambda}}$ = 155 radians/sec.; ratio natural frequency to square root of eigenvalue.
units: cm.-kg.-sec.; length-force-time.

Data for a typical turbine blade are given in Table III. The coordinate system of Fig. 2 applies to these data. The coefficients a_{xx}, a_{yy} and a_{xy} are calculated from Eqs. 68.

The self and mutual kernels are calculated from Eqs. 91 by the cinema integraph.[5] The results are in Table IV.

TABLE IV.

Self and Mutual Kernels of Twisted Turbine Blade.

(Multiply values by 10^{-4})

$k_{xx}(s, v)$

v \ s	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0										
0.1	0	3									
0.2	0	7	26								
0.3	0	12	45	89							
0.4	0	16	64	133	210						
0.5	0	21	84	178	293	428					
0.6	0	26	103	223	375	564	769				
0.7	0	30	122	268	458	699	973	1265			
0.8	0	35	141	311	540	835	1179	1560	1956		
0.9	0	39	160	356	622	971	1384	1854	2361	2909	
1.0	0	44	180	401	705	1106	1590	2149	2767	3452	4166

$k_{yy}(s\ v)$

v \ s	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0										
0.1	0	0									
0.2	0	2	10								
0.3	0	3	16	27							
0.4	0	4	23	43	76						
0.5	0	6	30	59	109	164					
0.6	0	7	37	75	142	225	321				
0.7	0	8	44	91	174	284	424	592			
0.8	0	9	50	106	206	344	525	754	1001		
0.9	0	11	57	122	238	403	626	916	1252	1612	
1.0	0	12	64	138	271	463	727	1078	1504	1991	2543

$k_{xy}(s, v)$

v \ s	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0										
0.1	0	2									
0.2	0	3	13								
0.3	0	4	19	33							
0.4	0	6	25	49	84						
0.5	0	8	32	66	118	176					
0.6	0	9	39	84	152	237	342				
0.7	0	10	45	101	186	298	446	597			
0.8	0	12	51	117	219	361	549	758	996		
0.9	0	15	58	133	253	423	652	920	1238	1587	
1.0	0	16	64	150	287	484	756	1082	1482	1943	2440

Generating functions are selected as

$$\left.\begin{aligned} g_{x0}(v) &= k_{xx}(0.7, v) \\ g_{y0}(v) &= k_{yy}(0.7, v) \end{aligned}\right\}. \tag{93}$$

Iteration of these generating functions in Eqs. 89 gives

$$\left.\begin{aligned} g_{x1} &= \left(s \int_0^1 k_{xx} m g_{x0} dv + \int_0^1 k_{xy} m g_{y0} dv\right) \Big/ m_0 \\ g_{y1} &= \left(s \int_0^1 k_{xy} m g_{x0} dv + \int_0^1 k_{yy} m g_{y0} dv\right) \Big/ m_0 \end{aligned}\right\}. \tag{94}$$

The constant s has been introduced as a scale factor, since the relative magnitudes of the generating functions g_{x0} and g_{y0} are arbitrarily established in Eqs. 93.

The appropriate value for s is found by making each of Eqs. 94 yield the same value for λ. For calculations of λ define B, C, D, E, F, and J as follows:

$$B = \int_0^1 g_{x0}(z) dz,$$

$$E = \int_0^1 g_{y0}(z) dz,$$

$$Cs + D = \int_0^1 g_{x1}(z) dz,$$

$$Fs + J = \int_0^1 g_{y1}(z) dz.$$

Hence for each iteration λ is given by

$$\lambda = \frac{Bs}{Cs + D} - \frac{E}{Fs + J}.$$

From this relation the following quadratic is obtained for calculations of s:

$$s^2 + (BJ - CE)s/BF - DE/BF = 0.$$

The positive root of s corresponds to the first mode and the negative root to the second mode.

Successive iteration and calculation of scale factors as in the foregoing leads to the eigenvalues and eigenfunctions for the first and second modes. The approximate eigenvalues are given by the first iteration, but additional iterations are required for the eigenfunctions to become stabilized. The resulting eigenvalues and eigenfunctions for the turbine blade are given in Table V. It is of interest to note that the eigen-

functions satisfy the orthogonality relation of Eqs. 84, although this relationship was not used in the foregoing iterations.

TABLE V.

Normal Modes of Twisted Turbine Blade.

s	First Mode		Second Mode	
	$x_1(s)$	$y_1(s)$	$x_2(s)$	$y_2(s)$
0.0	0.000	0.000	0.000	0.000
0.1	0.012	0.005	−0.018	0.001
0.2	0.047	0.023	−0.070	0.021
0.3	0.104	0.050	−0.146	0.040
0.4	0.183	0.095	−0.217	0.090
0.5	0.284	0.155	−0.289	0.164
0.6	0.408	0.234	−0.330	0.266
0.7	0.544	0.329	−0.362	0.423
0.8	0.690	0.432	−0.358	0.598
0.9	0.843	0.545	−0.340	0.787
1.0	1.000	0.661	−0.312	0.996

$\lambda_1 = 14.93.$ $\lambda_2 = 98.7.$

$\omega_1 = 596$ radians/sec. $\omega_2 = 1541$ radians/sec.

$f_1 = 95$ cycles/sec. $f_2 = 245$ cycles/sec.

Experimental observations of the first two natural frequencies for the turbine blade give the following:

First mode: 93 to 102 cycles/sec.
Second mode: 225 to 253 cycles/sec.

The calculated values in Table V check these observations and were made without prior knowledge of the experimental values.

7. CONCLUSIONS.

Integral equations are highly effective for the solution of problems in vibrations. Because of the closeness of the integral equation formulation to the physical picture, much simplification results when problems are studied directly with integral equations rather than by differential equations. The variations of physical properties or discontinuities cause no particular difficulty when they appear under the integral sign. Boundary conditions are contained in the Green's function and are handled there when the beam is subjected to the elementary unit-concentrated force. Once taken care of in the construction of the Green's function, the boundary conditions require no further consideration.

Much emphasis is placed on practical methods of solving the integral equation. If a mathematical machine, such as the differential analyzer, is available for evaluation of the integrals, the solution by successive approximations is easily obtained. On the other hand, because of the

rapid convergence of the successive approximations, an ordinary calculating machine is an economical tool.

When modes higher than the fundamental are required, the method of successive approximations must be used with care, or the solution will converge to the fundamental mode. Perhaps the application of the so-called *perturbated kernel* is the preferred method of solution, particularly if the integrals are evaluated only approximately. When the perturbated kernel is used, the importance of accurate physical data is immediately apparent. Since the kernel contains much of the physical data for the problem, it generally is found that the perturbated kernel is controlled largely by the tolerances on the physical data. This difficulty indicates no defect in the method of solution, but instead it boldly shows the engineer the accuracy of the solutions for the higher modes, rather than masking the accuracy under an elaborate calculation procedure.

The twisted turbine blade is an interesting problem to show the power of the integral-equation method in dealing with continuous systems where coupling exists. The agreement between the calculated natural frequencies and the experimental values probably is better than generally can be expected for so complex a problem. Yet where a large disagreement is found, the difficulty probably is caused by inaccurate physical data. The construction of the Green's function for the turbine blade shows the value of the bending-moment Green's function as an aid for construction of the final Green's function. The derivation of the orthogonality condition for the turbine blade is a special case of a development that may be required when higher modes are to be calculated for very complex systems. Of course, no orthogonality condition is required if only the fundamental mode is to be calculated.

In conclusion, the integral-equation approach to vibration problems is of particular value when there are variations of the physical properties of the vibrating member. As applied to beam vibration, the integral-equation method applies when simple bending theory is used, or when consideration is given to effects of rotary inertia, shearing force, longitudinal inertia, gyroscopic moment, etc. The technique is to consider the influence of each factor on the beam deflection when that factor is *acting alone*, and then by superposition to add all the individual influences to form the integral equation of the vibrating beam. The resultant equation generally is solved most directly by the method of successive approximations.

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Experimental Gas Turbines Being Constructed.—Two experimental gas turbines, one a 5000-kw. machine for electric power stations, and the other a 4800-hp. unit, are being constructed by the General Electric Company.

Alan Howard, G-E turbine-generator design engineer at Schenectady who headed design and development of the TG-100 Propjet, and TG-180 jet engines now powering the latest U. S. fighter planes, revealed that the company is also concentrating its facilities on gas turbine development in the nonaircraft field.

Shop tests of the 4800-hp. turbine are scheduled to begin soon. The stationary power plant is in the design stage and will not undergo factory test until later.

The 4800-hp. gas turbine will burn Bunker "C" fuel, although work is being carried on in the use of pulverized coal in order to permit more diversified application of gas turbines. The 4800-hp. unit being constructed is approximately 19 feet long and weighs between 2½ to 4 pounds per horsepower.

"We are making drawings," Howard said, "for a 5000-kw. plant suitable for central station or industrial applications. We believe this type of plant, when developed, will prove to have very competitive all-around econony and characteristics when compared with other types of plants in similar ratings."

In this stationary gas turbine, which also will be powered with oil, air flows through two compressors, an intervening intercooler, a regenerator and then into six combustion chambers. Nominal inlet temperature to the turbine is 1500° F., with the gases expanding first through a two-stage turbine which drives the high-pressure compressor Turbine and compressor run at a constant speed of approximately 8700 r.p.m.

The rest of the expansion occurs through the single-stage low-pressure turbine which drives the low-pressure compressor.

R. H. O.

Trend of Power Transmission Economics Toward Operation of Large Generating Stations at High Circuit Loadings.—Power transmission systems are now entering a stage in their development where added generating capacity will be located at relatively greater distances from the utilization areas, S. B. Crary and I. B. Johnson of General Electric's analytical Division told A.I.E.E. members at recent meetings in Montreal and Quebec.

"Economics of power transmission will be toward development of large generating stations for connection to well integrated-systems so that transmission may be accomplished at high circuit loading and load factors," the G-E engineers said.

The two men indicated that operations of high-voltage systems with grounded neutrals materially reduces transmission costs by allowing for the use of the next lower level of transformer insulation as well as reducing transmission line expenses.

According to Crary and Johnson, the 360-kv. voltage level is about the most economical voltage for distances in the range from 300 to 600 miles when line compensation is used for power transfers of 300,000 to 350,000 kw. per circuit.

"230 to 287 kv. levels," they said, "appear to be most economical for heavily loaded long-distance transmission line conditions such as exist in this country today."

R. H. O.

PARTIALLY PLASTIC THICK-WALLED TUBES.[1]

BY

C. W. MACGREGOR,[2] L. F. COFFIN, JR.,[3] and J. C. FISHER.[4]

ABSTRACT.

A theory is presented for the partial plastic yielding of thick-walled cylindrical tubes acted upon by any combination of internal pressure, external pressure and end load when the material follows an arbitrary stress-strain law. The solution combines (a) the distortion energy theory of plastic flow and (b) the effects of elastic compressibility of the plastic material.

Numerical values for stresses and strains are given for certain special cases of internal pressure in thick-walled open-ended tubes for an idealized stress-strain law, and the results are compared with earlier approximate theories.

The maximum internal pressure which can be withstood by a thick-walled tube of a ductile metal considerably exceeds the pressure at which plastic flow begins. As the pressure increases from an initial value of zero, the entire tube is at first elastic, and the stresses at any location in the tube are given by the well-known expressions[5]

$$\left.\begin{aligned} \sigma_r &= -\frac{\left(\dfrac{R_1}{r}\right)^2 - 1}{\left(\dfrac{R_1}{R_0}\right)^2 - 1}\,p_0, \\ \sigma_t &= \frac{\left(\dfrac{R_1}{r}\right)^2 + 1}{\left(\dfrac{R_1}{R_0}\right)^2 - 1}\,p_0, \\ \sigma_z &= \frac{F}{\pi(R_1^2 - R_0^2)}. \end{aligned}\right\} \quad (1)$$

[1] This paper reports a portion of an investigation conducted for the Bureau of Ordnance, United States Navy. The work was done in the Research Laboratories for Mechanics of Materials, Mechanical Engineering Department, at the Massachusetts Institute of Technology. This material has been released for publication by the Chief of the Bureau of Ordnance. The opinions expressed herein are those of the authors and should be in no way interpreted as those of the Navy Department.

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[5] See Nomenclature for definition of symbols.

These equations are valid until the pressure reaches a value at which the metal at the bore of the tube becomes plastic (1).[6]

As the pressure increases above this critical value, a zone of yielding moves outward from the bore through the tube. More specifically, there is a cylindrical bounding surface called the plastic-elastic boundary which divides the inner and plastically deformed material from the

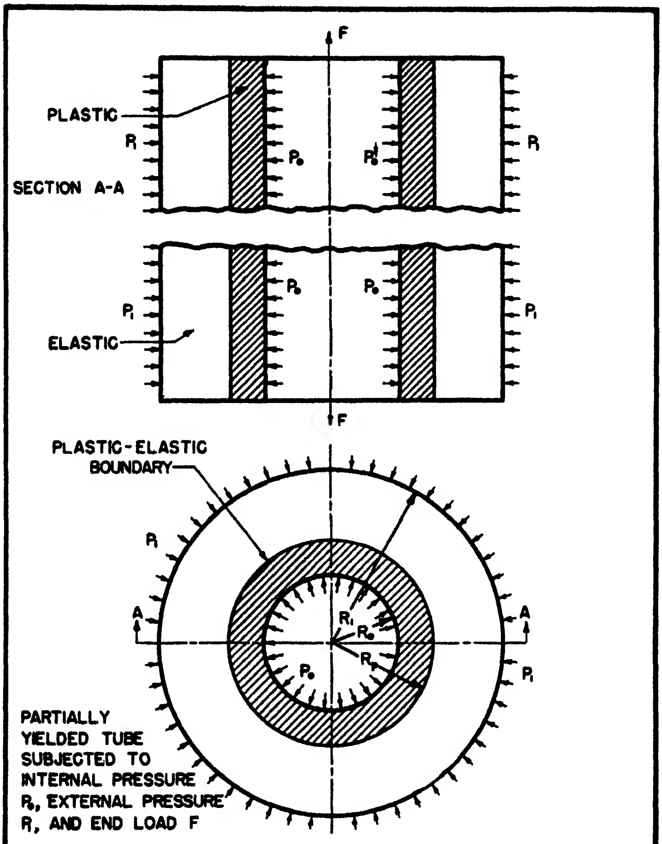

FIGURE 1

outer and still elastic material. At this stage the tube is said to be partially yielded. As the pressure is increased, the plastic-elastic boundary continues outward until it coincides with the outer surface, and the tube is then said to be completely yielded. A tube subjected to external pressure as well as internal pressure and end load will yield when the difference between the internal and external pressures is large enough. Such a tube, partially yielded, is shown schematically in Fig. 1.

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[6] Numbers in parentheses refer to Bibliography at end of paper.

The fact that the internal pressure in a thick-walled tube can rise considerably above the value for initial yielding before the tube becomes completely plastic is of practical interest. With regard to high pressure tubing and other high pressure equipment, it assures a factor of safety considerably greater than that based only upon initial yielding. For this reason it is of interest to know the ratio of pressures for complete to initial yielding for a tube of a given material. In some instances, as for example in gun design, it may prove desirable to provide for operation at pressures exceeding that for initial yielding. An accurate knowledge of the depth of yielding, the factor of safety and the expected deformations is of considerable importance.

If a tube is expanded so that the zone of yielding penetrates part way or in some instances entirely through the tube, and the internal pressure is then removed, desirable residual stresses may be produced. For certain residual stress distributions, it can be shown that upon the reapplication of pressure, the tube will not yield again until a pressure equal to the maximum expanding pressure previously applied has been reached. The elastic strength of a partially yielded tube thus may be considerably greater than that of an untreated tube of the same dimensions. This process has been used frequently in the fabrication of gun barrels and pressure vessels. It is often called "autofrettage" from the French for "self-hooping," since the partial yielding process produces desirable residual stresses at the bore similar to those produced by shrinking on the outer tubes of built-up guns.

Deformation of the tube in excess of the amount necessary to make it completely plastic is generally accompanied by large plastic strains. The assumption that the deformations of an element of material are negligible in comparison with the dimensions of the element, ordinarily made for elastic and small plastic strains, is no longer valid. For such large deformations a special analysis is required, making use of more general strain definitions (2).

For elastic strains, or for strains present in the plastic region of a partially plastic tube, the deformations are small enough so that the customary assumption that they are negligible with respect to the dimensions of a deforming element is well justified. The present discussion is restricted to a consideration of partially plastic tubes, including the elastic tube and the just completely plastic tube as limiting cases, and the ordinary expressions for strains are retained.

Several solutions have been published for the partially plastic tube subjected to internal pressure (3–5). Table I classifies them according to various assumptions used in their derivation.

As can be seen from Table I, the theory developed in this paper combines elastic compressibility of the plastic material with the distortion energy theory of plastic flow. It is applicable to a material having an arbitrary stress-strain curve and holds for any combination of internal

pressure, external pressure and end load. A double need for such a theory exists at the present time; first to permit closer design practice, and second to make possible a critical evaluation and comparison of the various more approximate solutions in use at present.

Table I.

Comparison of Several Theories for Partial Yielding of Thick-Walled Tubes.

Theory	Assumed Law of Yielding	Assumed Sum of Total Strains in Plastic Region	Assumed Axial Stress or Strain	Assumed Stress Strain Law	Assumed Loading
Nádai (3)	Distortion Energy	0	$\epsilon_z = 0$	$\sigma = \sigma_0$ in plastic region	Internal Pressure, End Load such that $\epsilon_z = 0$
Duguet (4)	Shear with Friction	Indeterminate	Indeterminate	Arbitrary	Internal Pressure
Macrae (5)	Maximum Shear	Indeterminate	Indeterminate	Arbitrary	Internal Pressure
Theory given in this paper	Distortion Energy	$\frac{1-2\nu}{E}(\sigma_r + \sigma_t + \sigma_z)$	(derived)	Arbitrary	Any Combination of Internal Pressure, External Pressure, End Load

STRESS ANALYSIS FOR PARTIAL YIELDING OF A THICK WALLED TUBE.

The following assumptions are made concerning the plastic or partially plastic thick-walled tube in which the elastic strains in the plastic region are considered:

(i) The sum of the total strains is

$$\epsilon_r + \epsilon_t + \epsilon_z = \frac{1 - 2\nu}{E}(\sigma_r + \sigma_t + \sigma_z).$$

(ii) The ratios of the principal strain differences to the corresponding principal stress differences are equal.

(iii) The axial strain is constant throughout the tube.

(iv) There is a unique relationship between the effective stress and the effective strain.

(v) The tube is acted upon by internal pressure, external pressure and an end load.

Assumption (i) takes into consideration the compressibility of the material, since in all real materials the volume does not remain constant, but increases with increasing mean stress. In a purely elastic metal the sum of the strains is equal to $(1 - 2\nu)/E$ times the sum of the stresses. When a metal becomes plastic it is assumed that the total principal strains are made up of two components, satisfying the rela-

tionship that the sum of the plastic components of the principal strains is zero, and the sum of the elastic components of the principal strains is $(1 - 2\nu)/E$ times the sum of the principal stresses as in a completely elastic material. This leads to the expression in assumption (i) for the sum of the total strains.

Assumption (ii) can be derived from the fundamental laws of plastic flow (3). Assumption (iii) assures that plane sections of the cross section remain plane during the expansion of the tube. This must be true except at sections very close to the ends of the tube.

The effective stress at a point in a body is defined as

$$\sigma = \frac{\sqrt{2}}{2} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}, \tag{2}$$

where σ_1, σ_2 and σ_3 are principal stresses. Similarly the effective strain is defined as

$$\epsilon = \frac{\sqrt{2}}{3} \sqrt{(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2}, \tag{3}$$

where ϵ_1, ϵ_2 and ϵ_3 are the principal strains. Experience indicates that for a given material and for values of ϵ less than about 0.2 a curve of σ versus ϵ is independent of the nature of the deformation. For such values of ϵ the relationship between σ and ϵ can therefore be determined by a tension test. Assumption (iv) can be expressed as

$$\sigma = f(\epsilon). \tag{4}$$

For a long uniform thick-walled tube, the principal strains may be expressed in terms of the principal stresses as

$$\left.\begin{aligned} E\epsilon_r &= \sigma_r - \nu(\sigma_t + \sigma_z) + D[\sigma_r - \tfrac{1}{2}(\sigma_t + \sigma_z)], \\ E\epsilon_t &= \sigma_t - \nu(\sigma_z + \sigma_r) + D[\sigma_t - \tfrac{1}{2}(\sigma_z + \sigma_r)], \\ E\epsilon_z &= \sigma_z - \nu(\sigma_r + \sigma_t) + D[\sigma_z - \tfrac{1}{2}(\sigma_r + \sigma_t)], \end{aligned}\right\} \tag{5}$$

where D is a function of the radius, and is zero when there is no plastic flow. Equations (5) may be written in simpler form as

$$\left.\begin{aligned} E\epsilon_r &= \omega_1\sigma_r - \omega_2(\sigma_t + \sigma_z), \\ E\epsilon_t &= \omega_1\sigma_t - \omega_2(\sigma_z + \sigma_r), \\ E\epsilon_z &= \omega_1\sigma_z - \omega_2(\sigma_r + \sigma_t), \end{aligned}\right\} \tag{6}$$

where

$$\left.\begin{aligned} \omega_1 &= 1 + D, \\ \omega_2 &\quad \nu + \frac{D}{2}. \end{aligned}\right\} \tag{7}$$

Equations (5) and (6) satisfy assumptions (i) and (ii) since from these equations it follows that

$$\epsilon_r + \epsilon_t + \epsilon_s = \frac{1 - 2\nu}{E}(\sigma_r + \sigma_t + \sigma_s)$$

and

$$\frac{\epsilon_r - \epsilon_t}{\sigma_r - \sigma_t} \qquad \frac{\epsilon_t - \epsilon_s}{\sigma_t - \sigma_s} \qquad \frac{\epsilon_s - \epsilon_r}{\sigma_s - \sigma_r}$$

A consideration of the equilibrium of an element of the tube shows that the following equation must be satisfied by the stresses (1):

$$r\frac{d\sigma_r}{dr} = \sigma_t - \sigma_r. \tag{8}$$

Introducing the variable x defined as

$$x = \log_e \frac{r}{R_1}, \tag{9}$$

equation (8) becomes

$$r\frac{d\sigma_r}{dr} = \frac{d\sigma_r}{dx} = \sigma_r' = \sigma_t - \sigma_r, \tag{10}$$

where the prime (′) indicates differentiation with respect to x.
Compatibility of the strains (1) requires that

$$r\frac{d\epsilon_t}{dr} = \epsilon_r - \epsilon_t \tag{11}$$

or, in terms of the variable x,

$$\epsilon_t' = \epsilon_r - \epsilon_t. \tag{12}$$

Substituting the expressions for the strains from equation (6) into equation (12) gives

$$a_t D' + \omega_1\sigma_r' + \omega_1\sigma_t' - \omega_2\sigma_s' = 0, \tag{13}$$

where a_t is one of the three quantities defined as

$$\left.\begin{aligned} a_r &= \sigma_r - \tfrac{1}{2}(\sigma_t + \sigma_s), \\ a_t &= \sigma_t - \tfrac{1}{2}(\sigma_s + \sigma_r), \\ a_s &= \sigma_s - \tfrac{1}{2}(\sigma_r + \sigma_t). \end{aligned}\right\} \tag{14}$$

As stated in assumption (iii) the axial strain ϵ_s is constant over the cross section, or

$$\epsilon_s' = 0. \tag{15}$$

Using the expression for ϵ_s from equation (6), equation (15) becomes

$$a_s D' - \omega_2\sigma_r' - \omega_2\sigma_t' + \omega_1\sigma_s' = 0. \tag{16}$$

The unique relationship between the effective stress σ and the effective strain ϵ

$$\sigma = f(\epsilon)$$

has been taken as the final condition which must be satisfied by the stresses and strains in the thick-walled tube. From equation (3),

$$\frac{9}{2}\epsilon^2 = (\epsilon_r - \epsilon_t)^2 + (\epsilon_t - \epsilon_z)^2 + (\epsilon_z - \epsilon_r)^2.$$

Differentiating with respect to x this becomes

$$9\epsilon\epsilon' = \frac{4}{E}(\omega_1 + \omega_2)(a_r\epsilon_r' + a_t\epsilon_t' + a_z\epsilon_z'). \tag{17}$$

In a similar fashion from equation (2)

$$2\sigma^2 = (\sigma_r - \sigma_t)^2 + (\sigma_t - \sigma_z)^2 + (\sigma_z - \sigma_r)^2.$$

Differentiating with respect to x gives

$$\sigma\sigma' = a_r\sigma_r' + a_t\sigma_t' + a_z\sigma_z'. \tag{18}$$

The ratio of equation (18) to equation (17) is

$$\frac{\sigma\sigma'}{9\epsilon\epsilon'} = \frac{a_r\sigma_r' + a_t\sigma_t' + a_z\sigma_z'}{\frac{4}{E}(\omega_1 + \omega_2)(a_r\epsilon_r' + a_t\epsilon_t' + a_z\epsilon_z')} \tag{19}$$

Equation (19) can, by means of the relationships given in equations (6), (7) and (14), be reduced to

$$\phi_1 D' + \phi_2 a_r\sigma_r' + \phi_2 a_t\sigma_t' + \phi_2 a_z\sigma_z' = 0, \tag{20}$$

where

$$\begin{aligned} \phi_1 &= \frac{4m}{9}(\omega_1 + \omega_2)(a_r^2 + a_t^2 + a_z^2), \\ \phi_2 &= \frac{4m}{9}(\omega_1 + \omega_2)^2 - 1, \\ m &= \frac{1}{E^2}\frac{\sigma}{\epsilon}\frac{\sigma'}{\epsilon'} = \frac{1}{E^2}\frac{\sigma}{\epsilon}\frac{d\sigma}{d\epsilon}. \end{aligned} \tag{21}$$

Equations (10), (13), (16), and (20) now form a group of four equations in the four unknowns σ_r', σ_t', σ_z' and D'. They are rewritten together as follows

$$\begin{aligned} &\sigma_r' && && && = \sigma_t - \sigma_r, \\ a_t D' + {} & \omega_1\sigma_r' + {} && \omega_1\sigma_t' - {} && \omega_2\sigma_z' && = 0, \\ a_z D' - {} & \omega_2\sigma_r' - {} && \omega_2\sigma_t' + {} && \omega_1\sigma_z' && = 0, \\ \phi_1 D' + {} & \phi_2 a_r\sigma_r' + {} && \phi_2 a_t\sigma_t' + {} && \phi_2 a_z\sigma_z' && = 0. \end{aligned} \tag{22}$$

These equations may be solved simultaneously for σ_r', σ_t', σ_z' and D' in terms of the values of a_i, ω_i and ϕ_i, which in turn are functions of σ_r, σ_t, σ_z and D. The solutions are

$$\sigma_r' = \sigma_t - \sigma_r,$$

$$\sigma_t' = \frac{[(\omega_1 - \omega_2)a_z^2 + \omega_1 a_r a_t]\phi_2 + (\omega_2^2 - \omega_1^2)\phi_1}{[\text{Den}]}(\sigma_t - \sigma_r),$$

$$\sigma_z' = \frac{(a_r - a_t)(a_z\omega_1 + a_t\omega_2)\phi_2}{[\text{Den}]}(\sigma_t - \sigma_r),$$

$$D' = \frac{(\omega_1^2 - \omega_2^2)(a_t - a_r)\phi_2}{[\text{Den}]}(\sigma_t - \sigma_r),$$

where

$$\left.\begin{aligned}
[\text{Den}] &= -[(a_t^2 + a_z^2)\omega_1 + 2a_t a_z \omega_2]\phi_2 + (\omega_1^2 - \omega_2^2)\phi_1,\\
\phi_1 &= \frac{4m}{9}(\omega_1 + \omega_2)(a_r^2 + a_t^2 + a_z^2),\\
\phi_2 &= \frac{4m}{9}(\omega_1 + \omega_2)^2 - 1,\\
m &= \frac{1}{E^2}\frac{\sigma}{\epsilon}\frac{d\sigma}{d\epsilon},\\
\omega_1 &= 1 + D,\\
\omega_2 &= \nu + \frac{D}{2},\\
a_r &= \sigma_r - \tfrac{1}{2}(\sigma_t + \sigma_z),\\
a_t &= \sigma_t - \tfrac{1}{2}(\sigma_z + \sigma_r),\\
a_z &= \sigma_z - \tfrac{1}{2}(\sigma_r + \sigma_t).
\end{aligned}\right\} \quad (23)$$

This system of differential equations is best solved numerically for any particular problem. The numerical solution is difficult since the boundary conditions can be satisfied only by trial and error methods. (These conditions for a given value of R_1/R_0 are the external and internal pressures, the external or internal tangential strain, and the end load.) A numerical solution can be obtained by assuming first that $\sigma_r = -p_1$ at the external surface of the tube. This satisfies the condition that the external pressure shall have a value p_1. Usually the value of p_1 is taken as zero. Reasonable values of σ_t and σ_z at the surface of the tube can then be assumed. The value of D corresponding to the required value of the external tangential strain can be determined from equation (5), and the numerical solution carried out by standard methods (2, 6). It will be necessary to try several combinations of values of σ_t and σ_z in order to determine which pair satisfies the other two boun-

dary conditions (i.e. to determine which pair of σ_t, σ_s values assumed at the surface of the tube will give simultaneously a preselected bore pressure p_0 and a preselected end load F).

It is evident that some method of simplification would be desirable in order to decrease the labor of solving the system of differential equations arising in this problem.

One of the greatest simplifications possible would be to assume an idealized stress-strain curve which could be made to fit the majority of actual stress-strain curves with a good degree of approximation. This

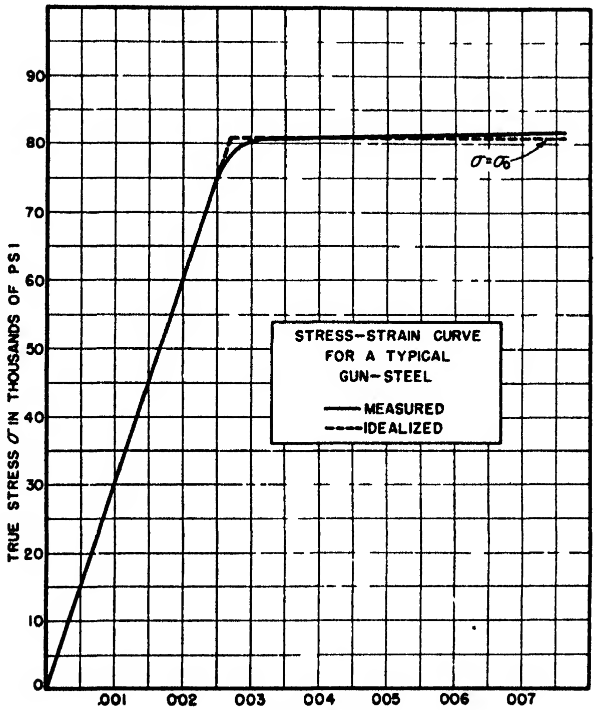


FIGURE 2

would make unnecessary a recomputation of the solution of the equations whenever a material with a different stress-strain relationship was considered. Further, a group of numerical solutions could be prepared in tabular form so that the solution for any particular set of boundary conditions could be obtained by interpolation. This method of simplification is adopted here.

Since steel is the material most generally used for thick-walled tubes, and since the stress-strain curve for steel frequently resembles the curve for a typical gun-steel given in Fig. 2, it is usually possible to simplify the stress-strain curve to two straight lines. The first, passing through the origin and having a positive slope, represents the relation-

ship in the elastic region, and the second, having zero slope, represents the relationship in the plastic region. This approximation is shown in Fig. 2. It must be borne in mind that the approximation breaks down for strains much larger than a few per cent. For partially yielded tubes, however, where the strains do not exceed this limit, the approximation is satisfactory.

The value of the effective stress σ for the tension test may be determined from equation (2) together with the fact that $\sigma_r = \sigma_t = 0$. Equation (2) reduces to

$$\sigma = \sigma_z.$$

Similarly knowing that

$$\begin{aligned} E\epsilon_r &= -\,\omega_2\sigma_z, \\ E\epsilon_t &= -\,\omega_2\sigma_z, \\ E\epsilon_z &= \omega_1\sigma_z, \end{aligned}$$

as obtained by setting $\sigma_r = \sigma_t = 0$ in equation (6), the expression for the effective strain ϵ in equation (3) reduces to

$$E\epsilon = \tfrac{2}{3}(\omega_1 + \omega_2)\sigma_z$$

for the tension test.

In the elastic region of the idealized stress-strain curve the quantity $D = 0$ and therefore $\omega_1 + \omega_2 = 1 + \nu$. Hence in the elastic region

$$E\epsilon = \frac{2(1+\nu)}{3}\,\sigma_z = \frac{2(1+\nu)}{3}\,\sigma$$

and

$$\sigma = \frac{3E}{2(1+\nu)}\,\epsilon.$$

The above expression is the analytical form of the function $\sigma = f(\epsilon)$ for the elastic region. From equation (21) it follows that

$$m = \frac{1}{E^2}\frac{\sigma}{\epsilon}\frac{d\sigma}{d\epsilon} = \frac{9}{4(1+\nu)^2},$$

from which

$$\phi_2 = \frac{4m}{9}(\omega_1 + \omega_2)^2 - 1 = \frac{4m}{9}(1+\nu)^2 - 1 = 0$$

and equations (23) reduce to

$$\left.\begin{aligned} \sigma_r' &= \sigma_t - \sigma_r, \\ \sigma_t' &= -\,(\sigma_t - \sigma_r), \\ \sigma_z' &= 0, \\ D' &= 0 \end{aligned}\right\} \qquad (24)$$

for the elastic portion of the stress-strain curve. Equations (24) integrate directly into the well-known equations for an elastic thick-walled tube subjected to arbitrary internal and external pressures and end load. Changing from the variable x to the variable r the solutions are

$$\left.\begin{aligned} \sigma_r &= A - \frac{B}{r^2}, \\ \sigma_t &= A + \frac{B}{r^2}, \\ \sigma_z &= C, \end{aligned}\right\} \tag{25}$$

where A, B and C are constants to be determined by the boundary conditions. Since $D = 0$, equations (23) become the ordinary elastic equations, for the first portion of the idealized stress-strain curve.

In the plastic region of the idealized curve, $\sigma = \sigma_z = \sigma_0 =$ constant, so that

$$m = \frac{1}{E^2}\frac{\sigma}{\epsilon}\frac{d\sigma}{d\epsilon} = 0,$$

from which

$$\left.\begin{aligned} \phi_1 &= 0, \\ \phi_2 &= -1. \end{aligned}\right\}$$

Equations (23) now reduce to

$$\left.\begin{aligned} \sigma_r' &= \sigma_t - \sigma_r, \\ \sigma_t' &= \frac{(\omega_2 - \omega_1)a_z^2 - \omega_1 a_r a_t}{(\text{Den})}(\sigma_t - \sigma_r), \\ \sigma_z' &= \frac{(a_t - a_r)(a_z\omega_1 + a_t\omega_2)}{(\text{Den})}(\sigma_t - \sigma_r), \\ D' &= \frac{(\omega_2^2 - \omega_1^2)(a_t - a_r)}{(\text{Den})}(\sigma_t - \sigma_r), \\ \text{where}\quad (\text{Den}) &= (a_t^2 + a_z^2)\omega_1 + 2a_t a_z \omega_2, \\ \omega_1 &= 1 + D \\ \omega_2 &= \nu + \frac{D}{2}, \\ a_r &= \sigma_r - \tfrac{1}{2}(\sigma_t + \sigma_z), \\ a_t &= \sigma_t - \tfrac{1}{2}(\sigma_z + \sigma_r), \\ a_z &= \sigma_z - \tfrac{1}{2}(\sigma_r + \sigma_t). \end{aligned}\right\} \tag{26}$$

The system of equations (26) can be solved numerically for σ_r, σ_t, σ_z and D as functions of $x = \log_e (r/R_1)$ taking as boundary conditions

$$\text{Elastic}\left\{\begin{array}{l} \sigma_r = \sigma_r \\ \sigma_t = \sigma_t \\ \sigma_z = \sigma_z \\ D = D(=0) \end{array}\right\}\text{Plastic}$$

at the plastic-elastic boundary (the point of discontinuous slope in the idealized stress-strain curve).

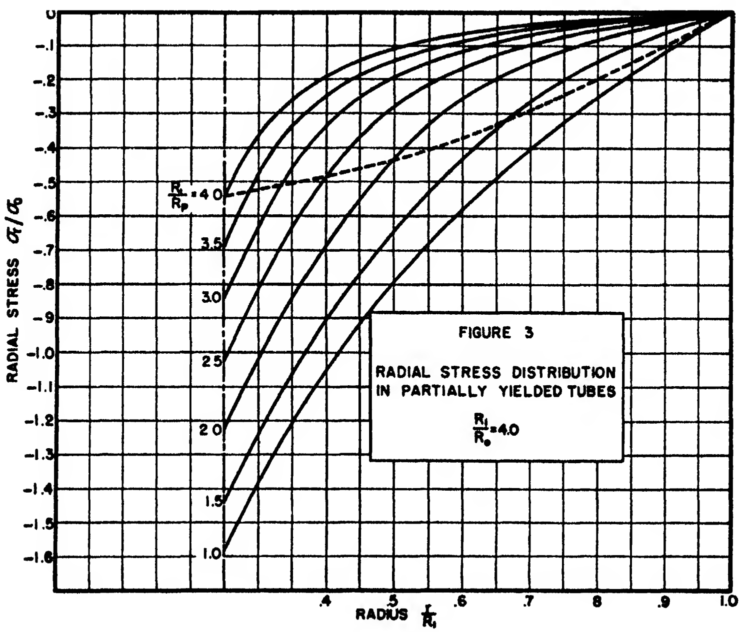


FIGURE 3
RADIAL STRESS DISTRIBUTION IN PARTIALLY YIELDED TUBES

No general solution has been found as yet for this system of equations. However, a large number of particular solutions for tubes with zero end load have been found by numerical methods, covering a range of values of R_1/R_0 from 1 to 4 for a number of depths of yielding. These solutions are arranged in tabular form in a report presented to the Bureau of Ordnance, Navy Department (7). A simple interpolation in the tables of this report gives values of σ_r/σ_0, σ_t/σ_0, σ_z/σ_0 and D as functions of x in the plastic region of partially yielded tubes for any desired values of R_1/R_0 in the range $1 \leqslant R_1/R_0 \leqslant 4$ and of R_1/R_p in the range $1 \leqslant R_1/R_p \leqslant R_1/R_0$. The stresses in the elastic region can be computed easily from the usual Lamé relationships once the stresses at the plastic-elastic boundary are known.

Although space does not permit the reproduction of these tables here, some idea as to the information they contain is given in Figs. 3–11. These figures show the stress distributions for several tubes with different ratios R_1/R_0 for a number of depths of yielding. A dashed line has been drawn in each figure in such a manner that the portion of a

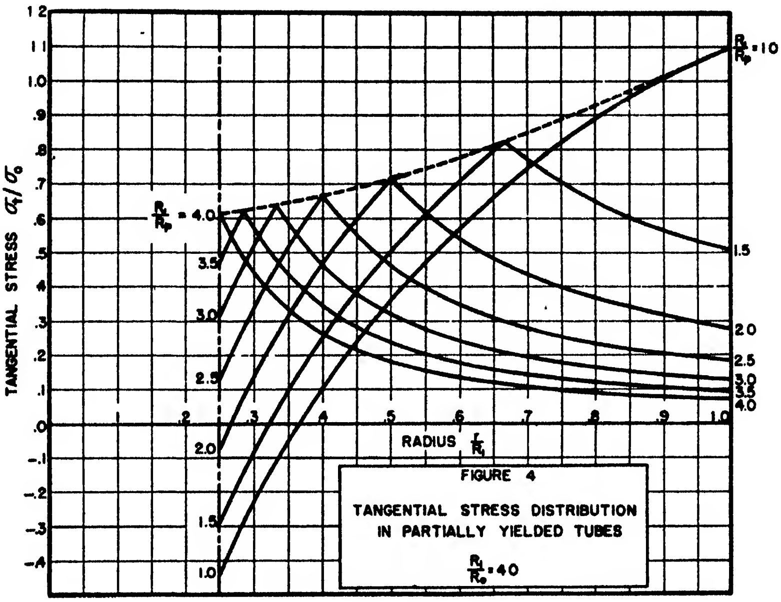


FIGURE 4

TANGENTIAL STRESS DISTRIBUTION IN PARTIALLY YIELDED TUBES

$\frac{R_1}{R_0} = 4.0$

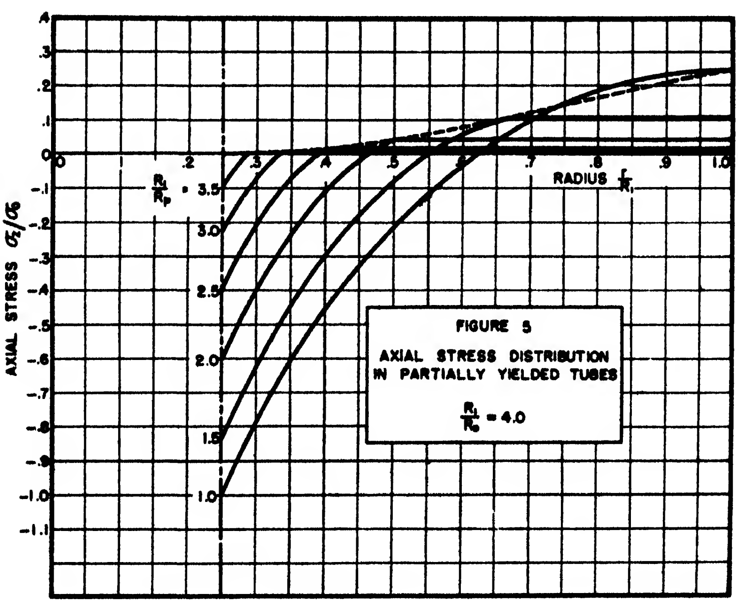
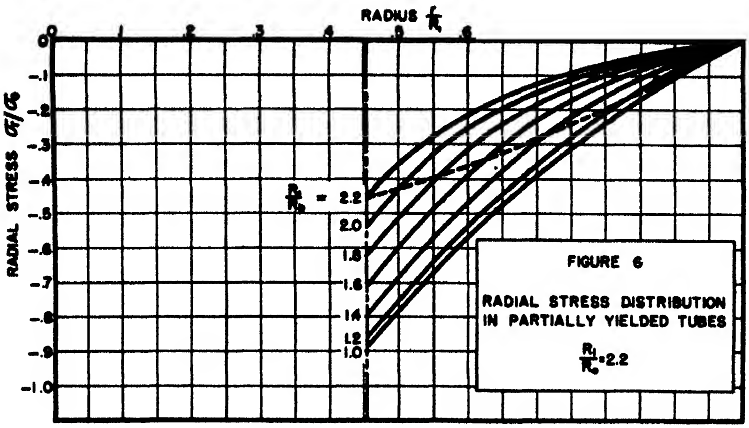


FIGURE 5

AXIAL STRESS DISTRIBUTION IN PARTIALLY YIELDED TUBES

$\frac{R_1}{R_0} = 4.0$


RADIUS $\frac{r}{R_1}$
RADIAL STRESS σ_r/σ_0
$\frac{R_1}{R_p}$ = 2.2
2.0
1.8
1.6
1.4
1.2
1.0
FIGURE 6
RADIAL STRESS DISTRIBUTION IN PARTIALLY YIELDED TUBES
$\frac{R_1}{R_o}$ = 2.2

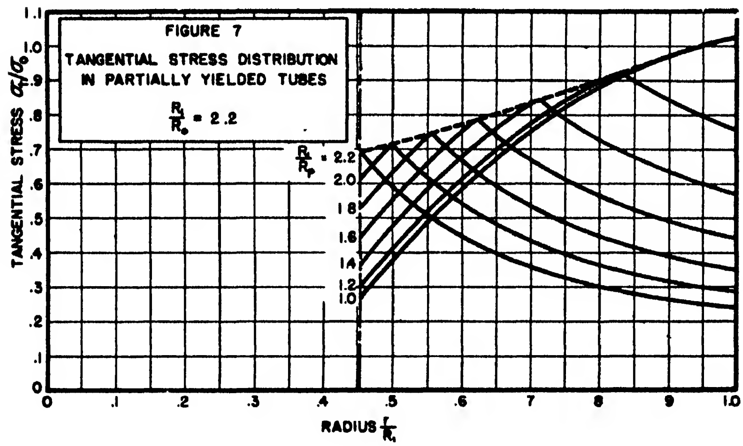
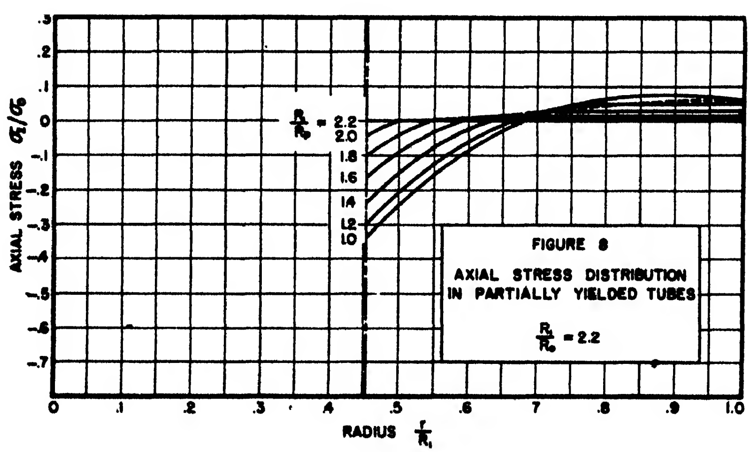
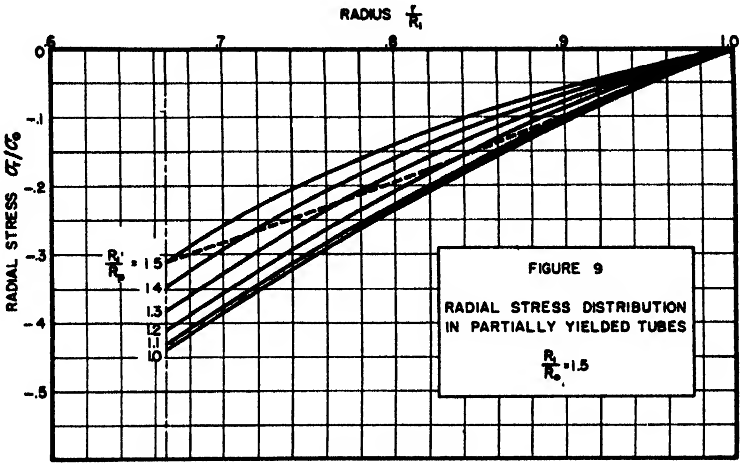
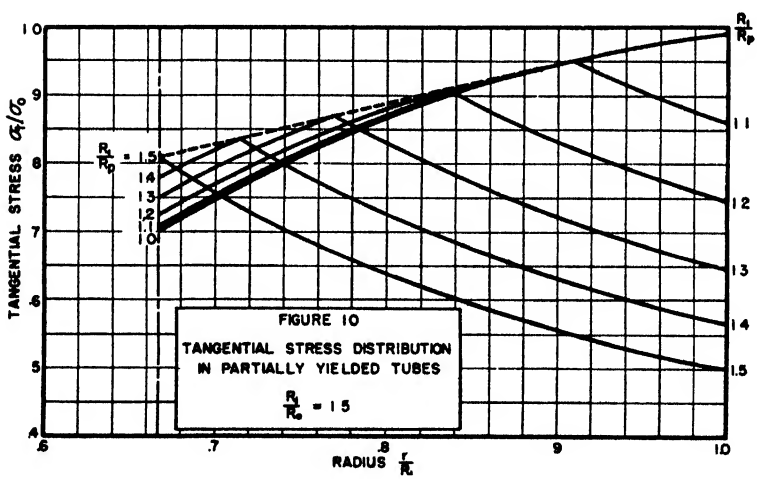


FIGURE 7
TANGENTIAL STRESS DISTRIBUTION IN PARTIALLY YIELDED TUBES
$\frac{R_1}{R_o}$ = 2.2
TANGENTIAL STRESS σ_t/σ_0
$\frac{R_1}{R_p}$ = 2.2
2.0
1.8
1.6
1.4
1.2
1.0
RADIUS $\frac{r}{R_1}$


AXIAL STRESS σ_z/σ_0
$\frac{R_1}{R_p}$ = 2.2
2.0
1.8
1.6
1.4
1.2
1.0
FIGURE 8
AXIAL STRESS DISTRIBUTION IN PARTIALLY YIELDED TUBES
$\frac{R_1}{R_o}$ = 2.2
RADIUS $\frac{r}{R_1}$

stress curve lying to the left of its point of contact with the dashed line represents the stress in the plastic region, and the portion lying to the right of the point of contact represents the stress in the elastic region.





The dashed lines may be considered as loci of the plastic-elastic boundaries for the family of stress curves in each figure. It is of particular interest to note the continuity of the stresses across each plastic-elastic boundary.

A few residual stress distributions are given in Figs. 12–14. The residual stresses have been computed assuming elastic recovery of the tube when the expanding pressure is removed. This is justified in the absence of an appreciable Bauschinger effect (5), and in the absence of residual bore stresses exceeding the yield strength σ_0. For strains as small as those present in partially-yielded tubes, the Bauschinger

effect is frequently negligible. However, an examination of the residual stresses at the bore of the tube in Fig. 14 shows that the residual effective stress exceeds σ_0. This indicates that this tube will yield again

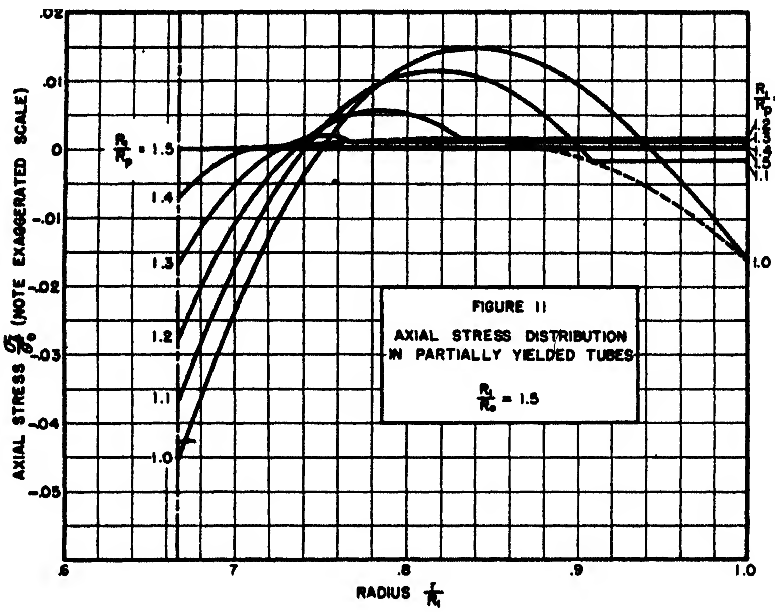


FIGURE 11
AXIAL STRESS DISTRIBUTION IN PARTIALLY YIELDED TUBES
$R_1/R_0 = 1.5$

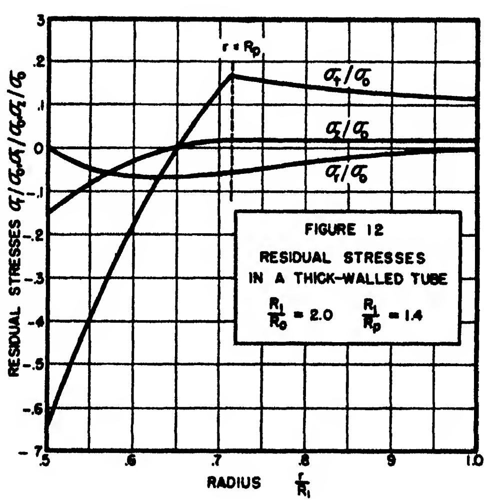


FIGURE 12
RESIDUAL STRESSES IN A THICK-WALLED TUBE
$R_1/R_0 = 2.0$ $R_1/R_p = 1.4$

upon removal of the expanding pressure, and that the actual state of residual stress will be altered from that shown in the figure.

As it is generally inadvisable to have the metal at the bore of a tube yield again upon removal of the expanding pressure, especially when the

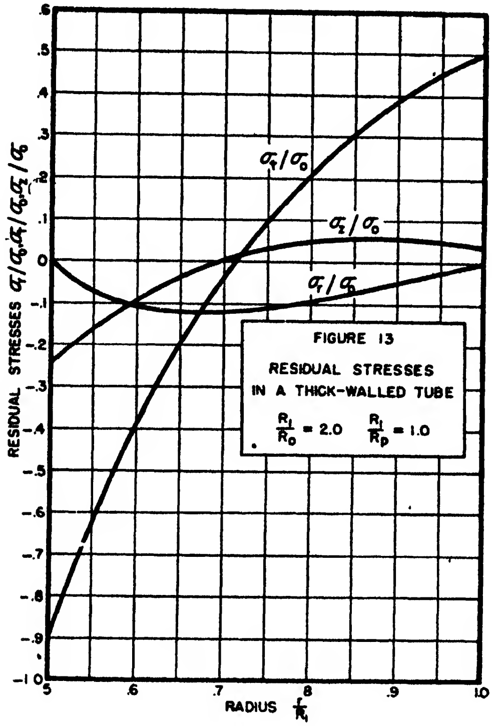


FIGURE 13
RESIDUAL STRESSES
IN A THICK-WALLED TUBE
$\frac{R_1}{R_0} = 2.0$ $\frac{R_1}{R_p} = 1.0$
σ_t/σ_0
σ_z/σ_0
σ_r/σ_0
RESIDUAL STRESSES $\sigma_r/\sigma_0, \sigma_t/\sigma_0, \sigma_z/\sigma_0$
RADIUS $\frac{r}{R_1}$

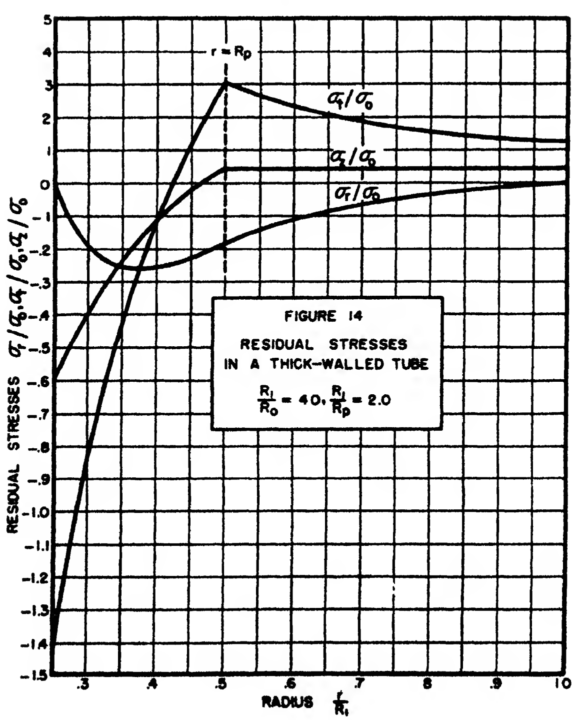
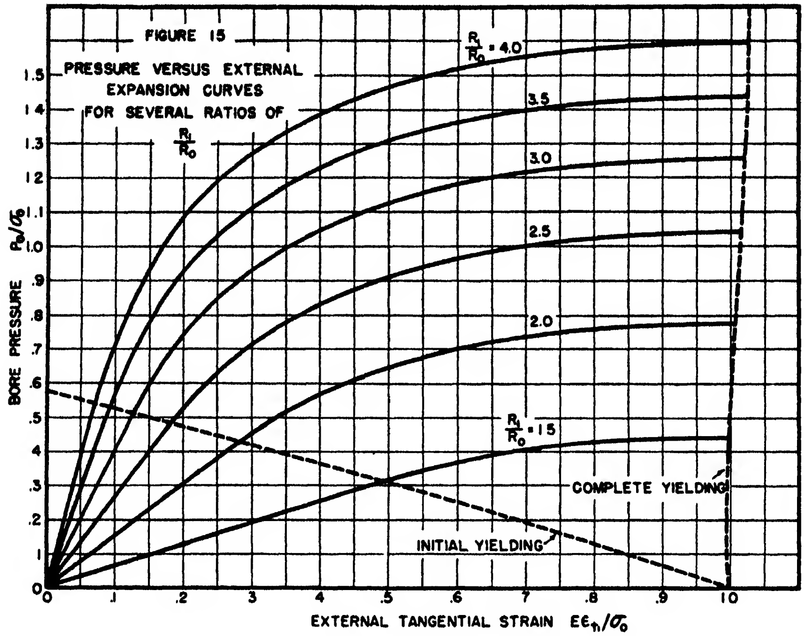
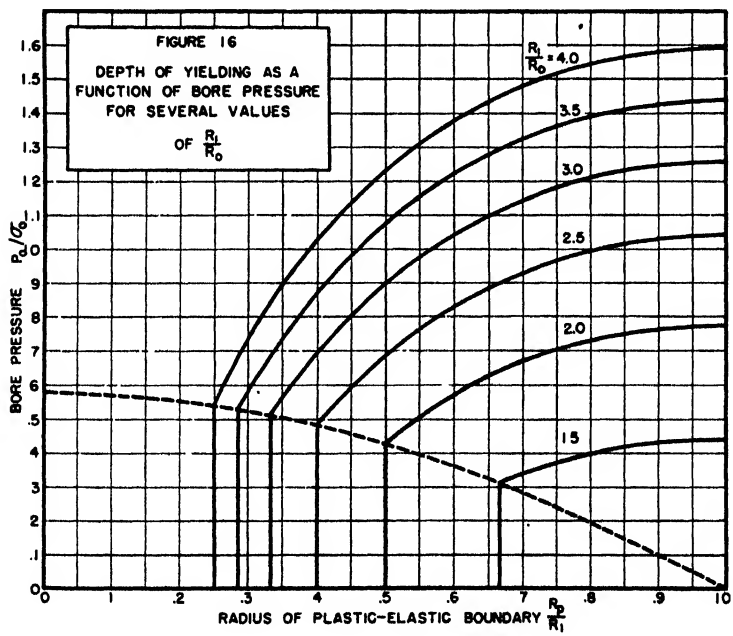


FIGURE 14
RESIDUAL STRESSES
IN A THICK-WALLED TUBE
$\frac{R_1}{R_0} = 4.0, \frac{R_1}{R_p} = 2.0$
$r = R_p$
σ_t/σ_0
σ_z/σ_0
σ_r/σ_0
RESIDUAL STRESSES $\sigma_r/\sigma_0, \sigma_t/\sigma_0, \sigma_z/\sigma_0$
RADIUS $\frac{r}{R_1}$

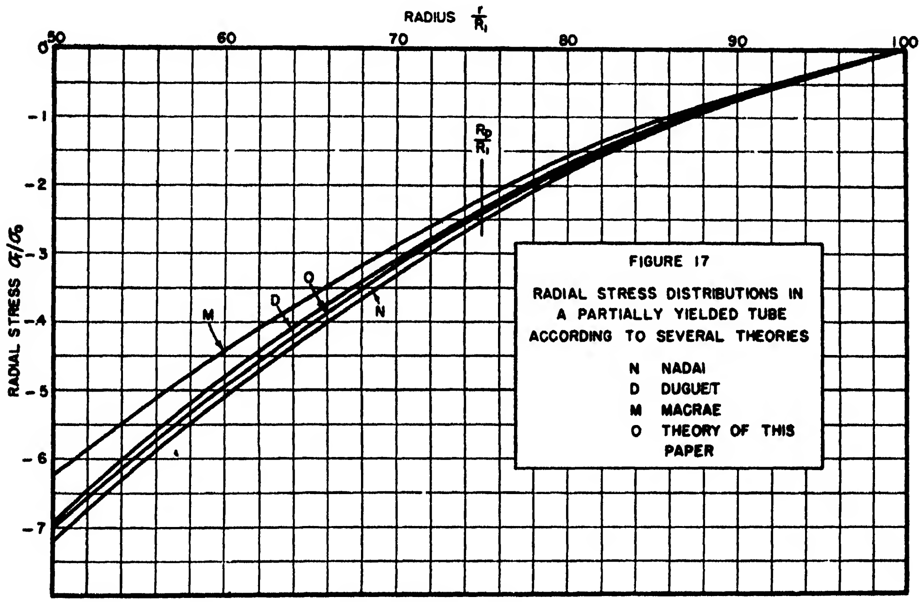




tube is to be subjected to cyclic applications of internal pressure, it is important to be able to determne accurately the state of residual stress in a partially yielded tube. If the residual effective stress at the bore computed on the assumption of elastic recovery of the tube does not

exceed σ_0, the residual stresses so found will be those actually present, and the tube will not yield upon removal of the expanding pressure. A negligible Bauschinger effect has been assumed in this discussion.

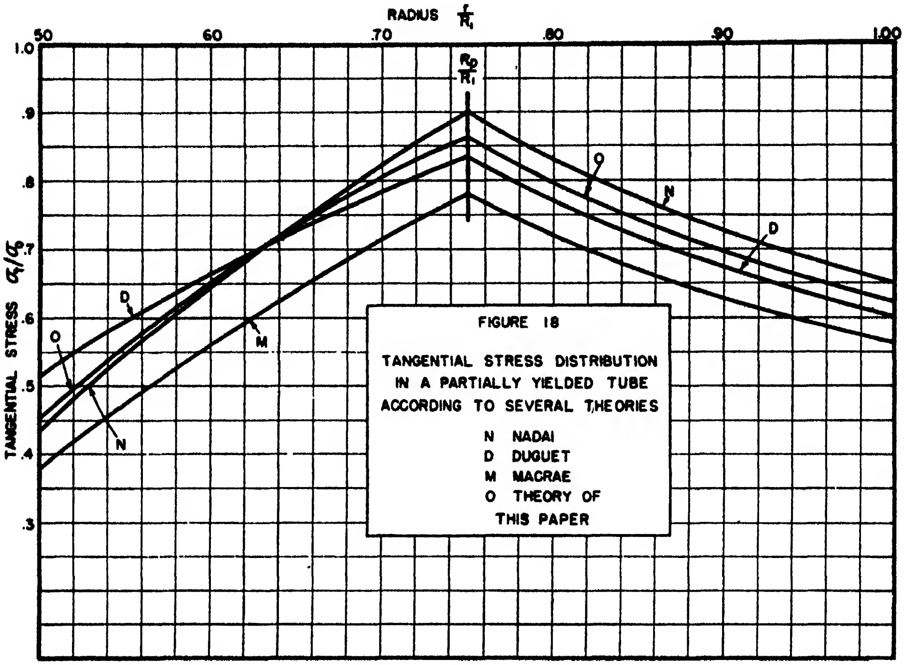
Figures 15 and 16 show further information which can be obtained from the same tables. Pressure vs. external expansion curves are given in Fig. 15 for tubes with several ratios of R_1/R_0. These curves are interesting in that they relate the variables most generally measured during the controlled plastic deformation of a thick-walled tube. Depth of yielding as a function of internal pressure and wall ratio R_1/R_0 is given in Fig. 16.

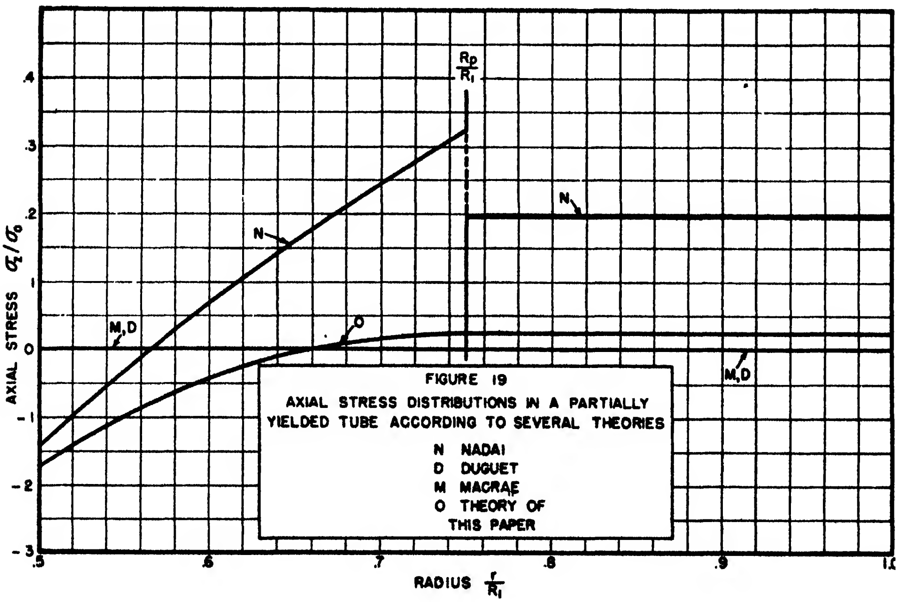


DISCUSSION.

Although a general survey of the various theories for the partial yielding of thick-walled tubes cannot be given at this time, a simple comparison will show the magnitude of the corrections introduced by the new theory. Figs. 17–19 compare the stresses predicted by the new theory with those predicted by the theories of Nádai, Duguet and Macrae. Fig. 20 compares the pressure vs. external expansion and pressure vs. axial strain curves predicted by these theories.

The theory of Macrae, based upon the maximum shear criterion of plastic flow, does not predict an axial stress distribution. The radial and tangential stresses corresponding to a given depth of yielding are significantly lower than those predicted by the theories based on the well-established distortion energy criterion of plastic flow, and the pressure vs. external expansion curve also falls lower for this theory.





The theory of Duguet, based on the concept of shear-with-friction similarly does not consider an axial stress distribution. The radial and tangential stresses agree fairly well with those of this paper, as does the curve of pressure vs. external expansion. A serious defect of the Duguet

theory becomes apparent when the residual stress system is considered after removal of the internal pressure. The sheár-with-friction theory of yielding assumes a compressive stress for flow equal to about 1.43 times the tensile stress for flow, and indicates that residual compressive tangential stresses numerically as great as 1.43 σ_0 can exist at the bore without yielding of the tube. A design based on this figure would undoubtedly lead to further yielding at the bore upon removal of the expanding pressure.

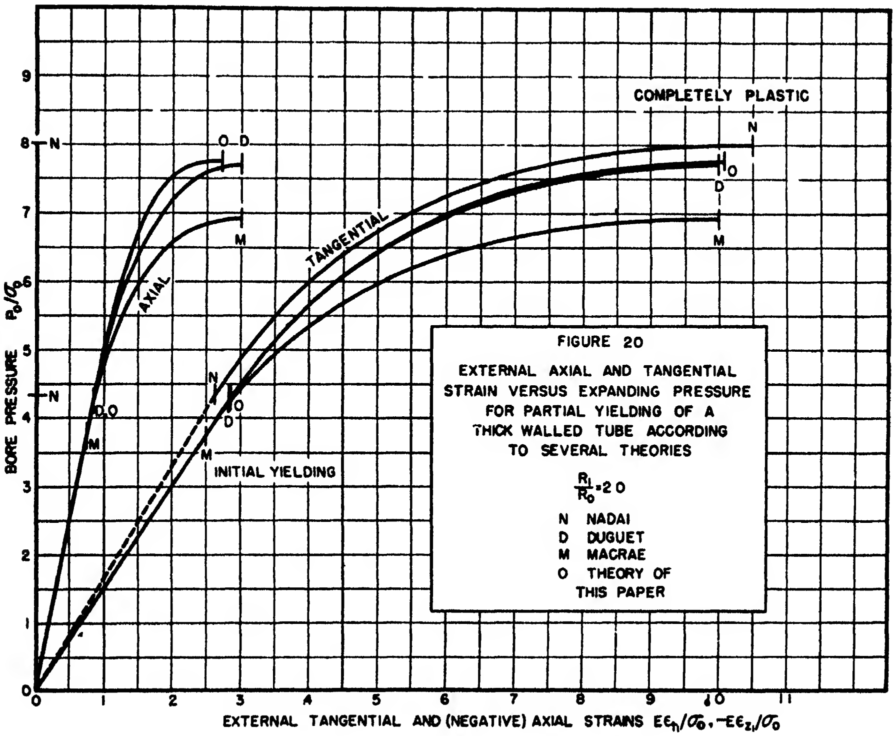


FIGURE 20
EXTERNAL AXIAL AND TANGENTIAL STRAIN VERSUS EXPANDING PRESSURE FOR PARTIAL YIELDING OF A THICK WALLED TUBE ACCORDING TO SEVERAL THEORIES

$\frac{R_1}{R_0} = 2.0$

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The theory of Nádai agrees fairly well with the theory of this paper except for the axial stress distribution and the axial strain.

From this comparison, it follows that the new theory checks the order of magnitude of the radial and tangential stresses obtained by the approximate theories of Nádai and Duguet. The favorable check lends more confidence to designs based on the older theories, wherever axial stresses and strains were not involved, and where residual compressive stresses greater than σ_0 were not predicted by the Duguet theory. Where the axial stress and strain are needed, however, and where assurance is required that the tube will not yield again on removal of the expanding pressure, and in general for all precise design, the new theory provides a satisfactory solution of the problem.

It should be noted that while the comparisons made between the theory of this paper and earlier theories were for partial plastic yielding of tubes under internal pressure and zero end load assuming an idealized stress-strain law, the theory presented herein is more general than earlier ones. The present theory includes the partial yielding of tubes loaded by any combination of internal pressure, external pressure and end load, and is valid for a material following an arbitrary stress-strain law.

SUMMARY AND CONCLUSIONS.

A new theory has been presented for the partial plastic yielding of thick-walled cylindrical tubes acted upon by internal pressure, external pressure and end load when the material follows an arbitrary stress-strain law. This solution combines (a) the distortion energy theory of plastic flow and (b) the effects of the compressibility of the plastic material. The resulting stresses and strains in all directions can be computed and no discontinuities of stresses or strains are present at the plastic-elastic boundary as occur in earlier solutions.

As examples, typical curves are included showing the variation of the stresses over the wall of the tube for various wall ratios and depths of yielding. These are for the special case of open-ended tubes under internal pressure and following an idealized stress-strain law. The general theory presented, however, is not necessarily restricted to these special conditions and is applicable to any combination of internal pressure, external pressure, end load and general stress-strain relation.

The special examples included compare the stress distributions and the pressure-external expansion curves as predicted by this theory with those reported by Nádai, Duguet and Macrae. Significant differences which will be of interest in precise design have been observed between the present solution and the earlier approximate solutions.

ACKNOWLEDGMENT.

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NOMENCLATURE.

A Constant of integration.
a_r $\sigma_r - \frac{1}{2}(\sigma_t + \sigma_z)$.
a_t $\sigma_t - \frac{1}{2}(\sigma_z + \sigma_r)$.
a_z $\sigma_z - \frac{1}{2}(\sigma_r + \sigma_t)$.
B Constant of integration.
C Constant of integration.
D Variable coefficient relating stresses and strains during plastic deformation.
E Modulus of elasticity.
Constant appearing in the shear-with-friction theory of plastic flow. Generally taken as 0.7.
F End load.
m $\frac{1}{E^2}\frac{\sigma}{\epsilon}\frac{d\sigma}{d\epsilon}$.
p_0 Bore pressure.
p_1 External pressure.
r Radius.
R_0 Bore radius
R_p Radius of plastic-elastic boundary.
R_1 External radius
x $\log_e \frac{r}{R_1}$
ϵ Effective strain, $\frac{\sqrt{2}}{3}\sqrt{(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2}$.
$\left.\begin{array}{l}\epsilon_1\\ \epsilon_2\\ \epsilon_3\end{array}\right\}$ principal strains (total of elastic plus plastic).
ϵ_r Radial strain (total).*
ϵ_t Tangential strain (total).*
ϵ_z Axial strain (total).*
ν Poisson's ratio.
σ Effective stress, $\frac{\sqrt{2}}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$.

$\left.\begin{array}{l}\sigma_1\\ \sigma_2\\ \sigma_3\end{array}\right\}$ Principal stresses

σ_r Radial stress.*

σ_t Tangential stress.*

σ_z Axial stress.*

σ_0 Yield stress (see Fig. 2).

ϕ_1 $\frac{4m}{9}(\omega_1 + \omega_2)(a_r^2 + a_t^2 + a_z^2)$.

ϕ_2 $\frac{4m}{9}(\omega_1 + \omega_2)^2 - 1$.

ω_1 $1 + D$.

ω_2 $\nu + \frac{D}{2}$.

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* A second subscript "0," "p," or "1" denotes the value of this variable at $r = R_0$, $r = R_p$, or $r = R_1$, respectively. Primes (′) indicate differentiation with respect to x.

INSULATION VARIABILITY.

A General Equation for Strength Distribution Curves.

BY

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An understanding of what follows requires a knowledge of the basic concepts of statistics and probability theory. Familiarity on the part of the reader with such concepts as distribution curves, standard deviation, coefficient of variability, and error function tables is taken for granted.

Earlier work by the author [1] dealt with the effect of area and inhomogeneity in lowering the breakdown voltage of systems of insulation. That paper dealt with mean values only. It was realized at that time that further progress in the field would require a knowledge of the forms of the distribution curves themselves and the changes that take place in them when units are connected in parallel. At that time, however, a solution to the problem was not clear.

The present paper deals with the actual change in strength distribution curve to be expected when units of insulation are connected in parallel. It also shows how such knowledge may be applied to specific problems.

THE GENERAL PROBLEM.

Given the breakdown distribution curve for a certain "lot" of insulating units, what kind of distribution will result when the units are tested in parallel, n units at a time?

SOLUTION.

Let $p(v)$ be the distribution function for the breakdown voltages (BDV's) of the basic insulating "units."

The probability of any unit failing above some given voltage v will then be given by the expression $\int_v^\infty p(v)dv$.

For n units in parallel the probability that all n units will fail above v is $[\int_v^\infty p(v)dv]^n$. This follows from the law of repeated independent events in probability theory.

[1] See "Insulation Variability, Its Influence in Determining Breakdown Voltages," presented at the Summer Convention of the A.I.E.E., June 1931. Also, "Breakdown Voltage as a Function of Electrode Area and Dielectric Homogeneity," JOUR. FRANK. INST., June 1931.

The probability that they will *not* all fail above v, i.e. that at least one will fail below v, then becomes $1 - [\int_v^\infty p(v)dv]^n$. But this is the condition for breakdown of the entire system of n parallel connected units. Therefore, letting $f_n(v)$ represent the distribution of breakdown voltages for the systems of n parallel connected units we have immediately the cumulative form of the required distribution

$$\int_0^v f_n(v)dv = 1 - \left[\int_v^\infty p(v)dv\right]^n. \tag{1}$$

INTERPRETATION.

This equation (1) is a basic equation. It is the starting point for the study and testing of inhomogeneous insulation. Its appearance, for a series of specific values of n, is shown in Fig. 1.

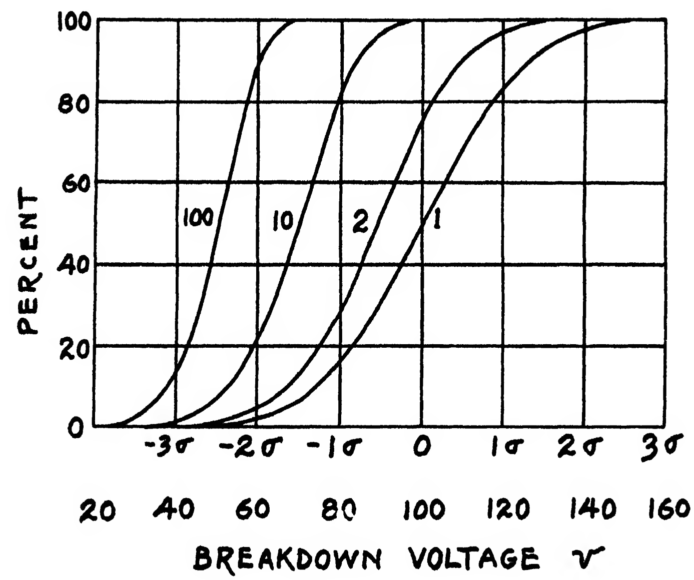

FIG. 1. Equation (1) plotted for different values of n for a normal distribution.

The curve marked (1) is the distribution of the parent population from which the "units" have been selected. It is a normal distribution with mean value at 0 and standard deviation σ equal to unity. Any voltage scale may be used. In this example, one extending from 0 to 160 volts has been chosen. This places the mean of the original distribution at 100 and thus allows one to read percentage changes directly. Such a particular choice makes the standard deviation $\sigma = 20$ voltage units and stands for a rather extremely inhomogeneous material. The ordinate scale is a simple percentage one. It has the usual meaning associated with cumulative distribution curves. For illustration, reading upward, from a voltage scale of 90, one strikes the original curve (1)

at a value of 30 per cent. This means that if the potential is raised to 90 volts, thirty per cent. of the original units will have failed. Similarly, a voltage of 100 corresponds to a percentage of 50, meaning that by the time a voltage of 100 is attained, one-half of the original units can be expected to have failed. Sixty-nine per cent. will have failed at 110 volts, 85 per cent. at 120, and practically all units will have failed at 150 volts.

Curves in Fig. 1 marked (2), (10), and (100) have been computed from eq. (1). Curve (2) represents the distribution of failures when samples are selected from the original lot and tested to destruction in parallel groups taken two at a time. Curves marked (10) and (100)

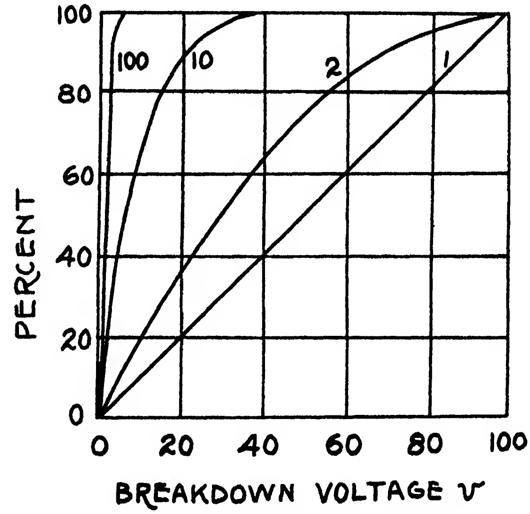

FIG. 2. Equation (1) plotted for different values of n for a uniform distribution.

are for systems of 10 in parallel and 100 in parallel respectively. Percentage of failures at different voltages may be picked off the curves as in the illustration for the original units. For example, at 60 volts approximately five per cent. of the two unit systems will have failed, twenty-one per cent. of the ten unit systems, and ninety per cent. of all the 100 unit systems. As remarked above the 60 volts might represent kilovolts or any other unit desired in a specific problem.

It is interesting to note that the derived distributions are no longer normal. This is the first time this has been shown.

Fig. 1 showed what happened in the case of an originally normal distribution. Fig. 2 shows what happens to the derived curves when the original lot is *not* normal. As a specific case a *uniform* distribution of original units extending from 0 to 100 was chosen. The curves marked (2), (10) and (100) in Fig. 2 give the results for breakdown tests made on systems of two units in parallel, ten units in parallel, and one

hundred units in parallel respectively. The curve marked (1) is the original uniform distribution. Here again it is clear that the derived curves are quite changed from the original form. As before, the voltage scale may be shifted in any way desired.

APPLICATIONS.

The general equation can be used to answer specific questions of the following type.

Problem 1. Units of insulation were tested for breakdown strength with the following results: Mean BDV 120 K.V., standard deviation 24 K.V., coefficient of variability 20 per cent. At what operating voltage will there be ten per cent chance of failure for systems consisting of 100 of these units in parallel?

Using curve (100) of Fig. 1, at the ordinate 10, the voltage is read as approximately 39 per cent. 39 per cent. of 120 is 47 K.V. *Ans.*

Note: Had the problem been for a more uniform type of insulation, it would have been necessary to change only the voltage scale of Fig. 1. Thus, had the standard deviation been only 6 K.V. instead of 24 K.V., 100 on the chart would be 120, 80 would be replaced by 114, 60 by 108, 40 by 102, etc. And the answer to the problem would have been approximately 101 K.V. instead of 47 K.V. This, incidentally, demonstrates the effect of uniformity in insulation. The more uniform the material the higher the allowable operating voltage.

Problem 2. What chance of failure has the above system if operated at 84 K.V.? 36 K.V.?

Solution: 84 K.V. is 70 per cent. of 120 K.V.
36 K V. is 30 per cent. of 120 K.V.

Here again the answers can be read directly from curve (100) of Fig. 1 at 70 and 30 on the breakdown voltage scale. They are approximately 100 per cent. or certainty for the first case of 84 K.V. and 2 per cent. for the second case of 36 K.V. This means the odds are roughly 50 to 1 that the system will not fail at an operating voltage of 36 K.V.

It can be seen further that chance of failure becomes negligible at operating voltages lower than 25 per cent. or 30 K.V.

SUMMARY.

The equation developed here is fundamental for an understanding of the behavior of inhomogeneous insulation, especially when the insulating material is of considerable extent in the form of continuous flat sheets, or cables, or discontinuous form as in pole-line construction. The "unit" may then be measured in terms of area in the case of sheets, and in terms of length in the case of cables. It is of basic importance in insulation research, testing, and operation. It furnishes the logical basis for the establishing of safety factors.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

STANDARDIZATION OF THE pH SCALE.

The pH unit, used to express numerically the degree of acidity or alkalinity of aqueous solutions, may be defined in a number of ways, each resulting in a slightly different value for the pH of a given solution. Consequently, several pH scales, based upon various definitions, have met with equal favor among chemists. In view of the increasing need in science and industry for accurate determinations of acidity, the National Bureau of Standards is recommending the universal adoption of a single standard pH scale, analogous to the International Temperature Scale. It is proposed that the pH assigned to solutions of buffer substances distributed by the Bureau as Standard Samples be taken as the fixed points on this standard scale.

In the preparation of many commercial products—for example, paper, textiles, dyes, ceramics, and beer—the rapidity and efficiency of the processes depend upon accurate control of the acidity or alkalinity of aqueous solutions. Such control is now a regulatory requirement in the preparation of certain medicines and in the manufacture of paper and leather for the Government. In sugar manufacture, the inversion of sucrose can be regulated at will or avoided entirely by holding the acidity within certain limits. Similarly, regulation of the acidity of electroplating solutions permits the character of the deposit to be controlled. Another application of particular importance is the avoidance of corrosion and embrittlement of boiler walls and tubes by regulation of the acidity of boiler water. The widespread losses due to underground corrosion are likewise effectively curbed in many cases by proper adjustment of acidity.

The several convenient pH meters now available commercially enable precise determinations of pH values in such varied media to be made with ease and rapidity, but these values are based upon a scale fixed by the pH assigned to the standards with which the instrument has been calibrated. The differences among scales of pH are the direct result of different procedures, definitions, and assumptions employed in arriving at the pH of the standard. The pH may be defined in one instance as the negative logarithm of the hydrogen-ion concentration or, again, of the effective concentration or "activity" of this ion. Often the pH value as defined by Sorensen in terms of the electromotive force of a galvanic cell with hydrogen and calomel electrodes is chosen. Al-

* Communicated by the Director.

though the differences among these scales rarely exceed 0.1 unit, the need for greater accuracy makes desirable the general adoption of a single series of consistent pH standards.

In an effort to encourage standard procedure in pH measurements, the National Bureau of Standards is now supplying four buffer materials in the form of Standard Samples of certified purity. These substances are acid potassium phthalate, potassium dihydrogen phosphate and disodium hydrogen phosphate (intended to be used together), and borax. They are being distributed at the rate of several hundred samples annually. The certificates furnished with these compounds specify the pH of certain aqueous solutions of the sample, which can provide fixed points on a pH scale.

In order to assign exact values to these fixed points, it was necessary to set up a scale based upon some suitable definition of pH. A consideration of the advantages and limitations of several scales led to a choice of a modified activity scale as most convenient and practical for general use. Although the activity of a single ionic species can be simply defined only in very dilute solutions, the influence of the hydrogen-ion activity in chemical equilibria is of far-reaching importance.

The pH of the NBS standards is derived from measurement of the electromotive force of cells without liquid junction, in which they are used as electrolytes. These cells are specially designed, utilizing the highly reproducible hydrogen and silver-silver chloride electrodes. Computation of pH is based upon several reasonable assumed relationships between ionic activities and mean activities. These assumptions are found to give identical values for dilute solutions. The scale thus obtained approaches a true scale of activity for solutions of low concentration; at higher ionic strengths it is best regarded as a consistent scale which necessarily rests upon an assumption not subject to experimental proof.

AUTOMATIC IONOSPHERE RECORDER.

A new instrument for automatic recording of ionospheric phenomena, which are of great practical importance in radio propagation, is now in operation at the Bureau's ionospheric research station at Stirling, Virginia. The prototype model of the recorder, developed by the Bureau's Central Radio Propagation Laboratory, was completed in time to participate in the recent Army Air Forces National Geographic Society Eclipse Expedition in Brazil. In addition to special records during the May 20 eclipse of the sun, valuable data were obtained on general ionosphere conditions. Plans are being made to install these recorders in all the ionosphere stations operated by the Bureau.

Long-distance radio transmission would be impossible if it were not for the ionosphere, a series of ionized layers in the atmosphere 50 to 250

miles above the earth that reflect radio waves back to earth. The new recorder will provide automatic and continuous measurements of the heights of the various layers and of critical frequency, that is, the maximum frequency that is reflected back to earth rather than passing off into space.

The Model C2 recorder was designed to utilize the so-called multifrequency technic of investigating the layers of the ionosphere. It is the first completely automatic recorder to use in continuous heavy duty the heterodyne pulse transmitter arrangement recently described by P. G. Sulzer.[1] In this arrangement the entire frequency range from 1.0 to 25.0 Mc./s. is covered continuously without switching bands. The receiver is tuned with the transmitter throughout its frequency range. Pulse transmissions are used similar to those employed in radar except with varying probing frequency. The frequency is plotted against the time delay of the echoes from the ionosphere, an interval that corresponds to twice the height of reflection. One sweep in frequency from lower to upper limits produces each ionosphere record in a time interval of as little as $7\frac{1}{2}$ seconds.

Two oscilloscopes provide indications corresponding in essential detail to the radar "A" and radar "B" scans. A special 35-mm. camera, driven by the motor that operates the transmitter oscillator, makes a continuous film record of the indications of the main oscilloscope. As the image of the sweep line is oriented at right angles to the direction of film travel, the graph of height reflection (time delay) versus frequency is recorded on height scales of 500, 1,000, and 4,000 kilometers. Simultaneous visual observation is possible by means of the monitoring oscilloscope.

A standard 16-mm. motion picture camera is provided to photograph the monitoring oscilloscope. When a series of 16-mm. film records, made at $7\frac{1}{2}$ or 15 second intervals over a long period of time, are projected, a striking accelerated version of the changes taking place in the ionosphere is observed.

The main cabinet of the new ionosphere recorder houses no moving parts, but encloses the transmitter mixer, transmitter wide-band amplifier, receiver mixer, receiver intermediate frequency unit with detector and video stages, pulsing and keying circuits for the transmitter, the transmitter fixed frequency unit, and all the power supplies for these units.

A smaller cabinet on rollers contains the recording and monitoring oscilloscopes, the recording 35-mm. camera, the variable frequency oscillator, the camera and oscillator drive motor, switches and their associated control, time base and sweep-generating circuits, and the necessary power supplies.

[1] See "Ionosphere Measuring Equipment" by P. G. Sulzer, July *Electronics*, 1946.

Other features of the Model C2 recorder are: a pulse receiver having large dynamic range and a differentiating circuit to minimize interference by CW (continuous wave) and broadcast stations, provision for automatic operation by a clock completely independent of power-line frequency, the use of hermetically sealed units to insure reliable operation in many climates, and regulation of every important d.-c. voltage to provide stability of power source.

The replacement of the old recording units in the ionosphere station network with an automatic and continuous recorder will assure a more comprehensive and reliable flow of worldwide ionosphere data into the National Bureau of Standards. To date, the principal emphasis in the study of ionospheric phenomena has been the practical use of the information in the prediction of radio propagation conditions. However, data on the ionosphere, which may be considered as a strategically located astrophysical radiation laboratory, will reveal more and more of the characteristics of radiation from the sun, and the physical conditions of the outer atmosphere, its temperatures and densities, mean free paths of electrons, recombination processes, and geomagnetic effects. These data may also lead to information on air circulation and other phenomena in the lower atmosphere (stratosphere and troposphere).

LIGHT WAVE OF MERCURY 198 AS THE ULTIMATE STANDARD OF LENGTH.

A new and better standard of length now exists in the wavelength of green radiation of mercury 198, an isotope transmuted from gold by neutron bombardment. In precision, reproducibility, and convenience, the new standard is superior to both the standard meter and the red line of cadmium, according to recent investigations by Dr. William F. Meggers of the National Bureau of Standards. Preliminary measurements by Dr. Meggers have shown an accuracy of one part in a hundred million of relative values and one part in a billion is theoretically possible.

Since 1889 the world's standard of length has been the "meter" distance between two lines on a platinum-iridium bar at the International Bureau of Weights and Measures in France. Fundamental measurements throughout all of science and industry are based on this standard, but it has several disadvantages. First, line standards are unsuitable in certain fields of measurement. Second, the intrinsic nature of lines ruled on surfaces—such lines are in effect small furrows—limits the precision attainable. Third, the meter is not readily reproducible.

Primarily because the standard meter does not afford sufficient precision in some fields, the red line of cadmium has been universally used for many years for precise measurements. However, the cadmium standard also has serious disadvantages. First, there is a fine structure

in the red radiation which prevents the line from being as sharp as desirable and thus limits the precision possible. Second, the cadmium standard requires excitation in a furnace which entails unwanted broadening of the spectral line because of relatively high temperature.

The green line of mercury 198 has none of the disadvantages of either the meter or the red line of cadmium. The normal human eye is far more sensitive to green than to red, an important consideration in visual adjustment of the interferometer with which lengths are measured and compared. All other characteristics desirable in a light wave standard—such as ability to be reproduced, absolute sharpness of the wavelength, intensity of the spectral line, life and convenience of maintenance—are possessed to a greater extent by mercury 198.

The future refinement of physical optics—for example, an accurate determination of the velocity of light—and the improvement of mechanical processes—for example, the ruling of better diffraction gratings—are dependent on the production and adoption of an ultimate standard of length superior both to the meter bar and to the wavelength of red radiation from cadmium. The nuclear reaction that now makes possible large scale transmutation and manufacture of pure elements not found in nature will also produce any desired quantity of the pure mercury from gold, and thus provide a material for a spectroscopic light source that emits light waves much more monochromatic than any emitted by natural elements. Theoretically, mercury isotope 198 should show interference patterns with retardations exceeding a million waves, and because it is possible with monochromatic lines to measure one-one thousandth of a wave, it is probable that the relative value of Hg^{198} wavelengths may eventually be determined with an accuracy of one part in a billion.

As long ago as 1927, the National Bureau of Standards recommended that the International Conference of Weights and Measures adopt a light wavelength, that of red radiation from cadmium vapor, as the primary standard of wavelength, and that the meter be defined in terms of this wavelength. The Conference objected that such a definition of the meter would menace the metric system, and explained that it was not a question of giving a true relation between the meter and the wavelength, but only a metric value of the latter which could be modified by future experiments. Strictly speaking, the world's primary standard of length is still the distance between two relatively wide lines drawn on a metal bar, despite the fact that practically all precise measurements of lengths in the 20th century have been made, and will continue to be made, with light waves.

The most monochromatic spectral lines are emitted by massive slow-moving atoms, and because mercury atoms are nearly twice as heavy as cadmium atoms and can be excited to radiate at less than half the absolute temperature, mercury lines are less than half as wide as

cadmium lines, other things being equal. Wavelengths from natural mercury cannot be used as standards of length because natural mercury consists of a fixed mixture of seven isotopes with atomic masses of 196, 198, 199, 200, 201, 202, and 204, and each isotope emits one or more spectral components none of which are exactly coincident. Consequently, the green line of natural mercury has sixteen components.

Because the effective wavelength of such a complex line observed interferentially varies with the phase relations of the various components, it is imperative to avoid complex lines in selecting a natural standard of length. This objectionable feature of mercury lines could be removed if a single isotope, for example Hg^{204}, could be separated from the rest, but up to the present it has not been practicable to isolate an isotope of natural mercury in sufficient quantity to make satisfactory lamps. However, this goal has now been achieved by transmuting gold (Au^{197}) into mercury (Hg^{198}). The feasibility of doing this was first demonstrated in 1940 by J. Wiens and L. W. Alvarez who reported that bombardment of gold by neutrons from a 60-inch cyclotron at the University of California produced enough mercury to be detected spectroscopically.

In 1942, the National Bureau of Standards purchased forty ounces of proof gold and enlisted the cooperation of the University of California to expose this gold to neutrons for one or more years. Unfortunately, World War II interrupted the experiment and only submicroscopic quantities of artificial mercury were made. The prospects were very discouraging until, near the end of the war, there were rumors of a secret source of neutrons thousands of times more effective than the largest cyclotron. In 1945, the National Bureau of Standards gold was transferred from California to Tennessee. The treatment this gold received was not disclosed but a year later the Bureau distilled from it about 60 milligrams of mercury, which was found from spectroscopic tests by Dr. Meggers, chief of the Bureau's Spectroscopy Section, to be pure Hg^{198}. Anticipating a considerable demand for Hg^{198} lamps, the Bureau has requested the Atomic Energy Commission to bombard some more gold with neutrons to produce one or more grams of Hg^{198} within a year. In the meantime, the available Hg^{198} has been used by Dr. Meggers in the preparation of several types of lamps which are being studied to determine the one most suitable for adoption as a standard.

In the design of a Hg^{198} lamp that will emit radiations suitable as ultimate standards of length a maximum is desirable in each of the following five characteristics: (1)monochromaticity, (2) reproducibility, (3) intensity, (4) life, and (5) convenience. It is perhaps obvious that some of these requirements conflict with others, and that it will be necessary to make compromises. It appears probable that either electrodeless tubes or Geissler tubes (similar to the ubiquitous luminous

signs), containing several milligrams of Hg^{198} and a small amount of argon gas will be useful for accurate measurements.

Employing an electrodeless lamp excited by high frequency radio waves, preliminary values of the wavelengths of a dozen Hg^{198} lines, ranging from the ultraviolet (3341 A) to the yellow (5791 A), have been measured by Dr. Meggers. Though publication of the observed wavelengths of Hg^{198} will be deferred until final values are in hand, these preliminary values, when tested by the combination principle of spectroscopy, appear to be correct within one part in one hundred million, whereas the best measurements made with natural mercury exhibit deviations of one part in one hundred thousand, due, no doubt, to the falsification of the wavelengths by the complexity of the lines.

Although cadmium and mercury are divalent chemical analogues, and therefore exhibit relatively simple and similar atomic spectra, whatever differences exist are invariably in favor of mercury. For example, the brightest line in the cadmium spectrum occurs in the blue-green (5086 A), whereas the mercuric analogue is in the green (5461 A) nearly coincident with the maximum sensitivity of the normal human eye. The red wave of cadmium (6438 A) is intrinsically only one-tenth as intense as the strongest line (5086 A), and is further handicapped by the fact that the eye is only one-seventh as sensitive for red as for green. Thus for the visual adjustment of interferometers the green line of mercury is seventy times as intense as the red line of cadmium. The mercury analogue of the cadmium red line is a yellow line (5791 A) which is always accompanied by another yellow line of shorter wavelength (5770 A) but nearly equal intensity. This yellow pair of mercury lines produces interference coincidences at intervals of 275 waves, and is happily heuristic for the whole order of interference without counting any fringes; it has no convenient counterpart in cadmium.

Mercury is the only heavy stable element that has an appreciable vapor pressure below zero degrees Centigrade, and therefore is unique among all elements in radiating, at low pressure and temperature, a relatively simple spectrum of extremely sharp lines provided isotopic structure is eliminated. The green line of mercury, rejected by Michelson 55 years ago on account of complex structure, has finally, by the production of mercury 198, been freed of its seven-isotope curse, and the green line of Hg^{198} now stands alone as the most nearly ideal standard wavelength that can ever be obtained from any atoms, natural or artificial. Coupled with the fact that adequate quantities of absolutely pure Hg^{198} are now obtainable by neutron bombardment of gold in chain-reacting piles, the unique properties of Hg^{198} force the conclusion that a progressive scientific world will eventually adopt the wavelength of green radiation (5461 A) from Hg^{198} as the ultimate standard of length.

The meter unit, the present unit of length, was created about 1790 to represent one-ten millionth of the earth's quadrant. In 1827, some natural philosophers meeting in Paris agreed that the meter could not be reproduced if the form of the earth were changed by collision with a comet. A Frenchman, Jacques Babinet, then proposed a light wave in a vacuum as a natural unit of length independent of the earth's dimensions. Later the same thought was expressed by German, Dutch, and British scientists, but the first pratical results must be credited to Americans, A. A. Michelson and E. W. Morley, who, in 1887, outlined "A method of making the wavelength of sodium light the actual and practical standard of length." Their method, involving the use of the optical interferometer devised by them for their celebrated experiments on the relative motion of earth and ethei, consisted of the measurement of a length and the counting of an equivalent number of interference fringes.

In 1889, Michelson and Morley described in detail a method of measuring the meter in light waves, and predicted that the brilliant mercury green line would in all probability be the wave to be used as the ultimate standard of length. Searching systematically for the radiation best suited as an ultimate standard, Michelson discovered in 1892, that the green light of mercury is complex, and discarded it in favor of the red light of cadmium. These classic investigations promptly led to Michelson's invitation to the International Bureau of Weights and Measures, where he performed his celebrated determination of the relation between the meter and the wavelength of cadmium red radiation. In the succeeding forty years Michelson's experiment was repeated a half dozen times and his result has been amply confirmed, considering the fact that the lines on the meter bar are ten to twelve wavelengths wide.

Indeed, it is the character of ruled lines themselves which limits the accuracy of wavelength-meter intercomparisons and there is therefore hardly any point to measuring the wavelengths of Hg^{198} lines relative to the meter. The wavelength of Hg^{198} green light can readily be measured relative to cadmium red light from ten to one hundred times more accurately than either relative to the meter. Adoption of the present provisional relation as exact, and subsequent substitution of Hg^{198} green for cadmium red appears to be the logical and expeditious approach to a better standard of length.

THE FRANKLIN INSTITUTE

ANNUAL MEETING, WEDNESDAY, JANUARY 21, 1948.

The Annual Meeting of The Franklin Institute was held on Wednesday evening, January 21, 1948, Mr. Richard T. Nalle, President, presiding. There were approximately 200 persons present.

The President stated that the minutes of the Stated Monthly Meeting for November were printed in full in the December issue of the JOURNAL OF THE FRANKLIN INSTITUTE. On motion, duly seconded, they were approved as printed.

The President also stated that the annual report from the Board of Managers, due to be presented at the Annual Meeting, would be presented at an adjourned meeting in the spring.

The report of the Secretary for the month of December was as follows:

Membership

Active	33
Associate	21
Student	36
Total Membership as of December 31	5424

The President then called on Dr. George S. Crampton to give the report of the tellers on the annual election for members of the Board of Managers, who were to serve for a period of three years.

Hiram S. Lukens
Orus J. Matthews
Morton Gibbons-Neff
George Wharton Pepper
M. M. Price
Charles S. Redding
James S. Rogers
Clarence Tolan, Jr.

Dr. George S. Crampton, Professor Charles M. Gay, and Mr. Ralph McClarren were the tellers for the election returns. The President declared the above members as duly elected to the Board of Managers. The President expressed his appreciation to the tellers for their service to the Institute.

The President then presented the speaker of the evening, Dr. W. F. G. Swann, Director of the Bartol Research Foundation of The Franklin Institute, Swarthmore, Pa. Dr. Swann spoke on "The Place of Science in Our Civilization." He traced the influence of science in its effect on the normal activities of man, its effect in removing the causes of unhappiness, and in providing him with more leisure. This raised the question as to how man can occupy himself with that leisure for the greatest good of the world and for his own happiness. Dr. Swann dealt also with the relative roles of utilitarianism and non-utilitarianism in the fields of man's activities, with the end point—the greatest happiness for all—as the ultimate objective.

HENRY B. ALLEN,
Secretary.

BY-LAWS.

ARTICLE I.

STOCK.

Section 1. The Real and Personal Estates of the Institute as held upon the First day of January, One Thousand Eight Hundred and Eighty-one, shall be valued at One Hundred Thousand Dollars, and shall be represented by Ten Thousand Shares of Stock of the par value of Ten Dollars each. Said shares shall be divided into two classes, *viz.:*

First Class. Shares not registered for use: on which no annual payment shall be charged or collected, and the holders thereof shall not have the privileges of members of the Institute, but may, if of legal age, vote at any annual election for managers upon the payment of One Dollar upon each share of stock on which they may desire to vote; provided, however, such shares have been held by the same person at least three months before such election.

Shares of the First Class may be converted into shares of the Second Class at the pleasure of the owners, provided the transfer be approved by the Board of Managers; but, when once so converted, they shall always continue in the Second Class.

Second Class. Shares registered for use: on which Twelve Dollars per annum shall be due and payable from resident members in advance on the first day of October in each year, except as hereinafter provided.

Non-resident holders of Second Class stock shall pay an annual fee of Five Dollars.

Section 2. The holders of Second Class stock shall be entitled to the use of the library, lectures and reading-room, and, if of legal age, to all other privileges of membership in the Institute, so long as they make the annual payment in advance; and shall, on the payment of One Dollar therefore, be entitled to a Certificate of Membership.

Section 3. If the annual dues for successive years remain unpaid at the expiration of two and a half years on any share of stock of the Second Class, such shares shall then become forfeited to the Institute; but such forfeiture may be remitted by a unanimous vote of the Board of Managers.

Section 4. Stock of the Second Class may be held in trust for persons not of legal age, and shall be liable to the payment of only one-half the annual fees due upon stock of Second Class held by persons of legal age; provided, that when such minors arrive at legal age, new certificates, subject to the full annual contribution, shall issue on payment of the customary fee.

Section 5. Certificates for the First Class stock may be issued for any number of shares in a single certificate; but every certificate for the Second Class shall be for one share only.

Section 6. No share of stock in the Second Class shall be transferred until all arrearages and fines are paid, and all books and tickets returned, and the transfer approved by the Board of Managers.

Section 7. All certificates of stock shall be signed by the President and Secretary; shall be issued by the Controller, and shall be transferable only on the books of the Institute by the owner, or his legal representative, on the surrender of the old certificate, and of a fee of twenty-five cents for each certificate.

Section 8. All subscriptions to stock shall be approved by the Board of Managers before the certificate can be issued.

ARTICLE II.

MEMBERS

Section 1. All persons interested in the purposes and activities of the Institute and willing to further them may become members when elected by the Board of Managers, or in a manner prescribed by the Board, except as qualified by Section 6 of this ARTICLE. Membership shall consist of the following classes:

1. Student members
2. Associate members
3. Active members
4. Sustaining members
5. Honorary members

The Board of Managers may establish other classes of members, provided that the privileges enjoyed and dues paid by such other classes are not inconsistent with the privileges and dues of the classes of members specifically provided for by these By-Laws.

Section 2. Student members shall be under twenty-five years of age and shall pay annual

dues of $2.00. They shall be entitled to unlimited free admission to the Museum and the Planetarium. Student members who are fourteen or more years of age may, if endorsed by a teacher or an Active member, have the use of the library upon the payment of additional annual dues of $1.00 but shall have no voting privileges or rights to hold office.

Section 3. Associate members shall pay annual dues of $5.00. They shall be entitled to unlimited free admission to the Museum and to the Planetarium, but shall have no voting privileges or rights to hold office. Upon the payment of $5.00 additional annual dues they shall have Family privileges as defined in Section 7 of this ARTICLE.

Section 4. Active members shall be not less than twenty years of age. They shall pay annual dues of $15.00. Active members, residing permanently at a distance of fifty miles or more from Philadelphia, shall pay annual dues of $7.50. Active members shall be entitled to use the library, to receive one copy of the Journal of The Franklin Institute, to vote and to hold office. They shall be entitled to all the privileges of Associate members and upon the payment of $5.00 additional annual dues shall be entitled to Family privileges.

Section 5. Sustaining members shall be not less than twenty years of age. They shall pay annual dues of not less than $50.00. They shall be entitled to all of the privileges of Active members and shall have Family privileges as defined in Section 7 of this ARTICLE free of additional charge.

Section 6. Honorary members shall be entitled to all of the privileges of sustaining members, except the right to vote and to hold office. They shall be nominated by the Board of Managers and shall be elected by four-fifths of the votes of the members present at any stated meeting of the Institute at which their nomination may be acted upon.

Section 7. Family privileges consist of the right to receive a card for each individual in the family of the member and resident with the member entitling the holder to unlimited free admission to the Museum and the Planetarium.

Section 8. Members belonging to classes of membership existing prior to the amendment of this ARTICLE shall be reclassified in a manner consistent with their former dues and privileges.

ARTICLE III.

PAYMENT OF DUES.

Section 1. The annual fees for membership shall be due and payable on the first day of each month, whichever is nearest to the date of election or as determined by the Board of Managers.

Section 2. Any member whose dues are more than two months in arrears shall have all the privileges of membership suspended until such time as all arrears are paid. Should the dues not be paid when they become six months in arrears the said member shall forfeit his membership.

Section 3. The Board may remit temporarily in whole or in part the dues of any member either by action in a particular case or by establishing regulations governing certain cases.

Section 4. Every person admitted to membership in the Institute shall be considered as liable for the payment of dues until he shall have resigned, been dropped or have been relieved therefrom by the Board of Managers.

Section 5. Resignations of memberships shall be made to the Board of Managers in writing, but need not be accepted until all dues and arrears up to date of resignation shall have been paid.

Section 6. The privileges and title of Associate member may be obtained for life by paying therefor in one year the sum of $100. From this payment may be deducted one-half of the Student or Associate Membership dues paid by the member during the preceding ten years, but in no case shall the deduction exceed $50.00.

Section 7. The privileges and title of Active Member shall be enjoyed for life or may be obtained for life by a member who has heretofore or who shall hereafter pay therefor, in one year, the sum of $300, except that a person residing permanently at a distance of twenty-five miles or more from Philadelphia may become an Active Member for life by paying therefor in one year the sum of $100. From this payment may be deducted one-half of the dues paid by the member during the preceding ten years, but in no case

shall the deduction exceed one-half of said payment. Associate Members for life may become Active Members for life by paying therefor in one year the sum of $200.

Section 8. Firms, corporations, associations or individuals may nominate and subscribe for the membership dues of groups of members of any class or classes, at the annual dues provided for, subject to the approval of the Board as to any particular nominee. If the dues of these nominees amount to $100 or more in the aggregate, the firm, corporation, association or individual shall be known as an Affiliate of the Institute.

ARTICLE IV.

MANAGEMENT.

Section 1. The Institute shall be governed by a board of twenty-four (24) Managers elected by the members.

Section 2. The officers, who shall be elected by the Board of Managers, shall be a President, an Executive Vice President, not more than five Vice Presidents, a Secretary, an Assistant Secretary, a Treasurer, and an Assistant Treasurer. The Board of Managers may elect such other officers as it deems necessary.

Section 3. At the annual meeting of the Institute, eight Managers shall be elected each year to serve for three years, provided that the Managers now elected, or who may hereafter be elected, shall continue to serve until their successors be appointed.

Section 4. All elections of the Institute shall be by letter ballot and no vote may be cast by proxy.

Section 5. Nominations for Managers shall be made in writing at the stated meeting in the month of December. Each nomination paper must be signed by at least two members, who shall certify that the candidate will serve if elected. After the nominations are closed, the President shall appoint three members, who are neither officers nor nominees, to act as tellers of election. The list of nominees shall be posted at the Institute and incorporated (with directions for voting) in a ballot to be sent to each member by the Secretary at least one week before the date of election. Each ballot shall be accompanied by a return envelope addressed "To the Tellers of Election," and provided with a space for the signature of the member voting.

Section 6. On the date of the annual meeting, and at an hour previously designated by their chairman, the tellers shall meet at the Institute, and shall count all legal votes that have been received by mail or placed in the ballot box before eight o'clock P.M.; and when the count is completed they shall report to the annual meeting of the Institute the total number of ballots cast, together with the number of votes received by each candidate. Thereupon the presiding officer shall announce the names of the candidates who received the plurality of votes, and shall declare them elected Managers of the Institute for the ensuing terms.

Section 7. At the organization meeting of the Board of Managers, the Board of Managers shall elect the officers provided for in Section 2 of ARTICLE IV to serve for one year; and may at said organization meeting or from time to time thereafter elect such other officers as it may determine upon, and shall determine and fix the compensation, if any, to be paid to the officers so elected or appointed by them. The officers who are in office immediately prior to the annual meeting shall continue in office until their successors are elected or appointed by the Board of Managers as herein provided.

Section 8. Vacancies occurring in any office may be filled by the Board of Managers by election or appointment of persons to serve until the next annual election.

ARTICLE V.

BOARD OF MANAGERS.

Section 1. The Board of Managers shall have general charge and control of the Institute and of the Benjamin Franklin Memorial and The Franklin Institute Museum, and shall consist of twenty-four members elected as provided in ARTICLE IV. The President, the Executive Vice President, the Vice Presidents, the Secretary, the Treasurer, the Chairman of the Committee on Science and the Arts, and the Chairman of the Library Committee shall be *ex officio* members. A quorum of the Board of Managers shall be nine of the elected and *ex officio* members. The Board of Managers may adopt such by-

laws, rules and regulations for the governance of their affairs as are not inconsistent with the Charter and these By-Laws.

Section 2. They shall present, through the President, at the annual meeting of the Institute, a report of the condition of the affairs of the Institute.

Section 3. They shall hold stated meetings in each month except in July and August.

Section 4. Special meetings may be called by the President at his discretion and shall be called by him on written request of the Executive Committee or of any seven members of the Board. In case of his absence or refusal to act, such special meeting shall be called by the Secretary.

Section 5. Members who have not attended five regular meetings in the twelve months prior to the stated meeting of the Institute in December, shall be reported thereat as having resigned, unless it be unanimously voted by the Board at that meeting that such member has been absent for sufficient reason.

Section 6. All vacancies on the Board of Managers shall be filled by the Board until the next annual meeting of the Institute.

Section 7. The Board of Managers may elect an Executive Committee consisting of five of its Members with such authority to act on behalf of the Board as the Board may delegate.

ARTICLE VI.

DUTIES OF OFFICERS.

Section 1. The President shall be the executive head of the Institute and, under the supervision of the Board of Managers, shall have general charge of the affairs of the Institute. He shall preside at all meetings of the Institute and of the Board of Managers and shall be, *ex officio*, a member of all standing committees of the Institute.

Section 2. The Executive Vice President, under the supervision of the President, shall have immediate charge of all affairs of the Institute other than those matters specifically delegated by action of the Board of Managers pursuant to these By-Laws to the Executive Committee and the Finance Committee. He shall prepare the annual budget, and submit it to the Board of Managers for approval. He shall be an *ex officio* member of all standing Committees of the Institute. He shall report to the President. In case of the disability of the Executive Vice President, the President shall designate another officer or other officers to assume his duties.

Section 3. In the absence of the President, the Executive Vice President shall exercise his duties. In the absence of both the President and the Executive Vice President, the Vice Presidents shall exercise the duties of President in order of their seniority in office.

Section 4. The Secretary shall be responsible for keeping the minutes of all meetings of the Institute and of the Board of Managers, shall keep the records of the Institute, and shall perform all the duties usually pertaining to the office of Secretary. He shall report to the Executive Vice President. In the absence or disability of the Secretary, the Assistant Secretary shall perform his duties.

Section 5. The Treasurer shall have custody of all monies received from the committee on Finance together with all monies received by the Institute from dues, admissions, and other operations of the Institute, depositing them, in the name of the Institute, in such bank or banks as the Board of Managers shall direct and disbursing them by checks signed as the Board of Managers shall likewise direct. He shall keep accounts of the receipts and disbursements and shall report thereon to the Board of Managers as directed. He shall have general supervision of the accounts of the Institute and shall render such financial statements as directed by the Board of Managers. He shall give bond to an amount and with such surety as the Board of Managers shall determine. The Treasurer shall report to the Executive Vice President. In the absence of the Treasurer, the Assistant Treasurers shall, in the order of their seniority in office, perform his duties.

ARTICLE VII.

BOARD OF COUNCILLORS.

The Board of Councillors shall consist of not more than fifty members and shall embrace, as *ex officio* members, the Governor of Pennsylvania, the Chief Justice of the Supreme Court of Pennsylvania, the Mayor of the City of Philadelphia, the President of the City Council of Philadelphia, the President of the Board of Public Education and the

Superintendent of Schools of the City of Philadelphia, the Provost and the President of the University of Pennsylvania, the President of Commissioners of Fairmount Park, the President of the American Philosophical Society, the President of the Poor Richard Club, and the President and Secretary of The Franklin Institute, which two latter shall respectively be Chairman and Secretary of the Board of Councillors. Additional members shall be nominated by the President or Secretary of the Institute and elected by the Board of Managers, at such intervals as the Board of Managers may see fit. The Board of Councillors shall meet at the call of its Chairman or of the Board of Managers for the consideration of any matter that pertains to the welfare of the Institute. The Secretary shall from time to time send to the members of the Board of Councillors a report of the activities and accomplishments of the Institute.

ARTICLE VIII.

COMMITTEES OF THE INSTITUTE.

Section 1. There shall be the following Standing Committees of the Institute:

1. Bartol Research Foundation Committee.
2. Biochemical Research Foundation Committee.
3. Committee on Science and the Arts.
4. Endowment Committee.
5. Finance Committee.
6. Library Committee.
7. Meetings Committee.
8. Membership Committee.
9. Museum and Memorial Committee.
10. Publications Committee.
11. Committee on Research.

Section 2. Each standing committee of the Institute shall consist of the number of members determined on by the Board of Managers and composed of members of the Board of Managers and/or non-members as may be desirable, except that the Committee on Science and the Arts shall consist of not less than sixty nor more than seventy-five members, and except that the Bartol Research Foundation shall consist of not less than ten nor more than fifteen members as provided for in Section 7 of this ARTICLE.

Section 3. The members of each committee shall serve one year, except that members of the Committee on Science and the Arts shall serve three years.

Section 4. The members of each committee, other than *ex officio* members specifically provided for by these By-Laws, shall be appointed by the President and approved by the Board of Managers at the organization meeting of the Board following the annual meeting except that each year but one-third of the total number of members of the Committee on Science and the Arts shall be so appointed and approved.

Section 5. The Chairman of each Committee shall be designated by the President, except that the Library Committee and the Committee on Science and the Arts shall select their chairmen. A chairman who is not an elected or *ex officio* member of the Board of Managers shall have the privilege of the floor at meetings of the Board but shall not have the right to vote.

Section 6. Each Committee shall report monthly to the Board of Managers through its Chairman. A copy of their report shall be in the hands of the Executive Vice President one week before the monthly meeting of the Board.

Section 7. The Bartol Research Foundation Committee shall consist of not less than ten nor more than fifteen members, including the President and the Executive Vice President of the Institute and the Director of the Bartol Research Foundation as *ex officio* members and nine members of the Institute of whom not more than four shall be members of the Board of Managers. The Committee shall, after consultation with the Director of the Bartol Research Foundation and subject to the approval of the Board of Managers, collaborate with the Executive Vice President in carrying out the purposes and determining the policies of the Bartol Research Foundation. It shall appoint the Director and the Staff of the Laboratories, subject to the approval of the Board of Managers. The Director of the Bartol Research Foundation, to whom all other employees of the Foundation shall report, shall have charge of the scientific activities of the Foundation and shall report on all matters to the Committee or to the Executive Vice President, as the Committee may elect.

Section 8. The Biochemical Research Foundation Committee shall report to the Board of Managers from time to time on the condition of the affairs and operation of said Foundation and shall be available to the Director of the Biochemical Research Foundation and its Advisory Council for collaboration and advice.

Section 9. The Committee on Science and the Arts shall investigate current discoveries, inventions and other achievements in the sciences and their application in the mechanical and industrial arts with a view of affording such recognition as may lie within the power of the Institute to bestow. The Committee shall exercise independent judgment therein. In all matters of a general or administrative nature the Committee shall collaborate with the Executive Vice President.

Section 10. The Endowment Committee shall, in collaboration with the Executive Vice President, encourage the gift of funds to the Institute.

Section 11. The Finance Committee shall have the custody and control of all the securities and investments of the Institute with full power and authority to buy and to sell, and to invest and reinvest the same; including the power to satisfy mortgages and extinguish ground rents, and to direct the placing of all such insurances on Real Estate held for investment as it may deem necessary, and to make such improvements, repairs and alterations to such Real Estate as it may deem necessary. It shall have power to authorize the proper officers of the Institute to execute the necessary papers to effect all purchases, sales and assignments of property other than real estate; to transfer registered securities; to subscribe to bond-holders' agreements to plans of reorganization involving any securities held by the Institute or in which it has an interest; and to do all such acts as are necessary in pursuance of the foregoing powers. It may, with the approval of the Board of Managers, appoint a Trust Company of the City of Philadelphia to act as Fiscal Agent under the direction of the Committee.

The Finance Committee shall keep a record of all its acts and proceedings, which shall be communicated to the Board of Managers.

Section 12. The Library Committee, in collaboration with the Executive Vice President, shall be of counsel and advice in the purchase of books and publications suitable to and consonant with the purposes of the Library; and in other matters pertaining to the Library.

Section 13. The Meetings Committee shall, in collaboration with the Executive Vice President, secure for presentation before the Institute papers dealing authoritatively with subjects of import in the fields of science, engineering and industry.

Section 14 The Membership Committee shall, in collaboration with the Executive Vice President, promote the several memberships defined in ARTICLE II of these By-Laws.

Section 15. The Museum and Memorial Committee shall, in collaboration with the Executive Vice President, be of counsel and advice to the Director of the Benjamin Franklin Memorial and The Franklin Institute Museum, and with the scientific staff of the Museum concerning the activities and the exhibits of both the Museum and the Memorial.

Section 16. The Publications Committee, in collaboration with the Executive Vice President, shall be of counsel and advice with regard to all publications of the Institute other than publications of the Bartol Research Foundation.

Section 17. The Committee on Research shall, in collaboration with the Executive Vice President, be of counsel and advice to all research activities of the Institute except those of the Bartol and Biochemical Foundations.

Section 18. Each committee shall organize and adopt rules as it sees fit, subject to the provisions of these By-Laws and the approval of the Board of Managers.

ARTICLE IX.

MEETING.

Section 1. The Institute shall hold stated meetings on the third Wednesday of each month, except in June, July, August and September, at 8:15 P.M.

Section 2. Special meetings shall be called by the President upon the order of the Board

of Managers or within ten days upon the written application of twenty-four voting members of the Institute.

Section 3. Twenty-four members shall constitute a quorum at any stated or special meeting.

Section 4. The Annual Meeting of the Institute shall be held on the third Wednesday of January of each year at 8:15 P.M. Elections at Annual Meetings shall not result from the poll of less than forty-eight ballots cast in accordance with ARTICLE VI, Section 6, of these By-Laws.

ARTICLE X.

AMENDMENTS.

These By-Laws may be altered or amended at any stated meeting of the members of the Institute, provided notice in writing, signed by two members, of the proposed alteration or amendment, shall be given to the Board of Managers two months prior to the said meeting, except that amendments to ARTICLE I, relating to capital stock, must be ratified subsequently by a majority of the stock represented at a meeting specially called for this purpose.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, January 14, 1948.)

HALL OF THE COMMITTEE,
PHILADELPHIA, JANUARY 14, 1948.

MR. WALTER C. WAGNER in the Chair.

The following report was presented for final action:

No. 3187 }
No. 3188 } Franklin Medal.

This report recommended the award of two Franklin Medals; one to Wendell Meredith Stanley, of Princeton, New Jersey, "In recognition of his discovery that a virus can be a nucleo-protein rather than a living organism, thereby establishing a significant turning point in virus research; and also of his development of an effective centrifuge-type influenza vaccine by means of which the intensity of future influenza epidemics may be greatly lessened"; and one to

Theodor von Kármán, of Pasadena, California, "In recognition of his outstanding engineering and mathematical achievements, particularly those relating to the development of advanced aerodynamic conceptions which have directly influenced the progress of aeronautical design, and for his unusual leadership whereby some measure of his own genius is constantly instilled in those who work with him."

JOHN FRAZER,
Secretary to Committee.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents plus ten cents for each order and postage.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M., Wednesdays and Thursdays from 2 P.M. until 10 P.M.

RECENT ADDITIONS.

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PATRICK, W. L., AND OTHERS. German Glass Industry. 1947.
PHILLIPS, F. C. An Introduction to Crystallography. 1946.
REILLY, J., AND W. N. RAE. Physico-Chemical Methods. Volumes 1 and 2. Fourth Edition. 1943.
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LIPPERT, FREDERICK G. Accident Prevention Administration. 1947.

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NEWTON, JOSEPH. An Introduction to Metallurgy. Second Edition. 1947.

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NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Elegy Written in (and for) a City Laboratory Shortly before Moving to the Country

(addressed to Dr. McDonald in recognition of the seventh anniversary of the removal of the laboratories of the Biochemical Research Foundation from temporary quarters in the loft and in parts of the 2nd, 4th, and 5th floors of the Integrity Trust Building, 133 South 36th Street, Philadelphia, to permanent quarters in Newark, Delaware, on the third Monday in December, 1940).

The clock now marks the hour when tasks may cease.
Experiments draw quickly to a close.
South new commuters speed by car and train
And leave the Lab to rabbits and repose.

Now twinkle distant buildings through the dusk,
And all the Lab with stillness is replete,
Save where the cockroach weaves his hurried flight
And muffled honks drift upward from the street.

Save that on yonder whirring thermostat
The regulator lamps switch off and on,
And the night watchman, flashlight in his hand,
Makes his accustomed, solitary round.

Beneath these rugged beams, these skylights too,
Where rooms in labyrinthine maze extend,
Each in his transitory bit of space.
Soon o'er their tasks no more the Staff shall bend.

The supersonic oscillator's screech,
The signal system with its raucous calls,
The din of generator and of lathe
No longer shall resound within these walls.

Here soon no more the blazing still shall burn,
Or conf'rence members ply their busy wits,
Famed visitors address the Seminar,
Nor ultracentrifuges fly to bits.

The books, the benches, all the apparatus
That Allen, Schroeder, Ely e'er okayed
Await alike the inevitable hour
With little gadgets Dietz and Kelley made.

Full many a fact of once potential worth
Shall with its journal be discarded too.
Full many a reprint's doomed to burn unread,
And waste its data on the chimney flue.

Far from the madding crowd's ignoble strife
A fair new structure nears its opening day.
There, amid sylvan groves and verdant fields,
We'll keep the noiseless tenor of our way.

Yet e'en these paper walls, this narrow stair,
These few dark rooms which never see the sky,
This humble setting for equipment rare,
Implore the passing tribute of a sigh.

For who, to dumb forgetfulness a prey,
The meanest habitation e'er resigned,
Left the fond scenes of earnest hours of toil,
Nor cast one longing, ling'ring look behind?

From some one man each institution stems,
In some one soul first glowed the vision clear.
Remembering this Lab, remember too
The one whose vision gathered substance here.

Haply some one among you may exclaim:
"Oft have we seen him at the close of day
Gaze from this window till the town lights shone,
Wondering, 'Can I meet the next payday?'

"Broad was his planning, and his purpose clear.
All that he lacked was ample funds to spend.
He gave to setbacks (all he owed) a sneer
And gained from heaven (more than he'd hoped) a friend."

L. K.

BOOK REVIEWS

THE TECHNOLOGY OF ADHESIVES, by John Delmonte. 516 pages, drawings, illustrations, 15 × 23 cms. New York, Reinhold Publishing Corporation, 1947. Price $8.00.

The importance of adhesives is increasing rapidly, with a strong trend toward the synthetic resin developments. This is seen in their applications which were stimulated a great deal by World War II. These include a number of new large scale applications of boat, aircraft and building construction. And aside from the wood field, adhesives for metals show much promise. The unity of aluminum alloys and the joining of steel with joint shear strengths of 3000 p.s.i. are an accomplished fact. Ceramics, glass, textiles, rubbers, and solid organic plastics are being bonded with adhesives which form glue lines stronger than the materials themselves. Confidence has been inspired in designers and engineers in the permanence of the bonding quality. There is much to be done in further development of adhesives that may revolutionize fabrication processes. This book is a treatise on the subject in its present state of knowledge.

There are twenty chapters in all, covering the many types of adhesives including phenolic resin, urea and melamine, polyvinyl resin, polystyrene and acrylic resin, rubber, cellulose derivatives, protein substances, vegetable and animal glues and sodium silicate. A good foundation is laid in the introduction to the work where chemical and physical classifications of adhesives are shown and considerable part of the book is devoted to theories of adhesive action. They are grouped here also as chemical and physical, but it is pointed out that there is much basic information still necessary to explain many unusual phenomena which arise in establishing an adhesive bond. Following this, attention is directed to adhesives for various applications such as the joining of wood, organic plastics, metal and rubber, tapes, papers, clothes, foils and for inorganic materials such as chemical stoneware, mica plate, glass, etc. The last chapter is on tests and specifications, reference being made to the specifications developed at Wright Field, Forest Products Laboratory, Naval Aircraft Factory, and the Bureau of Standards during the war. While most of this work related to wood, some referred to the bonding of metal parts. A review is given of other sources. There is a subject index in the back of the book adding greatly to its use as a reference.

The book is an up-to-date treatment of adhesives, their many kinds, compositions, uses, with theories, chemical and physical principles and characteristices. Where limited knowledge or differences of opinion exist it is so stated. The book gives an excellent exposition presented in a logical manner and arranged for ready reference.

R. H. OPPERMANN.

TUNGSTEN, K. C. Li and Chung Yu Wang. Second edition. 430 pages, tables and illustrations, 15 × 24 cms. American Chemical Society Monograph No. 94. New York, Reinhold Publishing Corporation, 1947. Price $8.50.

The special characteristics of tungsten for heat resistance and acid proofing have made tungsten a very important metal, both in time of war and in time of peace. During the last quarter century, no other metal in such comparatively small quantity has had so profound an influence upon the conduct and development of industries as has tungsten. In spite of the discovery of its various new uses during recent years, the future of tungsten still holds much promise. The book at hand, an American Chemical Society Monograph, is a treatment of the various aspects of the subject giving the state of present knowledge of it.

After a brief reference to the history of tungsten, the geology of tungsten is taken up including reference to deposits in various countries in the world, and this is followed by a description of principles and processes adopted for ore dressing. A section devoted to the metallurgy of tungsten points out that this is largely empirical and is based mainly on patents. The treatment here is divided into subheadings of decomposition of tungsten ore, purification of

tungstic oxide, and production of tungsten powder. The physical and chemical properties of tungsten are next given. Attention is given to the compounds of tungsten and oxygen. The progression then is through analysis of tungsten, the industrial application of tungsten, substitution of tungsten and the economics of tungsten. Appendices include the terms of purchase of tungsten, ores, Chinese contracts, the post-war tungsten situation and references relating to tungsten alloys. In the back there is an extensive subject index.

The work is a combination of digested literature and expert knowledge of the subject. It is well divided and arranged in logical order, easy to read, and readily adaptable as a reference with many helpful and useful tables, diagrams, curves, drawings and illustrations. It should stimulate interest of those who desire to know the present state of knowledge of the subject, as well as future possibilities.

R. H. OPPERMANN.

FUNCTIONAL ANATOMY OF THE MAMMAL, by W. James Leach. First Edition. 231 pages, illustrations, 15 × 24 cms. New York, McGraw-Hill Book Co., Inc., 1946. Price $2.50.

For various reasons it is practical, useful and necessary to use bodies of animals when studying anatomy. Many students properly ask why the body of the domestic cat is so commonly studied rather than some other type of mammal. The reasons of size and availability are obvious but from the biological standpoint the cat may be used as a fairly representative type, occupying a position near the middle of the list of classification of mammals. From the purely structural standpoint, man is not far removed from the position of the cat. Terminology and methods learned in the dissection of the cat are readily applicable to the human body.

A large part of the material in this book deals with the structure of the common domestic cat. The book is designed to make anatomical terms really intelligible by integrating a rather specific laboratory study of the cat with text material of a more general character with emphasis on man. Functional organization is stressed rather than how each of the parts is constructed as a separate unit. After a chapter on general considerations which includes a list of terms and definitions and sufficient background information, there is discussed the skeletal system, superficial dissection of the cat, a study of the voluntary muscles and a survey of the internal anatomy. Following this, there are sections devoted to the alimentary system, the respiratory system, the vascular system, the urogenital system and the nervous system. The latter part of the book is devoted to special sensory apparatus and the endocrine organs. An appendix gives a method of laboratory preparation and preservation of materials which is unusual. A subject index is in the back.

The presentation is clear and readily adaptable to a laboratory study. It meets its designed purpose satisfactorily and should be of value to medical students as well as students in experimental physiology, pharmacology and veterinary practice.

R. H. OPPERMANN.

GERMAN RESEARCH IN WORLD WAR II, by Leslie E. Simon. 218 pages, drawings and illustrations, 15 × 24 cms. New York, John Wiley & Sons, Inc, 1947. Price $4.00.

Those who have followed the changes in arms during the recent war have frequently seen new arms brought to the battlefield by the Germans and have wondered through just what channels the main ideas passed before they became actualities. How extensive was their research and to what extent was it especially organized for war work? How did they define basic research, technical research, engineering design, development, and production? Where were the weak spots in their system?

The author of this book, Colonel Leslie E. Simon, U. S. Army, was sent to Europe to go to German scientific establishments as soon as possible after their capture, examine what evidence there was and question personnel, in an effort to determine the structure and functioning of German research. Generalizations of this information and case histories which are permissible from the viewpoint of military security are given in this book. In presenting his story Colonel Simon gives considerable background as reasons for situations. Thus in the beginning he tells of various factors or influences which had important effects on research such as quantity and

quality of apparatus and equipment, character of research staff, degree of independence of research and extent of non-scientific direction. Also an adherence to a defined and consistent terminology has been necessary, descriptions of laboratories and concepts of psychological influences, and general conditions which prevailed in the German war ministries before the war. With this as preparation, a description of the German research organization and its functioning, a version of its accomplished research, and a basis for analysis and criticism of the whole pattern of the German scientific effort are presented.

The value of this work lies in the fact that it can be regarded as a study of a completed laboratory experiment, pointing out the inabilities of the military to evaluate and promote or reject the projects of civilian scientists, as well as the successes which were achieved. A wide variety of subjects is covered including ballistics, aerodynamics, instruments and measurement techniques, the wind gun, sound as a weapon, and explosive powered vortices. The book has much in it for the non-technical as well as the technical man interested in accomplishments and methods of organized research.

R. H. OPPERMANN.

PHYSICAL CONSTANTS OF HYDROCARBONS, by Gustav Egloff. Volume IV. American Chemical Society Monograph Series. New York, Reinhold Publishing Corporation, 1947. Price $17.50.

This well known series of books has been added to by the present Volume IV which is actually a continuation of Volume III, and an evaluation of the physical constants of polynuclear aromatic hydrocarbons. The compounds included under this classification are fused ring hydrocarbons having at least one aromatic ring. The nomenclature used is mainly the "Ring Index" but complications arising in the nomenclature of many derivative compounds necessitated departures from this system. It is chosen with the intention of offering pictures of the structure which may be readily visualized. The order of tabulation also offered a problem which has been admirably solved. All compounds having a common empirical formula are classified in one group. These groups are numbered with Roman numerals and listed in order of decreasing hydrogen content. Except for sections entitled miscellaneous and a few sections in which general classification is simplified by listing together such similar compounds as anthracene and phenanthrene derivatives, the compounds of one section have a common nucleus.

It is stated in the work that although thousands of polynuclear aromatic hydrocarbons have been prepared, these compounds represent only a minor fraction of the theoretical possibilities. A great number of the larger polynuclears have been synthesized in recent years, particularly in connection with cancer research and other physiological problems. The present study also shows that the published data for many of the known compounds are insufficient for the identification of these compounds on the basis of properties alone.

This American Chemical Society Monograph is a useful addition to an already well known series.

R. H. OPPERMANN.

PATENT NOTES FOR ENGINEERS, by C. D. Tuska. 165 pages, illustrations, 15 × 23 cms. Princeton, N. J., RCA Review, 1947. Price $2.50.

With this volume the RCA Review Department of RCA Laboratories Division introduces a new *Engineering Book Series*, to include subjects of general engineering interest which do not fit into their various other publications.

Mr. Tuska, Director of the Patent Department of the RCA Laboratories Division, has prepared these notes on patents aided by other members of the department. The stated purpose of the volume is "to bridge the technical gap between engineers, research workers and inventors generally, and their patent attorneys." To achieve this end the author has pointed out the need for proper protection of an invention by patenting, even for the inventor who, wishing to dedicate his invention to the public good, must patent it to prevent someone else from doing so and securing its benefits.

Since there is no definition of invention in either the United States Constitution or the Federal Statutes, there is presented a lengthy discussion of statutory invention as defined negatively by the courts. Specific cases are presented with citations from the court decisions. However even these "negative rules" have their exceptions which are pointed out. Warning is given that not all the "negative rules of invention" have been presented and that particular cases may well have to be decided by the Patent Office and the courts as to the presence or absence of "patentable invention." Various essentials of statutory invention are enumerated and discussed with examples.

The succeeding chapters consider certain definite points of great importance in the protection of inventions and for which the inventor is responsible. These are the disclosure of the invention, diligence in reduction to practice and the securing of witnesses to both the disclosure and the successful demonstration of the invention. In the discussion of the prosecution of patent applications and the handling of interferences, if such arise, the importance of these precautionary measures is amply demonstrated with adequate citation of cases. A concluding chapter considers the matter of patent ownership and use, especially as relating to employee and employer.

This is not a manual of procedure for the inventor as relating to patents, although it offers many practical suggestions, but rather a discussion of the legal aspects back of those suggestions. Such a presentation should make the inventor well aware of the need for taking the necessary precautions to adequately protect his invention.

G. E. PETTENGILL.

WIND-TUNNEL TESTING, by Alan Pope. 319 pages, illustrations, 14 × 21 cms. New York, John Wiley & Sons, Inc., 1947. Price $5.00.

In the development of new airplanes wind-tunnels play a vital role. Although there have been many papers on the various components of the tunnel and on corrections to be applied to the data secured, there has been no one comprehensive work on wind-tunnels. The present volume aims to fill this gap in aeronautical literature.

The many types of wind-tunnels are reviewed briefly. An accompanying list gives data respecting more than one hundred tunnels throughout the world, the listing being as complete as security restrictions permitted.

In attacking the problem of design, the author limits himself to the general utility tunnel. His treatment of the propellor-flow straightener system is precise but complete. The determination of the flow characteristics of the tunnel after construction are detailed. These include angular-flow variation in the jet, velocity variation in the jet, longitudinal static-pressure gradient and turbulence.

The various load measurements of the model that are required are noted with a detailed description of the different types of balances employed and nine different kinds of mountings. The actual testing procedure is discussed in full with sections on the various components of the plane as well as the complete model.

The important factor of wind-tunnel-boundary corrections receives full attention with downwash corrections discussed for various types of jets. The difficult problem of extrapolation to full scale is also considered. Auxiliary testing equipment, the use of small wind-tunnels and notes on wind-tunnel-model construction are additional material treated.

Numerous worked examples, suggested problems and ample references add to the value of the book for the student and the engineer.

G. E. PETTENGILL.

VAT DYESTUFFS AND VAT DYEING, by M. R. FOX. 323 pages, illustrations, 14 × 21 cms. New York, John Wiley & Sons, Inc., 1947. Price $5.50.

With the rapid growth of the synthetic dyestuffs and rayon industries, the whole art of dyeing has become exceedingly complicated. The present volume is an attempt to deal in detail with one aspect of the subject—vat dyestuffs and their application.

The book is essentially a practical one. After two introductory chapters dealing with the indigo and related dyes and the anthraquinonoid vat dyestuffs, the author discusses different dyeing methods. Agents to be used and the properties of the dyestuffs are considered as well as various fastness tests.

The application to animal fibers and cellulosic fibers is fully treated with indication of which dyes to use with which materials. The application of the indigosol dyestuffs is treated separately. Although the author does not pretend to offer a detailed account of textile printing, the chapter devoted to it is the longest in the volume. The descriptions and illustrations of machinery should prove of value. Non-textile uses of vat dyestuffs are touched on briefly, and there is a chapter on the methods of identification of vat dyestuffs. A useful feature is the extensive listing of commercial vat dyestuffs grouped into equivalent, or analogous, products.

The entire work reveals the author's practical knowledge of the subject and with the many examples should be a valuable text for anyone working in the field—student or practicing dyer.

G. E. PETTENGILL.

THE STRANGE STORY OF THE QUANTUM, An Account for the General Reader of the Growth of the Ideas Underlying Our Present Atomic Knowledge, by Banesh Hoffmann. 239 pages, illustrations, 14 × 20½ cms. New York, Harper & Brothers, 1947. Price $3.00.

In his telling of the complex story of the quantum theory, Dr. Hoffmann has used a wealth of analogies and similes with which to stimulate the interest of the reader. Presented as a struggle between the wave and the particle theories, the account often becomes involved and these artificial devices of the author prove helpful to the layman.

The book does not aim to present the mathematics involved, but only to serve as a "guide to those who would explore the theories by which the scientist seeks to comprehend the mysterious world of the atom." And as the author warns the story of events and discoveries is indeed complex, with many outlandish ideas, some good and some bad. The quantum theory was indeed the beginning of a new era. The persons mentioned run the gamut of those who have played a part in fitting together the fragments of the present picture. The first investigators Planck, Einstein, Bohr are followed by De Broglie, Heisenberg, Dirac and Schrodinger among others. There is no need to repeat here all that the volume contains but if you like your science served up with a dash of spice, humor and satire you will enjoy this discussion of the development of the quantum theory.

G. E. PETTENGILL.

THE CHEMISTRY AND TECHNOLOGY OF WAXES, by Albin H. Warth. 519 pages, drawings, tables and illustrations, 15 × 23 cms. New York, Reinhold Publishing Corporation, 1947. Price $10.00.

The English term *wax* is derived from the Anglo-Saxon *weax*, which was the name applied to the natural material of the honeycomb of the bee. When a material of similar nature was found in plants it also became known as *weax* or *wachs*, and later wax.

In modern times the term wax has taken on a broader meaning and is applied to all wax-like solids and liquids found in nature, and to those which occur individually in waxes, such as hydrocarbons, acids, alcohols and esters.

The literature on the subject of waxes is abundant, but widely scattered. In this volume the author brings together and correlates much material that is not available to one lacking the facilities of an extensive library.

After a brief discussion on the chemistry of waxes, the various subdivisions are dealt with. These include the natural waxes, fossil and earth waxes, lignite paraffins, petroleum waxes, synthetic waxes and wax compounds. Chapters on emulsifiable waxes, methods for determination of constants, and on wax technology conclude the work.

An appendix containing tables of physical constants of waxes is attached.

HENRY N. MICHAEL.

THE RARE-EARTH ELEMENTS AND THEIR COMPOUNDS, by Don M. Yost, Horace Russell, Jr., and Clifford S. Garner. 92 pages, drawings and tables. New York, John Wiley & Sons, Inc., 1947. Price $2.50.

Including lanthanum, fifteen chemically similar elements between barium and hafnium on the periodic table are known as rare earth elements. This region has been the least investigated by the spectroscopists. The book at hand presents the principal chemical and physical properties of the rare earth elements and their compounds, and shows the concordance of the current theories with these properties. The six chapter headings are indicative of the coverage. They are Electronic Structures and Oxidation States of the Rare Earth Elements, Paramagnetic Properties of Rare Earth Compounds, Absorption Spectra of Rare Earth Compounds, Evidence For the Existance of Element 61, Separation of Rare Earths, and Chemical and Physical Properties of the Rare Earths. The appendices are Nuclear Properties of the Rare Earth Elements, General Physical Constants, and The Periodic System of the Elements. Many references are made to the literature. In the back are name and subject indexes.

This 86-page treatment contains a fund of information digested from various sources and coupled with expert knowledge of the authors.

R. H. OPPERMANN.

PUBLICATIONS RECEIVED.

Alexander Dallas Bache, Scientist and Educator, 1806–1867, by Merle M. Odgers. 223 pages, 14 × 21 cms. Philadelphia: University of Pennsylvania Press, 1947. Price $2.75.

Improvising Supervision, by Frank Cushman and Robert W. Cushman. 232 pages, 13 × 19 cms. New York: John Wiley & Sons, Inc., 1947. Price $2.50.

Theory of Limit Design, by J. A. Van Den Broek. 144 pages, drawings, 14 × 22 cms. New York: John Wiley & Sons, Inc., 1947. Price $3.50.

Energy Unlimited, The Electron and Atom in Everyday Life, by Harry M. Davis. 273 pages, illustrations, 14 × 22 cms. New York: Murray Hill Books, Inc., 1947. Price $4.00.

Techniques of Observing the Weather, by B. C. Haynes. 272 pages, tables, drawings and illustrations, 14 × 22 cms. New York: John Wiley & Sons, Inc., 1947. Price $4.00.

Photographic Facts and Formulas, by E. J. Wall and Franklin I. Jordan. 364 pages, drawings and tables, 16 × 24 cms. Boston: American Photographic Publishing Co., 1947. Price $5.00.

Engineering Applications of Fluid Mechanics, by J. C. Hunsaker and B. G. Rightmire. 494 pages, drawings and illustrations, 15 × 24 cms. New York: McGraw-Hill Book Co., 1947. Price $5.00.

Composition and Pictures, by Eleanor Parke Custis. 224 pages, illustrations, 22 × 29 cms. Boston: American Photographic Publishing Co., 1947. Price $6.00.

Electronic Circuits and Tubes, by Electronics Training Staff of the Cruft Laboratory. 948 pages, drawings, 15 × 23 cms. New York: McGraw-Hill Book Co., 1947. Price $7.50.

The Scientific Paper, How to Prepare It, How to Write It, by Sam F. Trelease. 152 pages, 13 × 19 cms. Baltimore: The Williams & Wilkins Co., 1947. Price $2.00.

Understanding Vectors and Phase, by John F. Rider and Seymour D. Uslan. 153 pages, drawings, 13 × 19 cms. New York: John F. Rider Publisher, Inc., 1947. Price $.99 (paper).

Introduction to Modern Physics, by F. K. Richtmyer and E. H. Kennard. Fourth Edition. 759 pages, drawings, tables and illustrations, 16 × 23 cms. New York: McGraw-Hill Book Co., 1947. Price $6.00.

Organic Syntheses, by R. L. Shriner, Editor. 121 pages, 15 × 24 cms. New York: John Wiley & Sons, Inc., 1947. Price $2.25.

Fundamental Electronics and Vacuum Tubes, by Arthur Lemuel Albert. Revised Edition. 510 pages, drawings and illustrations, 16 × 24 cms. New York: The Macmillan Company, 1947. Price $6.00.

Fundamentals in Chemical Process Calculations, by Otto L. Kowalke. 158 pages, tables and drawings, 15 × 22 cms. New York: The Macmillan Company, 1947. Price $2.80.

High Frequency Measuring Techniques Using Transmission Lines, by E. N. Phillips, W. G. Sterns, N. J. Gamara. 58 pages, diagrams, 21 × 28 cms. New York: John F. Rider Publisher, Inc., 1947.

CURRENT TOPICS.

Expanding Air Causes Sudden Temperature Drop.—A bottle of "pop" from the refrigerator, or a tiny balloon such as Junior makes by sucking on a broken piece of rubber, may have little to do with the weather, but each has been found capable of causing a snowstorm, the General Electric Company reported recently.

Or digging still deeper into Junior's treasure trove, that popgun he got for Christmas also can "make it snow," the company said.

According to Dr. Bernard Vonnegut, scientist of the General Electric Research Laboratory, supercooled water droplets in a cold-chamber cloud can be transfromed to snow simply by uncapping the bottle of "pop," bursting the toy balloon or shooting off the popgun, in the cloud.

In a similar experiment out-of-doors, Dr. Vonnegut reported he has been able to change supercooled fog in his backyard into snow by firing a popgun.

Although seemingly unscientific, the new methods for producing man-made snow each employ the same fundamental principle that sudden expansion of air will cause a severe drop in temperature of the air expanded.

"In each case, the temperature drops low enough to produce spontaneously millions of ice nuclei," Dr. Vonnegut said. "If these nuclei, as they are created, are dispersed into a supercooled cloud, or a cloud whose water droplets are liquid even though below freezing in temperature, the ice nuclei will grow at the expense of the water droplets, and snow will result."

The sudden drop in temperature of the expanded air, taking place in approximately 1/1,000th of a second, must go down at least to minus 31 degrees Fahrenheit, according to Dr. Vannegut, since previous experimentation has proved that ice nuclei form spontaneously only at that temperature or lower.

In tests with a toy balloon, Dr. Vonnegut desired an extremely small balloon, and thus he resorted to the favorite balloon trick of children, in which a tiny bit of rubber becomes inflated by sucking it.

A balloon about the size of a BB shot produced approximately 100,000,000 nuclei when burst into a supercooled cloud, Dr. Vonnegut estimated.

Experiments with a bottle of "pop," in which the air in the neck of the bottle was expanded by removing the cap, furnished an answer to the common mystery of why a bottle of liquid, kept in a refrigerator a considerable time, sometimes doesn't freeze until opened.

"The liquid in the bottle becomes supercooled in these instances," Dr. Vonnegut said, "and when the bottle is opened, the sudden temperature drop caused by expansion of air in the neck makes ice nuclei form. The nuclei then cause the liquid to change to ice."

Dr. Vonnegut said that an automobile tire blowout or the unhooking of couplings on a railroad car might produce snow artificially if atmospheric conditions were right. "If a tire blew out or a coupling were unhooked in a condition of supercooled fog, the sudden momentary drop in temperature caused by air expansion would produce ice nuclei, which in turn would seed the supercooled fog and create snow crystals."

The technique of expanding air to cause a sudden temperature drop constitutes a third means for producing snow investigated under a current snow research program being conducted by the General Electric Company in conjunction with the U. S. Army Signal Corps Engineering Laboratories at Bradley Beach, N. J. and the Army and Navy air forces.

Other means developed for making snow include scientist Vincent J. Schaefer's dry-ice technique, in which the seeding of a cloud with dry-ice causes ice nuclei to appear resulting in formation of snow, and the foreign-nuclei technique, in which certain materials, foreign to snow but resembling it in crystalline structure, actually "fool" water particles in a supercooled cloud by serving as nuclei and growing into snow crystals at the expense of the water particles.

R. H. O.

More Sugar from Graded Fields.—"Turtlebacking," as a descriptive term, gives a good idea of a new method of handling the drainage problem in some of the low lying and flat sugarcane fields in Louisiana. On some soils, at least, the U. S. Department of Agriculture finds the increased sugar yield from a single crop more than pays the expense of "turtle backing." The Soil Conservation Service is investigating the possibilities on other types of soil.

In the Sugar Bowl country, drainage is a No. 1 problem of many plantations. Ditches have to be kept cleared. The material removed from the ditches tends to build banks or small leeves beside the ditches, hindering drainage from the fields and causing trouble in operating machinery.

Turtlebacking is also called cut crowning by agricultural engineers. As the popular name suggests, it consists in grading all the land between two lateral ditches so that there is a low crown half way between with water free to flow to the ditches. Where ditch banks have been raised by repeated clearing of the waterways, this may call for moving considerable soil to the slightly raised center of the turtle back. Once the grading is completed most of the maintenance can be done by ploughing and dragging when fields are replanted. In the main, turtle backing really amounts to restoring to the sugarcane fields the fine soil that the heavy rains wash into the drainage ditches.

In the first field on which turtlebacking was tested on the St. Delphine Plantation, the improved drainage resulted in just about doubling the corn yield and an increase in the yield of cane equivalent to more than 900 pounds of sugar to the acre, or more than $50 an acre in the value of the crop.

SCS technicians are experimenting with various combinations of plows and graders to find the most economical way to turtleback the sugar fields that need this treatment. R. H. O.

Synthetic Fibers Other Than Rayon Increasing Rapidly in Importance.—Synthetic fibers other than cellulose, in their infancy before World War II, are becoming increasingly important in the textile economy of the United States, according to a survey just made at the United States Department of Agriculture's Southern Regional Research Laboratory. The survey, which brings together the first figures to be made public on the consumption of these fibers, is part of a study of trends in the consumption of fibers in the United States, being made by Robert B. Evans and Barkley Meadows in connection with the research program on cotton utilization of the Bureau of Agricultural and Industrial Chemistry.

Until about 1935, rayon (synthetic fiber made of cellulose) was the only manufactured fiber in commercial production in this country. Its production, following one or two unsuccessful attempts, was started in the United States in 1911 and has increased so steadily and rapidly that as a textile fiber rayon now ranks second, quantitatively, only to cotton, with consumption increasing from 482 million pounds in 1940 to 875 million pounds in 1946.

Prior to 1940, the total consumption of all noncellulosic synthetic fibers (or synthetic fibers other than rayon) was so small in amount that it can be considered inconsequential. Thereafter, however, consumption increased rapidly, climbing steadily from 4.5 million pounds in 1940 to 53.3 million pounds in 1946. Although even the latter quantity is small as compared with the amounts of cotton, rayon, and wool consumed, it is already greatly in excess of the consumption of silk or flax.

Glass fiber is the oldest of the noncellulosic man-made fibers. Its production began in about 1936, but the output was quite limited during the first three or four years. Production of Vinyon, a fiber made of Vinylite synthetic resin, began about two years later, but is still relatively small. The commercial production of nylon yarns began in December 1939, following a short period of pilot-plant operations.

Manufacture of the casein fiber, Aralac, was initiated on a pilot-plant basis in 1939, with production in a commercial-size unit beginning in 1941. Saran, another fiber made from synthetic resin that was introduced commercially in 1940, might be termed a "borderline" case as a textile fiber since thus far it has been made commercially only in monofilament form; textile yarns are generally made up of many fibers or filaments. Since it frequently is woven and used for textile purposes, however, Saran has been included in the compilation given below. Certain other synthetic fibers such as soybean fiber and fiber made of acrylate resins, although in a pilot-plant stage of development, have not yet been produced on a true commercial scale and have not been included.

The accompanying table shows total consumption of synthetic fibers other than rayon in the United States for the years 1940–46, as indicated by reports from manufacturers. Separate figures for each specific fiber cannot be given since they would disclose the operations of individual firms.

Consumption of Synthetic Fibers Other Than Rayon [1] in the U.S., 1940–46.

Year	Quantity 1,000 pounds
1940	4,471
1941	11,663
1942	23,743
1943	37,200
1944	47,368
1945	49,292
1946	53,329

[1] Including domestic sales of nylon yarn and Aralac staple, estimated consumption of Saran as a textile fiber, production of Fiberglas continuous filament yarn and staple, and consumption of Vinyon in the United States.

Compiled from confidential reports of manufacturers except Fiberglas total for 1940, which was partially estimated.

R. H. O.

Plant Scientists Report on Chemical Quackgrass Killer.—Steady progress in the field of chemical weed eradication, according to plant scientists of the U. S. Department of Agriculture, promises ultimate control of many of the farmer's worst field pests. Latest advances have to do with the possibility of control of grassy weeds—such as quackgrass. The weed grasses are not controlled by the now well-known 2, 4-D.

Recently claims have been made for the effectiveness of IPC (isopropyl-N-phenyl carbamate) as a quackgrass killer. British scientists during the war found this substance, which is in the class of hormone-like chemicals or plant-growth regulators, checks or kills some kinds of cereals, members of the grass family, and that many broad-leaved plants are not injured by it. This led to the assumption that at least some grasses, including weed grasses, might be killed by applications of IPC, and now researchers at the Plant Industry Station, Beltsville, Md., announce favorable results from its use on mature quackgrass (with stolons or runners) and on quackgrass seedlings.

Limited experiments conducted at the Station since 1945 show that well established first-year quackgrass plants grown from seed and other quackgrass plants developed from well established stolons were killed by applying IPC to the surface of the soil. The experimenters, John W. Mitchell, P. C. Marth and L. W. Kephart, applied the chemical dry both outdoors and in the greenhouse, at varying rates, from 5 pounds to 60 pounds to the acre, using sand as a carrier. Even at the lowest rate of five pounds to the acre (in the greenhouse) the growth of shoots from stolons and of seedlings was completely checked. In 6 weeks after treatment outdoors at the rate of 10 pounds of IPC to the acre all growth was dead—not only the sprouts but the runners. In the greenhouse the stolon sprout growth was completely checked in 3 weeks and the stolons killed in 2 months with as little as 5 pounds to the acre.

Experience in the tests indicates, according to Mitchell, Marth, and Kephart, that the effect of the chemical on the plant is primarily from absorption through the roots. Consequently applications to the soil surface have been more effective than applications to the leaves. Experimentally, sand has proved a good material with which to mix the chemical as it readily sifts down to the soil surface.

The IPC (isopropyl-N-phenyl-carbamate) can be purchased as a fine powder from some chemical supply houses. The product may soon be found in packages under special trade names.

So far experience with this grass killer indicates the need for care as it will interfere with the growth of useful grasses for some time if the soil is dry. It is inactivated in moist soil, say the research men, at about the same rate as 2, 4-D, but so far little detailed knowledge has been built up on proper practice.

The experimenters warn that little is known so far about what effects may be produced by IPC on most broad-leaved plants. However, results at the Plant Industry Station have confirmed the English observations and that of others that some broad-leaved plants, including sugar beets and some weeds, have shown no signs of injury by it. They have observed no toxic effects to persons using IPC, but recommend reasonable care until more is known about effects on humans and animals.

R. H. O.

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A STUDY OF PATENT POLICIES IN EDUCATIONAL INSTITUTIONS, GIVING SPECIFIC ATTENTION TO THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.*

BY

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PART I.

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* A paper presented in the graduate seminar in electrical engineering at Massachusetts Institute of Technology by the author.

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While engineers and scientists employed in industrial laboratories have always been conscious of the importance of patents, their colleagues in university laboratories have only recently become convinced of any need for patenting their discoveries. The amount of research performed in universities has increased many times in recent years, and the influence of this research upon American economic life has become so immediate that scientists and administrators have been forced to work out patent policies governing the use of inventions made under university auspices. The problem of forming such patent policy involves both the *external* relations of the university with industry, government, and the general public and the *internal* relations between the university and its staff members. Because only the university can give equitable consideration to the best interests of all the parties concerned, upon it alone falls the responsibility for proper management of campus inventions.

University scientific workers commonly fear the practice of patenting because of the possible effect of commercialism upon the academic atmosphere of the campus laboratory and upon the traditional non-profit, tax-free nature of the university. On the other hand, the university's chief purpose for holding patents is to encourage public use of its inventions; university patents can protect the manufacturer who takes the initial risks of developing and commercializing the invention. Patents also prevent misappropriation of university discoveries by others than the true inventors, and can earn income for support of continued research. In deciding whether to take patents, educational institutions must weigh such advantages and disadvantages. University scientists and administrators now generally accept patents as proper means for protecting laboratory discoveries and for encouraging their public use.

Before 1930 a few universities took patents and developed them for income. Single spectacular discoveries, such as Insulin at the University of Toronto and Vitamin D at the University of Wisconsin, were

patented to control the quality of these public health products and to earn profits for the university. After 1930 financial necessity, as well as tremendous growth in the absolute magnitude of the research performed in universities, turned academic interest to patents. By 1948 nearly all educational institutions have established policies toward patents, and several more are actively exploiting patents for profit. The University of Wisconsin has been notably successful financially, having collected seven million dollars from one patent in fifteen years; but it has fallen into practices which were ruled to be violations of anti-trust laws. The patent experience gained by other universities in less spectacular ways indicates fairly definitive answers to several important questions of patent policy. Among these questions are: how shall the equities in staff inventions be divided between the university and the inventor? how shall inventions from outside-sponsored research be handled? what is a proper reward for the inventor? what form of licenses shall be given to manufacturers? and what organization for patent management can best serve the interests of the university and of the public?

The patent policy formulated by the Massachusetts Institute of Technology in 1932 reserves to the Institute rights to all inventions toward which the Institute has made substantial contributions in money or staff time. A Faculty Patent Committee determines the equities in inventions. An administrative Committee on Patent Management, using Research Corporation of New York as its agent, directs the commercialization of Institute patents. Research Corporation, a non-profit organization originally formed to earn research funds by the Cottrell electrostatic-precipitation patents, has thirty-five years of experience in managing patents assigned to it by universities.

The chief internal problem of university patents is one of dividing the equities in inventions. The university has both a legal and an ethical right to inventions toward which it contributes, and it can manage patents more advantageously than can the individual faculty member. The inventor should receive consideration, however, through financial reward by a percentage of the royalties and through his participation in managing the patent on his invention.

In meeting the external problem, the university must balance considerations of public relations against the desire to earn income for support of scientific research. Even though exclusive licenses would frequently advance the general welfare, public opinion limits the university to giving only nonexclusive licenses. Thus, the only way available to the university for maximizing patent income is to increase efficiency in patent management. Research Corporation has demonstrated that it can induce manufacturers to take its patents on a non-exclusive basis and can administer patents in keeping with a university's

of operation of campus laboratories throughout the country. Even in 1940, before wartime research expenditures began, some $32,000,000 was spent annually by universities for research in the natural sciences.[5] These figures cannot indicate the research that is conducted as an adjunct to teaching.

Both industry and government have long realized that educational institutions have the staff, the equipment, and the environment particularly suited to profitable research. Since the War, industry, having more funds for research than ever before, and the government, realizing the importance of scientific research to the national defense, are pouring great sums of money into university laboratories. In past years philanthropists attracted by the non-profit nature of the university have contributed to the support of scientific research that will benefit the public in more general ways.

In view of the magnitude of campus research the implications of university patent policy in our economy are far-reaching. Universities, in one extreme, might become through control of technological advances the holding companies of American industry. Or, unbeknown to them, in their patent laxity they could be made the tools for the concentration and monopolization in industry. Or, as a third potential danger, in reaction from the rigors of the commercialism outside campus laboratories, universities might withdraw entirely from direct responsibility for contributing the technological life-blood to the American economic society. In the triangular area delimited by these three apexes of danger, university scientists and administrators are working out research policy.

B. *Organization and Source Material of This Study*

The object of this paper is the analysis of the patent policies and procedures at American universities and colleges with view to applying this experience to the specific problem at the Massachusetts Institute of Technology. As far as is warranted by the data available an answer will be attempted for the two questions of which the patent problem is the specific manifestation. How shall the university control the use of its discoveries and what is the proper relation between it and its research workers?

Chapter II will trace the development of university patent policies historically to indicate the motivation for their initiation and will compare and analyze separate university policies in regard to several logical

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[5] Ref. 3, p. 80. The above-mentioned research funds of universities are lent significance by comparison with the $234,000,000 and $70,000,000 spent by industry and government, respectively, for technical research in 1940. However, governmental and industrial research was almost exclusively applied research. About $23,000,000 is estimated as spent in pure research by universities in 1940, as compared with only $16,500,000 total expenditures in that field by industry and government. (See ref. 3.)

subdivisions of the patent problem. With the experience of American universities generally as background, Chapter III will treat the patent developments at M.I.T., summarizing the Institute patent policy in subdivisions similar to those used for comparing other universities in the preceding chapter. Chapter IV will turn to procedural detail in explaining and analyzing how the Institute policy has been carried out; the management of specific patents will be studied to illustrate the M.I.T. policy and to permit more complete analysis. The final chapter will make an overall evaluation of university patent policy, drawing such conclusions as are possible with particular application to the Institute.

The data for this study have by necessity come largely from such diverse original sources as talks with individual administrators of university patent policies and numerous publications of individual universities. Only two general analyses of university patent policy have been published, one by Dr. Archie M. Palmer [6] in 1934 and another by Dr. Andrew A. Potter of Purdue University [7] in 1940. Both studies were limited in coverage and are now out of date. Realizing the importance of the patent problem with the magnified scale of postwar scientific research, the National Research Council has recently conducted a comprehensive survey of patent policy in educational institutions under the direction of the same Dr. Palmer mentioned above. Through the kindness of Dr. Palmer this writer has had access to the files of data collected for the National Research Council survey.

Before attacking the central consideration of the paper, two foundation stones must be laid. A resumé of the workings of the American patent system, particularly of Patent Office procedure, is necessary to an understanding of the problem faced by universities. Secondly, it is necessary to recognize frankly the traditional fear of patenting among scientific workers at universities, to analyze the basis of that fear, and to consider what compensating factors impel universities to protect their discoveries.

C. *The American Patent System* [8]

An intelligent analysis of the patent problems of universities is impossible without a rudimentary knowledge of the American patent system—the mechanics of its operation, its relation to research and inventions, and its economic and social results. To weigh its merits and faults is beyond the scope of this paper since a complete evaluation of the American patent system would involve complex economic, political, sociological, philosophical, and historical considerations. However,

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[6] Ref. 40.

[7] Ref. 27.

[8] This abbreviated statement of the operation of the patent system is based on Berle and DeCamp's "Inventions and Their Management" (Ref. 2).

one can profitably strive to answer the question: What is this patent system which is part of the economic and political environment in which university research is conducted?

Although the patent system has been under heavy attack in recent years, nevertheless, its essential features will probably remain for many years to come. The National Patent Planning Commission [9] two years ago reported, "It is the Commission's conviction that this recognition of the exclusive ownership of intellectual property has fully realized the intent of the Constitution on behalf of the public and should be continued." The Commission recommended several changes to improve the workings of the patent system, while preserving its intrinsic worth. Scientists and engineers whether in universities or in industry will do well to understand fully the mechanics of patenting.

1. *Fundamentals of the Patent System*

The Constitution laid down the objective of the patent system in stating:

> The Congress shall have the power to promote the progress of science and the useful arts by securing for limited times to authors and inventors the exclusive rights to their respective writings and discoveries.

In return for revealing the details of his invention to the public, the inventor receives a grant of monopoly for a period of seventeen years.

The American patent system sets out to work for the public good, that is, to create more inventions and make better use of them, by three means;

(a) By encouraging inventors through the profit motive.
(b) By causing complete and immediate disclosure of the details of inventions, and
(c) By providing financial protection for manufacturers in undertaking expensive industrial developments of inventions.

The primary reason for the patent system is the advancement of the public good, not reward to the inventor.

2. *The Statutory Law*

The Federal law provides that:

> Any person who has invented or discovered any new and useful art, machine, manufacture, or composition of matter, or any new and useful improvements thereof, or who has invented or discovered or asexually reproduced any distinct and new variety of plant other than a tuber-propagated plant . . . may . . . obtain a patent therefor.

Principles of nature, however basic to technological advance, are not patentable; nor are mere ideas. As a court decision has put it, "Invention consists in the conception of a function, and the selection of means whereby the function can be operatively carried out."

[9] Ref. 26, p. 743.

Even though an inventor may devise something new and useful before anyone else in this country knew of or used the particular device in question and before it was described in any patent or printed publication anywhere in the world, he may be refused a patent because of two so-called statutory bars: either publication anywhere or public use or salé in this country one year or more before the filing date of the inventor's application for the patent.

The patent itself contains, besides a grant from the government, drawings illustrating at least one embodiment of the invention, a specification summarizing the state of the art and describing the invention in detail, and a set of claims, each of which describes a novel combination from the use of which others than the inventor are excluded.

3. *Patent Application and Patent Office Proceedings*

The patent application is prosecuted in the Patent Office as a negotiation between the examiner representing the public and the attorney representing the inventor. The application consists of one or more drawings, the specification, and the claims, which the inventor hopes to be included in the issued patent. For it to be valid, the patent must conceal nothing.

An *interference* proceeding may be necessary in the Patent Office in order to determine the priority of invention between two or more individuals claiming substantially the same patentable invention. The burden of proof is on the "junior party," the last alleged inventor to file application for patent. The junior party's chance of success is estimated to be one in seven or eight; thus, early application is important. In interference proceedings complete and witnessed records kept from the date of conception on are important in establishing priority of conception and diligence in reducing the invention to practice. The procedure in handling interferences is an extremely technical matter and is the chief cause for delay in the issuance of a patent.

4. *Reissues and Disclaimers*

An inventor has the right to surrender within one year an issued patent which through an oversight omits an important claim or which makes too broad claims, and he may then apply for a "reissue" patent. The reissue will have the same expiration date as the original patent. The inventor may also narrow the claims of his patent by a "disclaimer," which must be granted by the Patent Office.

5. *Infringements*

Someone has said that a patent gives the right to litigate. Indeed, a patent gives *only* the right to prevent an infringement, that is, to exclude all others from making, using, or selling the patented invention.

In deciding an infringement suit the Federal courts must consider the scope of the patent in question and its applicability to the article that allegedly is infringing it. Infringement suits may be brought either to recover damages, or to obtain an injunction against further infringement. Damages are now paid in few cases. Infringement suits are expensive—$10,000 is the average cost—and are warranted only in cases where considerable infringement has occurred or is likely to occur.

6. *Conclusion*

This brief explanation of patent procedure is intended to point up some of the factors involved in the mechanics of patenting. Carroll L. Wilson [10] has correctly stressed the ever present danger of costly patent litigation. States he:

> The problem of drawing careful specifications and claims is one requiring a large amount of time and thought as well as familiarity with patent law and practice. The issuance of a good patent which has promise of some importance is usually the result of extensive negotiations with the Patent Examiner. Procedure in handling a patent during interference is complicated and likely to be lengthy and expensive. Because of the large percentage of litigated patents which are found invalid, there is no strong presumption of validity in an unadjudicated patent. This means that in order for a patent to be generally recognized as valid, expensive litigation is frequently necessary.

The complication and expense involved in prosecuting patents is a serious liability that faces universities and the individual staff member. Assurance that a patent can earn net income for anyone is small.

D. *Why Should Universities Take Patents?*

Without begging the question before all the available facts are presented, at least a partial answer must be essayed at this point to the traditional opposition to the patenting of university discoveries. What are the common arguments against university patents? In contrast to these arguments, what basic factors in the American patent system make it advisable for universities to concern themselves with patents?

1. *Common Fears of Patents on University Discoveries*

Frequently research men in university laboratories think of patents as being unethical, or at least a bit off-color. Because of altruism or because of their interpretation of professional ethics, they object to the personal monopoly that is attendant on the practice of patenting; [11] this objection they then confuse with the decision not to patent at all. They think the publication of the details of their discoveries is sufficient.

Secondly, men interested in research often fear that secrecy, competition, and personal jealousies detrimental to the free-to-all atmosphere of their laboratories will be introduced with the practice of patent-

[10] Ref. 38, pp. 13–14.

[11] Dr. Bush's statement, ref. 39, p. 84.

ing.[12] Only two years ago Dr. Rabi,[13] Columbia University physicist and Nobel prize winner, testified before a Senate Committee that "a patent-minded colleague in our department would in time find that he has few scientific friends. We like to discuss matters freely and it gives us the jitters to feel that someone is going to rush off and patent some idea which comes up." Closely allied with this fear that patents will cause dissension in the community of scientists is a mistaken belief [14] that patents place a legal stricture on other scientists who subsequently do fundamental research in the same field.

A third common fear expressed by scientists [15] is that patenting university inventions will direct research away from fundamental research and into lines that promise quick and sure commercial reward. To quote Sir Walter Fletcher,[16]

> It will be difficult for them [research workers] not to feel that the university [which exploits patents] will be more inclined to reward by pay or promotion him who makes some addition to knowledge of an immediately profitable kind rather than him who works for the sake of knowledge itself.

This redirection of research aims might come about in either of two possible ways: by motivating scientists to enter developmental research in hope of obtaining patent profits or by causing university research funds to be allocated to projects with the best commercial possibilities. The first possibility is not very dangerous, for certainly the motivation of university scientists is too deep to be affected particularly by the illusory prospect of profits from patents. The second possible method by which patents might redirect university research aims is more dangerous, but can be consciously avoided in administration of research funds. Some scientists fear, furthermore, the commercialization attendant upon patenting of university discoveries will result in scientific slipshodness or outright dishonesty in evaluating competing products if such dubious practices will pay off.

Finally, both the professional status of the scientist and the position of the university in public opinion might be harmed, some believe,[17] by court litigation, competition with industrial laboratories, and other difficulties that might accompany patents. As one writer [18] has expressed it,

> One of the greatest glories [of scientific research] is its intellectual integrity and independence—but can this reputation continue unsullied in the clash of competitive sales campaigns of patented commodities, infringement suits, and other contentions of the marketplace in which the financial interest of the research institution is on one side of the dispute?

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[12] Ref. 14, p. 11 and ref. 42, p. 4.

[13] Ref. 46, p. 976.

[14] Ref. 7, p. 332.

[15] Statement of Dean Langsdorf, Washington Univ., ref. 42, p. 3.

[16] Ref. 13, p. 548.

[17] Ref. 17, p. 21; ref. 24, p. 1317; ref. 40, p. 114.

[18] Ref. 18, p. 548.

The public might express very directly its disdain for the university which actively seeks patent royalties by letting it live on its earnings, by withholding contributions for research, and perhaps by revoking the tax exemption privileges of the universities. If events ever took this turn, educational institutions would become nothing more than business corporations and ultimately the value of a staff member would be judged by the commercial value of his discoveries.

These arguments have been expressed too frequently and too sincerely to be ignored. Only study of the patent experience of educational institutions will show if the difficulties envisioned have actually developed. However, there are factors intrinsic in the American patent system which answer some of the objections listed above and which make university patenting especially desirable for the public good.

2. *Reasons for Universities to Hold Patents*

Failure to patent, in the first place, will effectively hinder public use of many inventions. Contrary to the popular belief, an invention or scientific discovery is not given to the public simply by telling the public about it. Elihu Thomson, a famous inventor in the electrical industry and once Acting President of M.I.T., said cryptically, "Publish an invention freely, and it will almost surely die from lack of interest in its development. . . . Patent it, and if valuable, it will be taken up and developed into a business." [19] To put an invention on the market, and thus to make it of practical benefit to the public, involves in many cases sizable financial outlay in developmental work, an expense which a manufacturer cannot undertake if all others can outstrip him in competitive position by avoiding these initial expenditures.

The following case illustrates how "dedication to the public" has a result opposite to that intended: L. R. Cleveland, a research fellow at Johns Hopkins University, invented a method for sterilizing fruit juices by treatment with oxygen under high pressure.[20] The National Research Council, donor of the fellowship, considered it valuable enough to patent, but then dedicated it to "the free use of the people of the United States"; the sterilization method has never been used.[21] This instance is cited solely as one case where a university discovery went unused when offered freely to all comers. Admittedly, other economic and technological considerations may have been factors in its neglect; cause and effect is difficult to establish completely in social events of this type. Nevertheless, the major reason for existence of the patent system, reaffirmed after careful study by the distinguished National Patent Planning Commission [22] some two years ago, is its encouragement of public

[19] Ref. 38, p. 1505.
[20] Ref. 28, p. 19.
[21] Ref. 39, p. 25.
[22] Ref. 26, p. 743.

use of invention through protection for venture capital. This reason operates with equal force, whether the invention is made on a university campus or elsewhere.

Secondly, the inventor's sense of ethics should demand that he concern himself with the quality of the product derived from his discovery. "In the case of a new drug or process the possibilities of dilution or contamination or distortion are endless, and by means of a patent the inventor can insure that whatever is offered as the invention is technically right." [23] The need for quality control is illustrated by the experience of Dr. Babcock,[24] the University of Wisconsin professor who invented the Babcock test, still widely used for determining the butterfat content in milk. The test is good only to the degree that the glass test bottles used are accurately calibrated. Babcock, believing that the people of the State of Wisconsin "were entitled to all the benefits derived from its use," did not patent the discovery, and so had no control over manufacturers who threw machines with uncalibrated glassware on the market and almost sent the test into the discard. To cite an instance of a public health discovery, the control of the quality of Insulin by a committee of the University of Toronto, where Dr. Best and Dr. Banting made the discovery, was possible through the patent; proceeds from Insulin licenses supported control laboratories, as well as additional research to improve the product.[25] At the time of the discovery no laws sufficient to protect the public in regard to the quality of Insulin existed in either the United States or Great Britain.[26] Patent control effected by a public institution fills a gap in the protection of the public welfare. There has never been any question as to the wisdom of the Insulin control.

Furthermore, the educational research man should not provide fertile ground for the "patent pirate," who does his research in the current scientific publications, and who hastens to patent the discoveries of others. Unfortunately the same exactness and completeness of description is not employed in scientific publication as is demanded in a patent specification.[27] As the result, a shrewd patent thief can establish his claims of "novelty," obtain a patent, and restrict use of the invention to his own profit. Patents taken by the university discoverer preclude the possibility of monopoly misuse. The foregoing arguments were summed up by President Compton [28] in his annual report of 1932: Responsibility does not "end with mere publication of a patentable, scientific discovery or invention."

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[23] Ref. 13, p. 543.

[24] Statement of Dr. H. L. Russell, Director of Wisconsin Alumni Research Foundation, ref. 39, p. 29.

[25] Ref. 4, pp. 85–86.

[26] Ref. 39, pp. 99–100.

[27] Ref. 28, p. 1319 ff.; ref. 24, pp. 22–23.

[28] Ref. 21, p. 101.

As a further argument for university patenting, the large university is in a better position to develop and protect the patent right than most individual staff inventors. The legal mechanisms which wealthy, established concerns may use to delay, hamper, and sometimes invalidate the patent rights of individual inventors are well known. As long as such inequality before the law remains a fact of our society, the individual inventor is fortunate to share the strength of an institution, which will administer his patent in accordance with aims similar to his own and will return to him a portion of the income.

Further advantages derived from patenting university discoveries will be investigated later in this study and are mentioned here only for completeness. By patenting its discoveries the university responsible for the original research protects itself from being stymied in further development and research in the same technical field. Also, income accruing from patent licenses may support further university research and education; and outstanding scientists that are often lost to higher-paid industrial positions are encouraged by participation in royalty earnings to remain in university work.[29] It has long been recognized that royalties from book sales protected by the copyright law have gone far in encouraging university professors to be productive in book-writing; unlike books which are easily commercialized by publishing houses, patents are put to commercial use only with difficulty.[30] In this patent commercialization the university can be of help to the individual staff inventor. A final incidental benefit of university patents comes from the discussion and dissemination of new ideas that accompany the complete disclosures made in patent specifications. In other words, patenting may constitute an important part of the university's publication responsibility.

CHAPTER II.

THE PATENT EXPERIENCE OF AMERICAN COLLEGES AND UNIVERSITIES.

At least two reasons make it advisable to study the development of patent policies in American universities generally before turning to a study of the specific development at the Massachusetts Institute of Technology. The experience of comparable institutions in their patent dealings will form a basis for critical analysis of the M.I.T. patent program. Through the years the men responsible for the patent policies at the Institute constantly exchanged experiences with other universities. Thus, the development of the M.I.T. patent policies was an outgrowth of the general university experience.

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[29] Dr. A. R. Olpin, formerly Director of the Ohio State University Research Foundation, makes a strong case for university patents in conjunction with cooperative research with industry. Ref. 24, pp. 22–23.

[30] Ref. 42, p. 2.

A. *Scattered Beginnings Before 1930*

In the beginning university patent policy was one strictly of "hands-off." Traditionally the university believed that the invention was the private property, and the patent the private concern, of the university staff member. All the forces of inertia, all the common fears [1] of patenting held by the academicians, caused the colleges to ignore patents or else to shrug them off as a personal problem of the inventor. As legal precedent for this stand the university could point to the policy, still almost universally accepted, that professors should be allowed to copyright their own books and to realize the profits from such copyrights.[2] Galileo, and not the University of Padua, with which he was associated, applied to the Doge of Venice in the 16th century for a monopoly on a device for lifting water.[3] To return from this slight excursion into Renaissance history—this same situation existed in all American universities at the beginning of the twentieth century, in spite of the fact that the colleges and universities of the last century spawned many patentable ideas such as the electro-magnet of Henry at Princeton, the Babcock butterfat test at the University of Wisconsin, and the telegraph-line loading coil invented by Prof. Michael Pupin at Columbia. The first two inventions were not patented; the third made a millionaire of its inventor.

Universities were called upon to formulate a positive policy toward patents in the first instance by spectacular circumstances. Major inventions were made on their campuses, inventions toward which they had contributed in equipment and faculty time, or inventions of apparent commercial value which the faculty member felt unable to manage in his own right. A second, more evolutionary, factor that directed university attention toward developing a general patent policy was the growth of university research laboratories. Industrial research and large university laboratories are phenomena of the twentieth century. As the university laboratories were doing more and more institutional research, a method for handling its results was demanded. The need for a general patent policy was particularly apparent in regard to research sponsored by industry.

1. *Patent Policy Developed for Single Spectacular Inventions*

One of the earliest spectacular campus inventions did not directly change university patent policy, but did cause the creation of the non-profit Research Corporation. This organization, since its formation in 1912, has exercised a constant directing force in the development of methods for managing the results of university research.

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[1] These fears were summarized in Chapter I.

[2] Ref. 42, p. 10.

[3] Ref. 28, p. 1319.

While an instructor in physical chemistry at the University of California in 1905, Frederick G. Cottrell[4] discovered the electrostatic precipitation method for recovering various products from flue gases. Mists and dusts were removed from the gases by applying the principle of attraction of oppositely-charged particles. The mist or dust was imparted a charge by passing through an intense electric field, as exists at the surface of a slender wire at high potential, and were collected on an oppositely-charged plate. The invention was first applied to recovery of sulfuric acid mists.[5]

During his summer vacations Dr. Cottrell, having formed the Western Precipitation Company with a few associates, began installing the electric precipitators. However, the business mushroomed so quickly that Cottrell found his commercial efforts were interfering with his desire to continue his scientific work. The University of California was offered the patents, but did not feel able to accept the assignment because of difficulties involved in such a commercial undertaking. Cottrell then offered the invention to the Smithsonian Institution, which in turn was instrumental in 1912 in organizing the independent, non-profit Research Corporation. Such distinguished scientists and engineers as Arthur D. Little, Elihu Thomson, T. Coleman Du Pont, and Charles A. Stone were among the founders.

Research Corporation, organized as a stock corporation whose shares may pay no dividends, set for itself two purposes: first, to build an efficient business organization to promote the precipitation process commercially, and to set an example for patent management in the public interest; and secondly, to apply all profits from this patent and others it might obtain to the support of further scientific research. As Dr. Cottrell himself has said, "The purpose in organizing Research Corporation was not merely to produce revenue for scientific research, but to act as a sort of laboratory of patent economics and to conduct an experiment in patent administration." [6]

Almost immediately heavy royalty returns from the Cottrell patents started to accumulate in a fund for financing university research. Grants were started in 1927 and among subsequent grants was one to M.I.T. to build the Van DeGraaff ten-million-volt electrostatic generator. Up to this year $1,250,000 had been distributed to some 52 universities all for research in pure science, and the Corporation[7] has recently announced that $2,500,000 more will be granted in the next five years. The University of California was one of the first universities to use the Research Corporation facilities in patent matters.

[4] Ref. 19, pp. 2–3.

[5] Ref. 11, p. 224. Dr. Cottrell described the trials and tribulations of the university inventor in a detailed and interesting article published in 1912. (Ref. 19.)

[6] Ref. 11, p. 224.

[7] Ref. 17, p. 34.

Two dramatic university discoveries in the twenties turned attention to the patent problem. They were preparation of Insulin at the University of Toronto (1921) and the conversion of ultra-violet energy into Vitamin D at the University of Wisconsin (1924). The University of Toronto has been authorized to own and exploit patents since 1906,[8] but not until the discovery made by Banting and Best were patents given particular consideration. The mechanism chosen to manage the Insulin patent, to control the quality of product sold by licenses and to apply royalties to further research was a special Insulin Committee within the University. In general, the University of Toronto has opposed taking patents for scientific discoveries in educational institutions,[9] but it has set up special committees to manage patents in two cases (in regard to Insulin and to an anti-silicosis invention) where control of public health discoveries appeared particularly necessary.

The Insulin patents [10] earned $500,000 gross for the University of Toronto in the year 1924 alone. The research men responsible have not participated in the income partly because of the prohibitive provision of the medical code of ethics and partly because of the belief at Toronto that giving a "cut" to the staff member degrades his research.

An entirely different method of patent management was undertaken at the University of Wisconsin. Dr. Harry Steenbock had developed methods of improving the antirachitic properties (Vitamin D) in food and drug products by irradiation with ultra-violet light from a quartz mercury lamp. Believing from a previous experience with patents on Vitamin A that the administrative machinery of the University was too cumbersome to handle patents effectively,[11] Dr. Steenbock was willing to hand over the patent rights to a non-profit organization formed in 1925 by interested Alumni under the name of the Wisconsin Alumni Research Foundation. The purpose of the Foundation was to commercialize the Steenbock and other patents and to build up from license royalties an endowment principal, the income from which could advance scientific research in any field at the University of Wisconsin. A control laboratory was established to fix standards of potency and to work out methods of test control for Vitamin D products, and some eight pharmaceutical firms were licensed to manufacture a standard product under the trade-name "Viosterol." By 1930, less than five years after organization, the Foundation had a gross income of about $100-per-day and had acquired several other patents, notably the iron-copper patent for the treatment of anemia, that have since proved profitable. The Foundation was already involved in the litigation, which has accompanied its financial success throughout 22 years of existence.[12] Patent

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[8] Ref. 40, p. 100.

[9] Statement of Lorne Hutchinson, Exec Secr. of the Insulin Committee, in ref 39, p. 100 and in ref. 15, pp. 330–332.

[10] Ref. 18, p. 60.

assignment to the Foundation by staff members was entirely voluntary, but such assignment was frequently resorted to by University inventors partly to avoid the business problems of patent development and partly to earn research funds for the University of Wisconsin. The inventor received at least 15 per cent of the net income. Prof. Steenbock [13] had received, up to 1940, $760,000 out of total royalties of $7,500,000 on the Vitamin D patents.

A third patent,[14] also in the field of public health, will be only mentioned in passing. Doctors George and Gladys Dick developed a scarlet fever antitoxin while associated with the John McCormick Institute for Infectious Diseases in Chicago. No patent application was made initially for a year after publication of the experimental results, but the officers of the United States Public Health Service notified the inventors that it lacked the legal powers to obtain the necessary quality control and recommended that the inventors assume responsibility for maintaining serum quality by means of a patent. The Scarlet Fever Committee was incorporated in 1925 with almost exactly the same functions and objectives as the Insulin Committee. The Dicks received no part of the patent royalties.

2. *Patent Policy Evolving From Growth of Institutional Research*

It would be untrue to say that educational institutions were not thinking about the patent problem after 1920. State universities, whose experiment stations were doing considerable applied research in cooperation with industry, were particularly concerned with the problem.

As early as 1913, the Engineering Experiment Station of the University of Illinois asserted that results of experimental work paid for from University funds belonged to the University and that the staff inventor might be required to patent his invention and assign it to the University. A committee of the Association of Land-Grant Colleges and Universities in 1922 found three land-grant colleges with definite patent policies.[15] The committee's report contained in substance the following specific suggestions:

(1) An institution has a right to inventions made by staff members in course of regular duties or at the institution's expense.

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[11] Ref. 48, p. 1; ref. 44, pp. 21–27.

[12] Discussion of the legal difficulties into which the Wisconsin Foundation has fallen within the past five years will be reserved for later in this chapter. In the above historical development the early influence of the Wisconsin experience upon university patent policy generally is emphasized.

[13] Ref. 44, p. 27.

[14] Ref. 7, pp. 327–30.

[15] Ref. 6.

(2) There may be cases where both inventor and institution have equity in the patent, a case which must be decided on its own particular merits.
(3) Only patents with no commercial value should be dedicated to the public.
(4) Universities should give job of patent management to an outside organization, as the Research Corporation, to avoid complications detrimental to the University.

At Purdue University just at the end of this trial period before 1930 another Research Foundation [16] was established with wider objectives than those of the earlier Wisconsin Alumni Foundation. Patent administration was only incidental to its primary purpose: "To promote *educational* purposes by encouraging, fostering and conducting scientific investigations and industrial research." [17] Organized as the results of inventions of commercial value in the agricultural and engineering experiment stations of Purdue University, the Purdue Research Foundation actively sought out research contracts with industry. It intended that the research problem posed by an industrial sponsor should be used as "a teaching mechanism for intensively training creative students." [18] Although the Foundation was willing to accept assignment of patents on inventions made at the University without Foundation sponsorship, most of the patent problems have come from the sponsored or cooperative research. The Purdue Foundation may be contrasted with the Wisconsin Foundation both as to the source of its patents and as to its breadth of functions.

In obtaining a large volume of contracts for research of genuine scientific importance the Purdue Foundation has been highly successful. Research was done for industry at cost, but all patents were assigned to the Foundation and "small" royalties were charged. Throughout its existence the Foundation has kept its eye on the educational goal in evaluating the patent situation, but in spite of this apparent nonchalance toward patent profits it has been financially successful.

3. *Concurrent Experience in Non-Profit Research Organizations*

In the period before the first World War several non-profit research organizations had been organized and were gaining experience in patent management, experience which was being watched by university administrators. These organizations are divided according to their purposes into two groups: first, those which sponsor research on behalf of private industry, and second, those that do research primarily for the benefit of the general public. Universities have counterparts of both

[16] Ref. 44, app: pp. 19–20.
[17] Article II, Articles of Incorporation, Purdue Research Foundation.
[18] Letter of G. Stanley Meikle, Director of Purdue Research Foundation, to Mr. J. R. Killian, Jr., Nov. 9, 1943.

types of research. The Mellon Institute of Industrial Research,[19] organized in Pittsburgh about 1907 in loose affiliation with the University of Pittsburgh, does research on specific problems of immediate practical interest to industrial corporations. In keeping with its declared function to be a research service organization for industry, all inventions in the field of the investigation belong outright to the donor of the research funds. The individual inventor, who makes a discovery as a Mellon Institute fellow, is considered in the same legal position as the employee of an industrial laboratory.

Research Corporation exemplifies the organizations acting primarily for the public. The Bartol Research Foundation of The Franklin Institute and the Carnegie Institution of Washington, organizations similarly concerned with patent management in the public interest, were both functioning before 1910.[20] Fellows of the Bartol Foundation are required by a 1926 plan to assign all patent rights to the Foundation, but the inventor receives 40 per cent of the patent profits. In contrast the Carnegie Institution as early as 1923 took the stand "that any new and useful inventions or discoveries which may result from researches financed by the Institution shall be dedicated to public use." The Institution will not receive profit from patents, although it does use patents for purposes of control and commercial promotion.

4. *Summary to 1930*

At the beginning of the depression decade the patent problem was clearly drawn and several methods of handling the problem were being worked out. Two different factors had turned university attention to the patent problem: first, a steady growth of research sponsored cooperatively by industry demanded a generally applicable policy toward resulting patents, and, secondly, spectacular inventions on university campuses demanded immediate concern with patent policy. The majority of universities had tacitly taken the stand that they had no equity in patents of staff inventions not even those inventions made through use of the time and facilities of their institutions. A few had taken the reverse stand and required assignment to the university or at least submission to a decision by a university committee. There were available three types of experience for managing college patents:

(1) By an outside non-profit agency, as the Research Corporation.
(2) By a separate legal entity, loosely affiliated with the university, as the Wisconsin and Purdue Research Foundations.
(3) By a special university committee as the Toronto Insulin Committee.

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[19] Ref. 32.

[20] Information summarized in this paragraph is taken from ref. 40, pp. 126–7.

The time was ripe for all universities conducting scientific research to formulate a definite patent policy.

B. *Experience since 1930*

Since the beginning of the thirties there has been an accelerated trend toward university development of patents on inventions made by staff members. Associated with this trend has been the growth of separate legal corporations affiliated with universities for the purpose of exploiting patents in their behalf. Dr. Palmer's 1934 study of university patent policy [21] found some 18 institutions developing patents; his current study,[22] as yet unpublished, shows nearly 200 universities and colleges with direct interest in patents. In at least 20 cases the separate research foundation manages patents for the university. Five other universities have a standing arrangement with Research Corporation for patent-management.

1. *Reason for a Definitive Patent Policy*

Several factors may be responsible for the stepped-up interest of universities in patents. First was economic necessity engendered by the Depression. Both endowment income and tax revenues were drying up; consequently, the almost unbelievable profits of the Steenbock and the Insulin patents were extremely appealing. The Wisconsin Alumni Research Foundation was able to turn over $325,000 to the University when the mother institution met hard times in 1933 and 1934.[23] Universities had, of course, considered the possibility of patent profits before, but they had usually spurned the proposition, believing that with the bad publicity that might accompany patent exploitation the university endowment might in the end actually suffer financially. This last fear, though still effective in the thirties, was overpowered by the compelling need for new sources of income. University administrators turned to working out methods of patent management that would not be detrimental to their public relations. The separate foundation was believed to be a partial solution.

A second factor was the growing belief, particularly in the state colleges, that where public funds spawned invention, the public (i.e. through the university) should receive the patent profits. Also, scientists were becoming aware of the advisability of patenting (as outlined in Chapter I) and, realizing their own limitations in the commercial field, they turned over their patents to the university or its agency for development, control, and profitable exploitation. Within a few years after 1930 many articles appearing in scientific journals, under such

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[21] Ref. 40.

[22] Interview of this writer with Dr. Archie M. Palmer, Washington, D. C., April 11–12, 1947.

[23] Ref. 48, p. 7.

titles as "Society's Need for Patents to University Research Workers" and "Should Scientific Discoveries be Patented?",[24] discussed the pros and cons of university patents.

A final, and perhaps most important, factor in the development of a generally-applicable patent policy was the increase in the quantity of university research, particularly research done cooperatively with industry. A steady stream of research from which many patentable discoveries might result demanded that universities develop a generalized policy toward patents. Purdue and Ohio State faced up to this problem in forming their Research Foundations; a similar motivation existed for other engineering colleges performing research cooperatively with industry. Numerous universities, because of the huge amount of institutional research that they are conducting with industrial and governmental sponsorship as an aftermath of the war, are currently formulating a patent policy.

2. *Motivation for Formation of University Research Foundations*

Several reasons prompted formation of the Research Foundation as a legal entity separate from the university:

(1) Many restrictions on universities, particularly on state institutions, prevented them from contracting directly with industry and from managing and commercializing patents through their regular business offices.
(2) Fear that the tax-exemption of educational institutions might be revoked if they too-obviously engaged in patent exploitation and other commercial practices impelled universities to use the legal artifice of a separate corporation which they in fact controlled.
(3) The foundation scheme put to use for the university talents of alumni and other interested business men and industrialists in managing patents or research. This was originally the case in the Wisconsin Alumni Research Foundation and was particularly true in the formation of the Ohio State Research Foundation, of which such men as Charles F. Kettering are trustees.

3. *University Patents and the Recent Interpretation of the Anti-trust Laws*

The influence of the Wisconsin Foundation in hastening development of university patent policies and management procedures has already been shown. Another side of the picture appearing during the past decade has had an appreciably different influence on patent policy. Since about 1940 the Foundation has been under increased attack by the Anti-Trust Division of the Department of Justice for alleged prac-

[24] Ref. 28; ref. 29.

tices in the restraint of trade, and in 1944 a circuit court of appeals invalidated the Steenbock patents, holding that the restrictive methods used in commercializing them were "unwarranted and against the public interest." [25] The Wisconsin Foundation has fallen into some of the pitfalls that beset a university in the management of patents. Its practices through the resultant bad publicity have done great harm to both the University with which the Foundation is indirectly connected and to the cause of an effective and beneficial university patent program.

Assistant Attorney-General Wendell Berge [26] has extensively documented the case against the Wisconsin Alumni Research Foundation. His charges may be summarized as follows:

a. The Wisconsin Alumni Research Foundation "has been the vehicle for creating a domestic monopoly resulting in division of fields, price fixing, control of container size, and limitation of potency of vitamin products--as a result of which the public has been charged excessive and arbitrarily high prices." A licensee of the Foundation describes it as being "merciless in beating out competition."
b. It has suppressed use of competing Vitamin D processes by licensing arrangements and by threats of patent litigation.
c. It entered international cartels to divide the world into non-competitive areas.
d. In spite of its semi-independent nature the Alumni Foundation has been used in the interest of the Wisconsin milk interests to forbid use of Vitamin D irradiation for oleomargarine and to strengthen the concentration of the condensed milk industry in the State of Wisconsin.[27]
e. Its practices have been detrimental to the advancement of science in
   (1) its lack of interest in vitamin research unless a commercial advantage could be obtained,
   (2) its use of "its licensing scheme to discourage research by its licensees," and
   (3) its attempt to suppress scientific data and truthful advertising in order to maintain its monopoly position.

The financial success of the Steenbock patents is largely attributable to the willingness of pharmaceutical, chemical, and food companies to come under the Wisconsin price "umbrella," share the field of sales by

[25] The three U. S. licensees of the Toronto Insulin Committee were charged in 1941 with price fixing in violation of the Sherman Act and they paid large fines rather than assume the costs of litigation. Ref. 16, p. 112.

[26] Ref. 1, pp. 82–111. Mr. Berge's chapter on the vitamin monopoly is interesting reading and presents in detail the facts substantiating the above summary.

[27] Ref. 23, p. 497.

license arrangement, and pay the fairly reasonable royalty of 3 to 10 per cent on a large volume of sales.

The Wisconsin experience points up a dilemma that will face many universities: can maximum commercial profits be obtained and the public welfare preserved at the same time? The Wisconsin Foundation set out originally on the acceptable objective of effecting "rigid control through which it is possible to protect the public and prevent unscrupulous commercialism from capitalizing on the Steenbock discovery." [28] The fact that the control was perverted in order to make money and to serve special interests shall not obscure the worthiness of the goal; it only increases the necessity for finding the middle ground.

C. *Present Patent Practice in American Universities*

With a background understanding of the past motivation for patent policy, the reader may now wish to know what universities are actually doing about patents. Because of the dearth of organized information about patent policies of universities and colleges, the extensive table of the Appendix* was prepared to survey present patent practices.[29] Although the patent problem has been broken into its several facets to facilitate study, the essential unity of the total patent policy of a given university should be remembered. Only with difficulty can the patent policy of a university be pushed into "boxes" for comparison with other universities in the vertical columns of a table; thus unwarranted simplification must be guarded against.

1. *Policy toward Patent Exploitation as an Income Source*

The most significant differences in the patent policies of universities arise in their attitude toward income from university-controlled inventions. To quote a monograph now being prepared by the Department of Justice,[30] "the great majority of the institutions studied appear to be willing to use patents to produce income, and a substantial number, particularly those which have established foundations, are engaged in an active program of obtaining patents and administering them for profits." Dollar-and-cents data on the success of universities in obtaining patent profits are not available. Although Dean Potter of Purdue,[31] who has gained considerable experience in patent management, has written that "the income accruing to educational institutions from patents is insignificant," present data, although still scanty, in-

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[28] Ref. 48, p. 4.

* The Appendix will follow Part II of this paper in the April issue of this JOURNAL.

[29] Note: All illustrative material used in this section (c) is taken from the Appendix, the sources of which are indicated completely there. Source references will not be given in this section for facts summarized in the Appendix.

[30] Ref. 44, p. 18.

[31] Ref. 27, p. 7.

dicate that the income is more substantial than he suggests. Although under these circumstances no definitive answer is possible, the indication is clear that several universities are receiving significant income from patents and that, as the present patents are brought into wider use and as more patents are gathered into the portfolios of universities, this income may be expected to increase.

Five universities, obviously a very small minority of all educational institutions, have expressed themselves as being vehemently opposed to profits from inventions. Fifteen years ago a faculty committee at the University of Pennsylvania adopted the resolution that "inventions or improvements made by [the University] . . . should not be restricted by the University, but should be announced to the world so that such benefits may be freely enjoyed by all, and *without pecuniary profit either to the University or to any one in its service*." [32] A later statement of policy by the University of Pennsylvania permits patenting of discoveries for profit outside the field of public health. In 1943 the Board of Trustees of the University of Chicago took an uncompromising position against patents in stating: "The University will not profit financially from research by means of patents, royalties, or licensing agreements. Members of the staff will not be permitted to receive direct or indirect financial return from patents."[33]

Harvard, Yale, and Johns Hopkins Universities adopt approximately similar attitudes toward institutional patents: [34]

a. No patents are taken by the university except for dedication to the public. Harvard and Johns Hopkins believe it undesirable for either a university, or any agency associated in the public mind with its name, to hold patents.
b. Patenting is the private concern of the individual staff inventor, except that "no patents primarily concerned with therapeutics or public health may be taken out by a member of the University."

Some universities, including those above, consider medical patents in a category by themselves. Dr. Fishbein of the American Medical Association [35] finds two major objections to receiving profit from public health discoveries. First, patent competition among universities, such

[32] Ref. 40, p. 124; ref. 38, p. 55. The italics are supplied by this writer.

[33] Ref. 44, appendix, p. 3. This action by the Univ. of Chicago was called a "blow to the American patent system" by the American Patent Law Association.

[34] Ref. 44, pp. 18–19. The Harvard policy was laid down by faculty vote in 1934 after embarrassing difficulties with a patent on the Drinker respirator, devised by a member of the School of Public Health. (Ref. 8, p. 424.)

[35] Ref. 12, p. 1317. A thorough discussion of the pros and cons of medical patents was made by a 1939 Conference on Medical Patents called by the American Medical Association (refs. 7 and 8).

as undoubtedly existed among three or four over Vitamin D patents, is particularly undesirable in this field. Secondly, public relations of universities are susceptible to harm from charges of profiting from the misfortunes of the public. Besides the Vitamin D patents, the scarlet fever patents held by the Scarlet Fever Foundation and the patent on the copper-iron treatment for anemia held by the Wisconsin Foundation have been sources of friction. In spite of the objections to public health patents, many universities have used them to raise funds to finance further research. Toronto [36] holds the patents on preparation of Insulin; St. Louis, Theelin (the female sex hormone); Minnesota, Thyroxine (active element of thyroid gland used to treat goiter, cretinism, etc.); Wisconsin, Cincinnati, Columbia, and M.I.T., various Vitamin D patents. The objections of Dr. Fishbein are fundamental only if the public interest is injured by the end results of medical patents; otherwise, he is only pointing out difficulties encountered in administering a profit-making policy. The public interest can be served by university patent income in two very important ways:

a. The financial incentive to the scientific man and the protection of the manufacturer in the developmental stage can increase the chance that health discoveries will be developed to practical use.
b. The profits may be applied to further research to insure a steady flow of public-health discoveries.

Only the small minority of universities now consider public health patents improper sources of income. It is noteworthy that the few universities opposing patent profits are private institutions as contrasted with land-grant colleges which have led in developing patents for profit.

2. *Patent Ownership* [37]

Although many universities accept control of patents for profit, several variant philosophies in regard to ownership of the original patent rights are current. A large number of universities and colleges have no definite patent policy, although few universities doing appreciable scientific research have failed by now to give it consideration. Another small group of institutions, including notably the University of California, Syracuse University, and Louisiana State University, expressly leave all patent rights to the inventor, but nonetheless these universities do in special cases receive and administer patents. The University of Texas, in a recent change of policy, leaves all ownership rights to the

[36] In fairness to the University of Toronto it should be noted that it discourages patenting scientific discoveries of any type; control of Insulin quality was considered more important than profit from its sale. (Ref. 39, p. 100.)

[37] This discussion does not involve ownership of patents arising from research sponsored by outside corporations or government. These patents demand special consideration.

staff member, but requires payment to the university of a small percentage of net proceeds from the individually-developed patents.

A third and more sizable group of universities also follows the general policy that all patentable inventions belong to the staff member responsible, but they actively encourage assignment of the patents to the university or its agency for administration. The University of Wisconsin has gathered a valuable group of patents in its associated agency, the Alumni Research Foundation, on this voluntary basis. Columbia University, University of Minnesota, California Institute of Technology, and Ohio State University belong also in this group. The success of these universities in receiving voluntary patent assignment derives principally from two factors:

a. Inventors desire to avoid the business problems of patent commercialization.
b. They desire that their discoveries be brought to public use in such a manner as to provide funds for further university research.

The trend among universities is to adopt a fourth type of ownership policy, one which requires faculty members to assign inventions resulting from their "regular duties on University time and at University expense." [38] This policy approximates the one which governs the relation of the scientist or the engineer to his employer in industry. Land-grant colleges have long followed this policy since its initiation at the University of Illinois in 1913. Lehigh University and Massachusetts Institute of Technology led the way among private institutions by working out similar statements of policy in the early thirties. The survey made for the Appendix indicates at least fourteen universities clearly accept this philosophy of ownership. The required assignment is usually effected through voluntary acceptance of a faculty or board-of-trustees policy; in almost no case does a written contract exist between the university and faculty members, nor is any form of compulsion exercised. As an exception, the University of Florida requires reporting of inventions made by a faculty member in his own field of research. The University of Florida also has an out-of-the-ordinary policy in re-

[38] The breadth of interpretation that might be given this phase is evident. As Mr. Spencer of Northwestern Univ. Law School, ref. 42, p. 10, said, "Generally speaking, faculty members do not make a rigid distinction between their own time and university time,—a distinction which is very easily made in the business world,—because it is common practice for a university professor to mark examination papers during the evening and to devote certain daylight hours to some enterprise of his own." In general, inventions outside any specific university project are interpreted as the property of the faculty member involved. Contrary to the implication of Mr. Spencer's statement, the distinction between their own time and company time is difficult for scientists in industry to make, and usually all patents, at least in the technical field of the company, become company property. Of course, those universities requiring assignment of inventions by staff members may interpret the equities involved more liberally than industrial concerns.

gard to patents resulting from student theses. It is nearly universally considered elsewhere that a student through tuition payment receives rights to the results of his research; however, the University of Florida requires assignment of the patent to itself and then divides the amount awarded the inventors in such a manner that two-thirds goes to the faculty member directing the thesis research, and one-third to the graduate student.

The trend toward assertion of university rights to patents should be studied against the background of the accompanying trend toward institutional research centers, as contrasted with uncoordinated individual research efforts. Patent management is only part of the greater whole of research administration. The trend is associated with recognition that, although patents must name individuals as inventors, it is increasingly difficult to say that one person, or even several, are solely responsible for the results of institutional research.

3. *Treatment of Inventions Resulting from Outside-sponsored Research*

In the treatment of the results of so-called "cooperative" research, the engineering experiment stations of land-grant colleges early met the need for a patent policy of general application. The universities were receiving funds from industry in which the patent system played a prime economic part.

Special treatment is given in nearly all cases to patents developed from outside-sponsored research. Nearly universally the university and the sponsor make a patent agreement before initiation of the research program; that agreement may frequently modify the rights of staff members and graduate students to their inventions. For example, both Carnegie Institute of Technology and the University of Wisconsin, while generally leaving full patent rights with the staff inventor, require assignment of sponsored inventions to the Institute or University.

Varied degrees of preferential treatment are given to the research sponsor. In several cases the patents are given outright to the sponsor. Full assignment appears justified if the research undertaken by a university is only a continuation of research initiated earlier by the cooperating firm, if full costs including overhead and administrative cost are paid by the sponsor, or if further developmental work is necessary before commercial application. Full assignment also appeals to universities which try to avoid the administrative detail and legal complications of patent management. The tendency, however, apparently is to give only partial rights to the cooperating company. These "partial rights" take any of several forms:

a. Remission of license fees. The minimum concession to the sponsor is usually a free unrestricted license in the technical field of the sponsored investigation. Carnegie Institute of Technology remits the license fee until such remitted funds add to twice the

total contribution by the sponsor or ten times his contribution in the year the patent was issued, whichever sum is greater.

b. Opportunity for the sponsor to purchase full patent rights by payment of a special patents charge which is frequently about 20 per cent of the total costs of the original research program, including overhead and administrative costs. Lehigh University, the University of Florida, University of Michigan, University of Colorado and Texas A. & M. are applying variations of this policy.

c. An exclusive license to the invention for a restricted period of time or a particular use to permit the sponsor to recoup his expenditures. A great many universities, including M.I.T., use this method to encourage research sponsorship.

Purdue University has been unusually successful in encouraging industrial contributions for research without relinquishing extensive patent rights.

4. *Patent Administering Agencies*

Since formation of the Wisconsin Alumni Research Foundation in 1925, comparable foundations have been organized at some twenty other universities, most of which are state institutions. The principal reasons for their establishment were reviewed earlier in this chapter. The foundations vary in the control exercised over them by the associated university. The Wisconsin Foundation and, in a lesser degree, the Ohio State Research Foundation are independent organizations which nevertheless act primarily in the interest of the associated university. However, nearly all the foundations formed during the past decade have centralized control in the faculty, administration, or board of trustees of the university. For example, the Foundation at the University of Connecticut authorized by State law in 1945 was organized directly under the board of trustees.

The foundations vary also in the breadth of their functions. The majority are chiefly patent-administering agencies; but some, as those foundations at Purdue and Illinois Institute of Technology, also carry on research or act as the go-between for the university and the outside contracting party.

The same motivation as that for forming legally-separate research foundations has caused some universities to use other agencies dissociated from the university for patent management. For example, the Research Corporation of New York at present administers nearly all patents of five universities: namely, M.I.T., Columbia University, Princeton University, University of California, and University of Arizona. Some ten other colleges use the Research Corporation for managing isolated patents. The Research Corporation can never remove all problems from the shoulders of the university; in fact in many

cases involving litigation or licensing policy it will consult the university concerned.[39]

In sharp contrast to the research foundation over which the university has only the slightest indirect control (as the Wisconsin Foundation) there are many educational institutions which control their patents directly. Some university administrators argue that an educational institution cannot and shall not shunt off its patent problems. Dr. Hutchison, executive secretary of the Insulin Committee, through which the University of Toronto has managed directly those valuable patents, has said cogently:[40]

> A non-commercial, scientific, academic organization has a very real responsibility to the public . . . in deciding in any instance as to whether or not a discovery or invention shall be patented. . . . After making that decision . . . it is the duty of the scientific institution itself . . . to control the outcome [of the decision to patent, assign, or license], and if it makes mistakes try to correct them, or at least to take responsibility for them, and not foist them off on some other organization.

The observed trend toward formation of legally-separate research foundations should be explained more as an attempt to gain legal flexibility in handling patents, rather than as one to avoid responsibility for patent management. Whether patents are managed through a research foundation or directly by the university administration much the same as other property, Dr. Hutchison's thesis apparently is now widely accepted, and very few research foundations exercise the independence from the parent university that the Wisconsin Alumni Foundation has enjoyed. The foundations formed in more recent years, as noted above, have been organized under the tight control of the university. It should be recognized that, in tying the research foundation closely to the university by means of officers acting in dual capacities, the legal insulation from litigation difficulties enjoyed by the University of Wisconsin is lost. Any court could doubtlessly break through the legal artifice of the separate corporation and impose its fine or other penalty directly on the university itself. Administration by an entirely dissociated organization, as Research Corporation, is in this respect preferable.

Besides the University of Toronto, St. Louis University has had considerable experience in direct management of patents. An administrative committee in the School of Medicine, the Committee on Grants for Research, has administered the Doisy patents for preparation of Theelin, the female sex hormone, since 1930.

5. *Reward to the Inventor*

With few exceptions the inventor shares in the profits from his discovery after he assigns the patent to the University or the associated

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[39] From this writer's talk with Dr. Joseph W. Barker, President of Research Corporation, April 10, 1947.

[40] Ref. 39, pp. 101–102.

patent-administering agency. This policy of inventor reward, quite different from the practice in the research laboratories of large industrial concerns, is specifically supported in the patent-policy statement of Virginia Polytechnic Institute:[41]

> The research worker must have displayed more than routine zeal in the attack of a problem and . . . he must have drawn on his own mental equipment and energy to an extraordinary extent [to make a valuable invention]. . . . Employees of large corporations may reasonably expect that their inventions, even if assigned under contract to their employers, will result in increased salary recognition. . . . It is more difficult to secure such recognition in academic circles and it is believed preferable [at V.P.I.] to allow the research worker to retain a direct interest in any inventions resulting from his work.

The University of Toronto and St. Louis University are exceptions in giving the inventor no reward. Their patents have been concerned with public health, and in keeping with the code of ethics of the medical profession the doctors making the discoveries have not wished to profit directly. Actually they have received their reward in liberal grants for further research and in improved laboratory facilities.

V.P.I. as well as Lehigh University and Wittenberg College gives 50 per cent of net profits to the inventor. They believe that equity in the university patents is shared equally by the inventor and the university.

The Research Corporation gives 7 per cent of *gross* receipts to the inventor. Its philosophy is that while some reward to the inventor is necessary, its prime purpose should be to earn profits for the furtherance of research, distributed by fixed proportion (usually about 50 per cent) to the university from which the patent came and by special grants-in-aid for particular research projects.

The great majority of colleges allot 15 to 25 per cent of *net* patent profits to the inventor. Many universities, while fixing a minimum percentage, leave the exact proportion of receipts payable to the inventor to the discretion of the board of trustees or other administering agency. Although the fixed stipend protects the university administrator from charges of favoritism and political manipulation in distributing patent profits and is generally more easily administered, the flexible rate of inventor reward has one great advantage in recognizing the varying classes of patents that a university may administer and the varying amount of equity which the inventor may have in the patent rights. As universities gain more experience in patent matters, they may well investigate whether more of those patents toward which they have contributed but little might be assigned to the university if they gave a larger proportional inventor reward in such cases.

6. *Licensing Policy*

Little information is available about the licensing policy of universities even though this policy is of paramount importance in the effect

[41] Ref. 44, App., p. 24.

of university patents on the public welfare. In general universities prefer to give only non-exclusive licenses, a policy followed by Research Corporation for 35 years. The complex system of restrictions applied to the Wisconsin Vitamin D licenses has no parallel in the policy of other universities. However, many educational institutions are willing to give exclusive licenses in special cases, such as the following:

a. Cases where only an exclusive license will bring the invention into public use. The Texas A. & M. Research Foundation on occasion grants an exclusive license with a precautionary provision for diligence by the licensee in using the invention.
b. Cases where quality control of the manufactured products is best assured by licensing one or only a few manufacturers. St. Louis University gave an exclusive license for a period of a year and a half on its Theelin patents, including in the license agreement provisions designed to control quality.

In the problem of how to license the patents it owns, the university faces a fundamental dilemma between a desire for profits from patent exploitation and a wish to serve the public in a competitive economy. Profits can be maximized only by restricting the number of licenses given. A middle-of-the-road policy may be found, but the dilemma will remain.

William H. Nicholls of Iowa State College [42] after considering this dilemma in specific reference to the Steenbock patents argues:

> The only solution *from the social viewpoint,* therefore, is probably to patent the results of technical research in public institutions [but only in order to be in position to prevent their misuse] and grant their use free or on reasonable terms to all who find them technically and economically feasible and will meet the desired standards of quality.

He defines "reasonable terms" as license fees from which the justifiable cost of quality maintenance could be recouped; in other words, all net income from patents is "dirty money." He would not accept the above-mentioned cases for exclusive licenses as being justifiable. Mr. Nicholls is not entirely sound in supporting his argument by an appeal to social ends. Judicious use of exclusive licensing may promote the public welfare in very specific ways: by bringing more inventions into public use, by effecting better quality control of the manufactured product, and by supporting further university research. Mr. Nicholls argues that exclusive licensing can be used validly to bring patents into use only if the firms in the given industry are small and financially weak and if free competition exists to protect the producer and consumer. Actually, even old well-established companies frequently will not develop patents without some degree of patent protection. That the university has a difficult problem in balancing the diverse factors in the interest of the public is agreed to by all.

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[42] Ref. 23, p. 498.

Also, in attempting to evaluate university licensing policy by an absolute criterion of its effect on competition, Mr. Nicholls overlooks the relative criterion which would compare the effect on competition of a university patent with the effect of an industrially-controlled patent on the same invention. There is little doubt that a licensing policy may be developed to yield income to universities without imposing the restrictions on competition inherent in an industrially-owned patent.

The one instance in which restrictive licensing has been used extensively to the detriment of the public good is a case in which a research foundation had a free hand to manage the patents as it wished. The present tendency to keep control of all patent-administering agencies tightly in the hands of the university is doubtlessly a trend in the right direction. The public consciousness of the university is greater than that of an independent research foundation, such as the Wisconsin Foundation. The university itself, an increasing number of patent administrators appear to believe, must assume direct responsibility for making and executing its patent policy. This statement does not imply, however, that a university may not use other institutions, such as Research Corporation, with objectives proved by long years of experience to be similar to its own, to perform the actual mechanics of patent management.

(To be continued.)

Rising Fertilizer Tonnage.—An estimate of 15 million tons of commercial fertilizers used in 1946 sets a new high record for the United States. A table prepared by A. L. Mehring of the Department of Agriculture shows that 1942 was the first year in which fertilizer tonnage was more than 10 million tons, and 1910 was the year when fertilizer passed the 5 million ton mark.

The rapid wartime increase in fertilizers was in contrast to the record in World War I. In 1914, the record shows a consumption of more than 7 million tons, which dropped to a little more than 5 million in 1916, and did not pass the 1914 figure again until 1920. In 1921 the figure dropped below 5 million. A year by year increase followed until 1930 with nearly 8½ million tons, followed by a "depression" slump to about 4½ million tons in 1932. Since then the trend has been sharply upward. Use of fertilizer more than doubled in the 10 years from 1936 to 1946, says Mehring.

R. H. O.

Sound Waves Inspect Metal. (*Compressed Air Magazine*, Vol. 52, No. 9.)—During the war, German technicians developed a supersonic method of locating bubbles in sheet metal too minute for detection by X-ray or magnetic processes. A report on the device prepared by Dr. G. J. Thiessen, a member of the National Research Council at Ottawa, Canada, states that the method tests sheet metal for imperfections by comparing the energy of supersonic vibrations before and after passage through the metal. Anything greater than the normal drop in transmission indicates the presence of a flaw. Experiments have shown that a bubble only one-hundred thousandth of a millimeter in diameter can cause a hundredfold drop in energy. Slag- or rust-filled bubbles do not cause a drop of like magnitude but, even so, are readily discovered.

The supersonic vibrations are generated by a 15-watt quartz-crystal sender that operates at 1,000 kilocycles and "wobbles" the frequency 100 cycles per second to prevent standing waves in the metal undergoing test as well as between the sender and receiver. The vibrations are transmitted by a metal block to a narrow chamber of running water which, in turn, passes them on by making contact with one edge of the test metal. A similar arrangement picks up the vibrations at the other edge and forwards them to the receiver. The sending and receiving units are attached to opposite ends of a long pair of tongs that straddle the sheet, move along its edges, and are kept in contact therewith by hand pressure or by springs in the tong handles.

Full details about this and other methods of inspecting metals are contained in a report, entitled "The Non-Destructive Testing of Materials and X-Ray Protection Methods," which can be obtained in photostat, microfilm ($1), or mimeographed form (25 cents) from the Office of Technical Services, Department of Commerce, Washington 25, D. C.

R. H. O.

CONSTRUCTION OF ALIGNMENT NOMOGRAM FROM EMPIRICAL DATA.

BY

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INTRODUCTION.

The equations that can be exactly represented by nomograms are only a few special cases of the vast number of functional relations which will never cease to appear in different forms while man's inquiry into the secrets of nature is going on. Modern scientific literature is full of such functional relations of which many have not been concisely and accurately represented by suitable equations and are left in the primitive form of tabulation of values. Whether in the form of an equation or in the form of a tabulation of values, they are functional relations for which nomographic representation is desirable. It is the purpose of this paper to describe a general method of constructing alignment nomograms from a tabulation of values. The resulting nomograms are necessarily approximations. So long as the approximations are close enough for practical purposes, it matters little whether they are theoretically correct or not. All graphical methods are based upon measurements and they can never be perfectly correct.

METHODS OF APPROACH.

Functional relation between three variables involved in a practical problem is generally continuous and single-valued. It may be represented by a family of curves showing the relation between any two variables while the third one takes different constant values, or by a surface in space.

When the family of curves can be transformed all at the same time into straight lines, the relation can be represented by a nomogram. Wertheimer [1] gave a graphical method of transformation for the possible case.

When the relation is considered as a continuous and smooth surface, a more general method known as surface fitting may be used to construct the nomogram with higher accuracy.

SURFACE FITTING.

In Fig. 1 is shown a surface representing a function of two variables

$$r = f(t, u)$$

[1] Albert Wertheimer, "The Graphical Transformation of Curves into Straight Lines and the Construction of Alignment Nomogram," JOURNAL OF FRANKLIN INSTITUTE, March, 1935.

or experimental data of a bi-variate relation according to the usual method of coordinate geometry. The three ordinate distances of any point on the surface represent the values of t, u and r that satisfy the condition of the relation.

When an equation can be represented by an alignment nomogram, its surface is said to be nomographic.

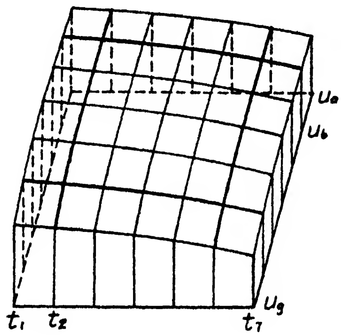


FIG. 1.

By surface-fitting is meant the process of substituting a surface of known character or more particularly a nomographic surface for a surface defined by a given equation or a given table of values. The object is to find a nomogram giving a surface which approaches everywhere or coincides in as many points as possible with the original surface. By a graphical method to be described, it is possible to construct a nomographic surface which possesses in common with the original surface four sections determined by two values of each of the variables t and u.

When two smooth surfaces possess four common sections such as at t_2, t_6, u_b and u_f shown in Fig. 1, it is very likely that they approach each other very closely at any other part within the region. As will be shown later, it is still possible to manipulate the nomographic surface so that an additional point common to the two surfaces may be created. The error of this method of approximation may be computed; and examples of many practical problems indicate that the errors are small.

METHOD OF CONSTRUCTION.

In actual construction the work may be started with a table giving the values of the function corresponding to given values of the variables as shown in Table I, where the symbol d_2 stands for the value of r at $u = d$ and $t = t_2$. Most of the values not required in the construction are not given. On a straight line r as shown in Fig. 2 a natural (or functional) scale is marked covering the values of r from the minimum to the maximum. Then choose any two suitable points to be used as

TABLE I

$u \backslash t$	t_1	t_2	t_3	t_4	t_5	t_6	t_7
u_a	a_1	a_2				a_6	
u_b	b_1	b_2	b_3	b_4	b_5	b_6	b_7
u_c		c_2				c_6	
u_d		d_2				d_6	
u_e		e_2				e_6	
u_f	f_1	f_2	f_3	f_4	f_5	f_6	f_7
u_g		g_2				g_6	g_7

the graduation points t_2 and t_6; locate the graduation points a_2 and a_6 on the r scale; and draw two lines from t_2 and t_6 and passing through a_2 and a_6 respectively. The meeting point of these two lines will be used as the graduation point u_a on the u scale. Similarly the values b_2 and b_6 are used to determine u_b and so forth until all the graduation points of the u scale are located and a smooth curve is drawn through them.

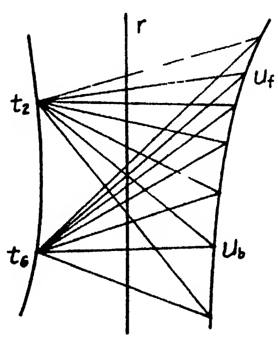


FIG. 2.

For the construction of the t scale, two suitable points such as u_b and u_f of the u scale may be used as the common points of projection. In this case, corresponding values of r in the rows u_b and u_f are used in locating the required points of the t scale.

Fig. 2, when used as an alignment nomogram, will give the correct values of the function at $t = t_2$, $t = t_6$, $u = u_b$ and $u = u_f$.

CONSTRUCTION OF SCALES BY RECTANGULAR COORDINATES.

It is still possible to plot the scales of this kind of nomogram by rectangular coordinates When the points t_2 and t_6 or u_b and u_f are too

near to the r scale, it becomes necessary to prolong two short lines to have an intersection at a great distance away. This means inaccurate location of the required points. In such case, the method of plotting with computed coordinates may be used. It is only the method of computation that needs some explanation.

The r scale may be assumed as the Y-axis and any line perpendicular to it may be taken as the X-axis as shown in Fig. 3. Then the X-ordi-

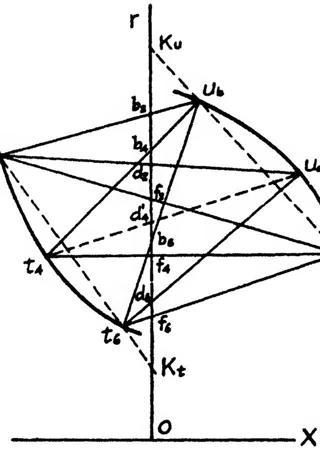


FIG. 3

nates of all the graduation points of the r scale are equal to zero; and their Y-ordinates depend upon the position of the origin. It is perfectly general to assume the origin at the point where the value of r is zero. Then the Y-ordinates represent the values of r according to the scale equation $Y = mr$ where m is the unit distance of the scale.

The coordinates of the initial points t_2 and t_6 may be measured from the figure. Nearest round values may also be assumed and used instead of the measured ones. Let the coordinates of t_2, t_6 and u_b be X_2, Y_2, X_6, Y_6, X_b and Y_b respectively. Since the Y-ordinates of the points of intersection of the t_2u_b and t_6u_b with the r scale are mb_2 and mb_6 respectively, their equations may be written as

$$\begin{vmatrix} X_2 & Y_2 & 1 \\ X_b & Y_b & 1 \\ 0 & mb_2 & 1 \end{vmatrix} = 0 \quad \text{and} \quad \begin{vmatrix} X_6 & Y_6 & 1 \\ X_b & Y_b & 1 \\ 0 & mb_6 & 1 \end{vmatrix} = 0$$

which give the values of X_b and Y_b as follows:

$$X_b = \frac{X_2X_6m(b_2 - b_6)}{X_2(Y_6 - mb_6) - X_6(Y_2 - mb_2)}, \tag{1}$$

$$Y_b = \frac{X_2mb_2(Y_6 - mb_6) - X_6mb_6(Y_2 - mb_2)}{X_2(Y_6 - mb_6) - X_6(Y_2 - mb_2)}. \tag{2}$$

These two equations may be used as the general formulae for computing the coordinates of the graduation points of the u scale. Similar equations may be derived for computing the coordinates of the graduation points of the t scale in terms of those of u_b and u_f and the values of r.

ERROR OF APPROXIMATION.

The continuous lines shown in Fig. 3 are those actually used in the construction of the scales of t and u. Their intersections with the r scale give the values of r in Table I. But the value d_4' determined by the dash line t_4u_d may not be the same as d_4, the true value of r at t_4 and u_d. The difference

$$D_4 = d_4 - d_4' \tag{3}$$

represents the error of approximation of the nomogram which may be found as follows:

The coordinates X_b and Y_b, given by (1) and (2), may be substituted into the equation

$$\begin{vmatrix} X_4 & Y_4 & 1 \\ X_b & Y_b & 1 \\ 0 & mb_4 & 1 \end{vmatrix} = 0$$

of the line t_4u_b. The resulting equation will be

$$X_4\{X_6(Y_2 - mb_2)(b_4 - b_6) - X_2(Y_6 - mb_6)(b_4 - b_2)\} \\ + Y_4X_2X_6(b_6 - b_2) - mX_2X_6b_4(b_6 - b_2) = 0.$$

Similarly, the lines t_4u_f and t_4u_d give

$$X_4\{X_6(Y_2 - mf_2)(f_4 - f_6) - X_2(Y_6 - mf_2)(f_4 - f_2)\} \\ + Y_4X_2X_6(f_6 - f_2) - mX_2X_6f_4(f_6 - f_2) = 0$$

and

$$X_4\{X_6(Y_2 - md_2)(d_4' - d_6) - X_2(Y_6 - md_2)(d_4' - d_2)\} \\ + Y_4X_2X_6(d_6 - d_2) - mX_2X_6d_4'(d_6 - d_2) = 0,$$

respectively. The substitution of X_4 and Y_4 found from any two of these three equations into the third one will give, on simplification, the following determinant:

$$\begin{vmatrix} \dfrac{b_6 - b_2}{b_4 - b_2} & b_4\dfrac{b_6 - b_2}{b_4 - b_2} & \dfrac{Y_2X_6 - Y_6X_2}{X_6 - X_2} - mb_6 \\ \dfrac{d_6 - d_2}{d_4' - d_2} & d_4'\dfrac{d_6 - d_2}{d_4' - d_2} & \dfrac{Y_2X_6 - Y_6X_2}{X_6 - X_2} - md_6 \\ \dfrac{f_6 - f_2}{f_4 - f_2} & f_4\dfrac{f_6 - f_2}{f_4 - f_2} & \dfrac{Y_2X_6 - Y_6X_2}{X_6 - X_2} - mf_6 \end{vmatrix} = 0.$$

Substituting

$$d_4' = d_4 - D_4$$

and

$$mK_t = \frac{Y_2X_6 - Y_6X_2}{X_6 - X_2} \tag{4}$$

into the above determinant and solving for D_4, we have

$$D_4 = \frac{\begin{vmatrix} b_6 - b_2 & b_4(b_6 - b_2) & (K_t - b_6)(b_4 - b_2) \\ d_6 - d_2 & d_4(d_6 - d_2) & (K_t - d_6)(d_4 - d_2) \\ f_6 - f_2 & f_4(f_6 - f_2) & (K_t - f_6)(f_4 - f_2) \end{vmatrix}}{\begin{vmatrix} b_6 - b_2 & b_4(b_6 - b_2) & (K_t - b_6)(b_4 - b_2) \\ 0 & (d_6 - d_2) & (K_t - d_6) \\ f_6 - f_2 & f_4(f_6 - f_2) & (K_t - f_6)(f_4 - f_2) \end{vmatrix}} \tag{5}$$

which gives the error of the nomogram for any combination of the values of t and u.

It can be shown that the value of K_t given by (4) is equal to the scale reading on the r scale at its point of intersection with the line t_2t_6. Since the points t_2 and t_6 on the t scale are arbitrarily chosen at the beginning of the construction, they may be made such that the corresponding value of K_t fulfils the condition that $D_4 = 0$. This gives

$$K_t = \frac{\begin{vmatrix} b_6 - b_2 & b_4(b_6 - b_2) & b_6(b_4 - b_2) \\ d_6 - d_2 & d_4(d_6 - d_2) & d_6(d_4 - d_2) \\ f_6 - f_2 & f_4(f_6 - f_2) & f_6(f_4 - f_2) \end{vmatrix}}{\begin{vmatrix} b_6 - b_2 & b_4(b_6 - b_2) & b_4 - b_2 \\ d_6 - d_2 & d_4(d_6 - d_2) & d_4 - d_2 \\ f_6 - f_2 & f_4(f_6 - f_2) & f_4 - f_2 \end{vmatrix}}. \tag{6}$$

Similarly the other value K_u at the intersection of the line u_bu_f with the r scale is given by

$$K_u = \frac{\begin{vmatrix} b_2 - f_2 & d_2(b_2 - f_2) & b_2(d_2 - f_2) \\ b_4 - f_4 & d_4(b_4 - f_4) & b_4(d_4 - f_4) \\ b_6 - f_6 & d_6(b_6 - f_6) & b_6(d_6 - f_6) \end{vmatrix}}{\begin{vmatrix} b_2 - f_2 & d_2(b_2 - f_2) & d_2 - f_2 \\ b_4 - f_4 & d_4(b_4 - f_4) & d_4 - f_4 \\ b_6 - f_6 & d_6(b_6 - f_6) & d_6 - f_6 \end{vmatrix}} \tag{7}$$

and other equations involving the same set of data in an arrangement with respect to the variable u may be obtained.

When the value of K_t used in the actual construction is based upon $D_4 = 0$, the resulting nomogram will give a surface that has an additional point in common with the original surface at $t = t_4$, $u = u_d$. Actual examples indicate that this extra point of contact greatly reduces the error of approximation in the surrounding area.

EQUATION FOR DIRECT CALCULATION.

The expression for the value of d_4' given by the nomogram may be obtained from the original determinant as follows:

$$d_4' = \frac{\begin{vmatrix} b_6 - b_2 & b_4(b_6 - b_2) & (K_t - b_6)(b_2 - b_4) \\ d_6 - d_2 & 0 & (K_t - d_6)(d_2) \\ f_6 - f_2 & f_4(f_6 - f_2) & (K_t - f_6)(f_2 - f_4) \end{vmatrix}}{\begin{vmatrix} b_6 - b_2 & b_4(b_6 - b_2) & (K_t - b_6)(b_4 - b_2) \\ 0 & (d_6 - d_2) & (K_t - d_6) \\ f_6 - f_2 & f_4(f_6 - f_2) & (K_t - f_6)(f_4 - f_2) \end{vmatrix}}, \tag{8}$$

which may be used to compute what are theoretically given by the nomogram. As in the construction of the nomogram, only the data on the four specific sections are used in the calculation.

TEST OF ACCURACY.

As it is desirable to know before the start of the actual construction how closely the given surface may be approximated by a nonogram, an equation for computing the error but without the presence of K_t may be developed under an assumption which is reasonable and also desirable. If it is assumed that the initial points t_2 and t_6 are equidistant from the r scale, we will have

$$X_6 - X_2 = 0,$$
$$K_t = \text{infinity},$$

and

$$\frac{1}{K_t} = 0.$$

This reduces (5) to

$$D_4 = \frac{\begin{vmatrix} b_6 - b_2 & b_4(b_6 - b_2) & b_4 - b_2 \\ d_6 - d_2 & d_4(d_6 - d_2) & d_4 - d_2 \\ f_6 - f_2 & f_4(f_6 - f_2) & f_4 - f_2 \end{vmatrix}}{\begin{vmatrix} b_6 - b_2 & b_4(b_6 - b_2) & b_4 - b_2 \\ 0 & (d_6 - d_2) & 1 \\ f_6 - f_2 & f_4(f_6 - f_2) & f_4 - f_2 \end{vmatrix}}, \tag{9}$$

which may be used to compute directly from the data the error corresponding to d_4. It is expected that the central point, the four corner points and the four middle points of the four edges should have large errors, if not the largest, for they are located far away from the lines of coincidence. The errors at these points may be computed with (9) and used as guidance in locating the position of the point of contact in order to determine the value of K_t.

INCREASE OF ACCURACY BY SUB-DIVISION OF THE REGION.

Obviously the efficiency of the four lines of coincidence in holding together the original surface and that of the nomogram will be decreased when

(a) the original surface is undulating or irregular, and
(b) the region to be covered is too large.

In either case, when the calculated error is larger than allowable, we may divide the given region into several smaller ones as shown in Fig. 4, and construct separate nomograms for the individual sub-regions. Even in that case it is sometimes still possible to arrange the diagrams compactly in one plate as shown in Fig. 5.

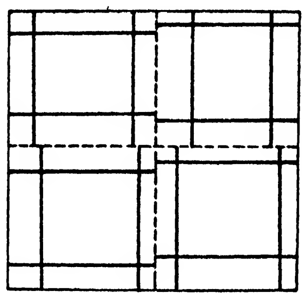


FIG. 4.

PRACTICAL CONSTRUCTION.

The final object in this method of construction is also a nomogram suitable for practical use; and the question of shape and dimension is still the main point of consideration in the actual process of construction.

As the error of approximation depends on the spacing and the location of the four lines of coincidence, and such lines must be assumed before the error can be computed, the designer has to make trials at the beginning. Examples indicate that the first trial calculation may be made with these lines somewhere at the outer eighth points as shown in Fig. 6 and the point of contact at center will give the value of K_t. Then the errors at other points may be calculated, and the accuracy of approximation of the given function by the proposed nomogram may be determined.

When the required accuracy cannot be reached by a single nomogram, further trials may be made with

(a) a functional transformation of the data as illustrated in Example 6 (see below), or
(b) a sub-division of the region as shown in Fig. 4 and illustrated in Fig. 5

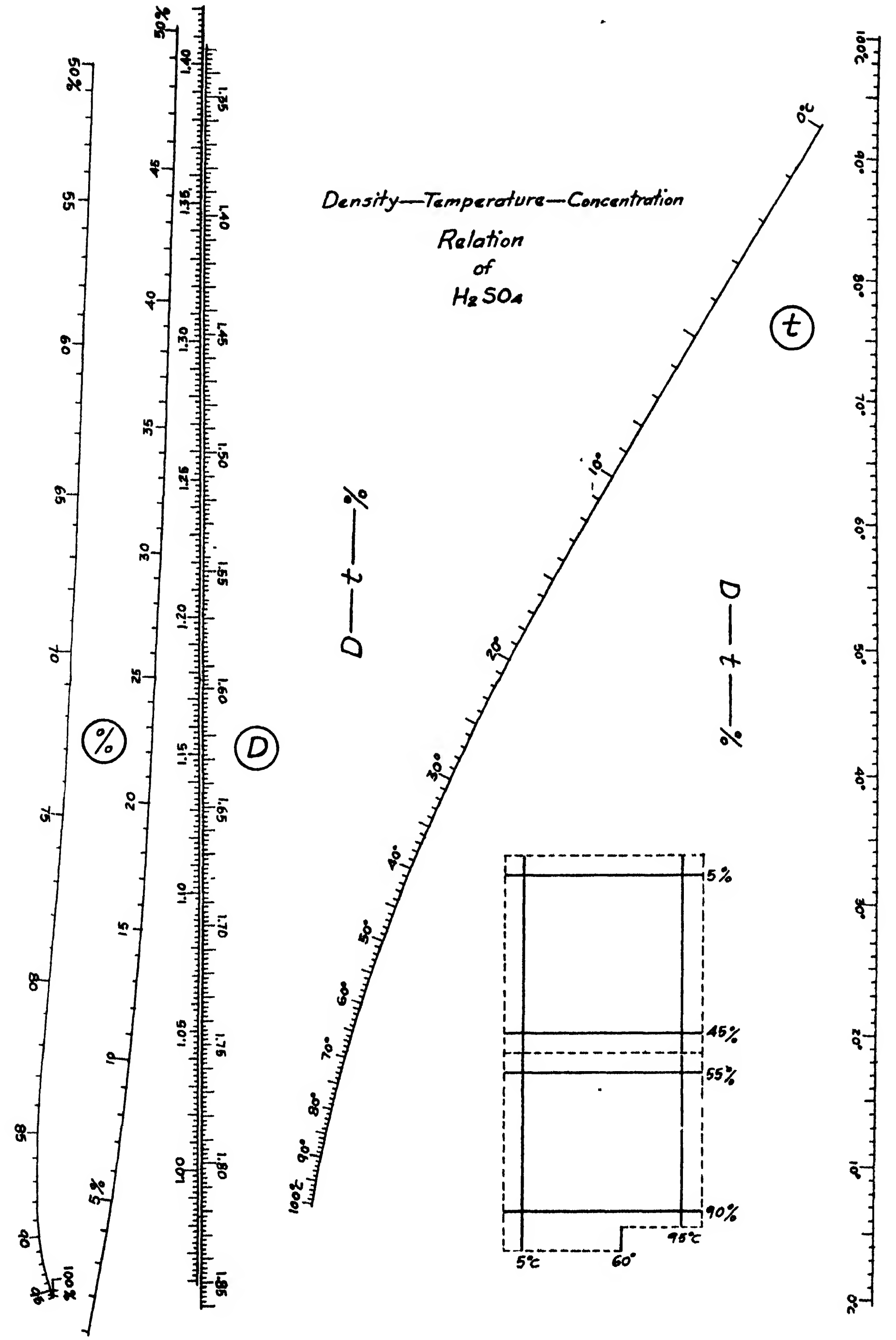

Density—Temperature—Concentration
Relation
of
H_2SO_4
D—t—%
%—t—D
%
D
t
5%
45%
55%
90%
5°C
60°
95°C

FIG. 5.

Let a_1 and g_7 be the minimum and maximum values of the function in Table I. The unit distance m for the r scale should be such that

$$m(g_7 - a_1) = L,$$

where L is the desired total length of the r scale. Of course, there is no need of great exactness and a nearest convenient length may be used for m.

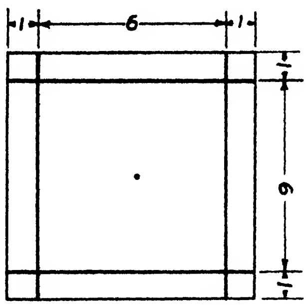


FIG. 6.

As the shape of the nomogram will be completely determined by the r scale and the location of the initial points t_2 and t_6 with respect to it, the positions of these two initial points should be subject to the following conditions:

(a) they give the calculated value of K_t, and
(b) they give a satisfactory figure of the diagram.

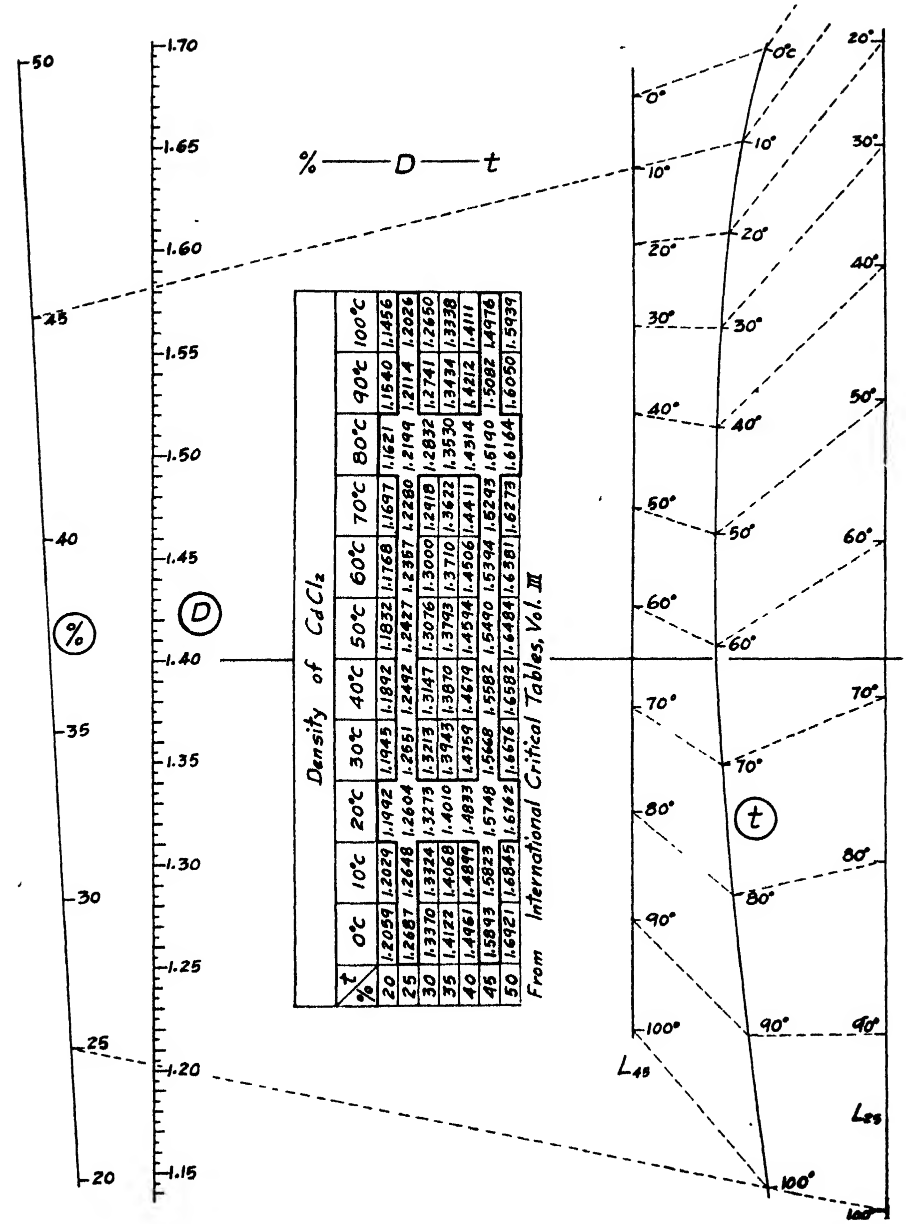
Usually the shape of the diagram is determined by the four graduation points t_1, t_7, u_a and u_g. Draw several lines all concurrent with the r scale at the point of reading K_t; and make trial sketch for the location of the points t_1, t_7, u_a and u_g with t_2 and t_6 on any one of these lines. This trial work will soon lead to a proper location of the points t_2 and t_6.

When the accuracy of the nomogram is higher than actually required, there will be no need to make the line t_2t_6 exactly pass the point of reading K_t on the r scale. In that case the designer has greater freedom in choosing the positions of the points t_2 and t_6.

When the initial points t_2 and t_6 are very near to the r scale, the method of plotting by rectangular coordinates or the method illustrated in Fig. 7 may be used.

TRANSFORMATION OF THE NOMOGRAM.

When it is necessary to use the calculated value of K_t in the location of the initial points t_2 and t_6, our freedom in choosing the coordinates will be limited; and when the shape of the nomogram constructed under this restriction is undesirable, the usual method of transformation based



Density of $CdCl_2$

t / %	0°C	10°C	20°C	30°C	40°C	50°C	60°C	70°C	80°C	90°C	100°C
20	1.2059	1.2029	1.1992	1.1945	1.1892	1.1832	1.1768	1.1697	1.1621	1.1540	1.1456
25	1.2687	1.2648	1.2604	1.2651	1.2492	1.2427	1.2357	1.2280	1.2199	1.2114	1.2026
30	1.3370	1.3324	1.3273	1.3213	1.3147	1.3076	1.3000	1.2918	1.2832	1.2741	1.2650
35	1.4122	1.4068	1.4010	1.3943	1.3870	1.3793	1.3710	1.3622	1.3530	1.3434	1.3338
40	1.4961	1.4899	1.4833	1.4759	1.4679	1.4594	1.4506	1.4411	1.4314	1.4212	1.4111
45	1.5893	1.5823	1.5748	1.5668	1.5582	1.5490	1.5394	1.5293	1.5190	1.5082	1.4976
50	1.6921	1.6845	1.6762	1.6676	1.6582	1.6484	1.6381	1.6273	1.6164	1.6050	1.5939

From International Critical Tables, Vol. III

FIG. 7.

on mathematical computation or graphical projection can still be applied in this case.

It can be shown that the error of the nomogram remains unchanged during the process of transformation.

METHOD OF PROCEDURE WITH EXPERIMENTAL DATA.

Data obtained from observations are contaminated with errors of measurement which can never be removed entirely. So the observed values of a bi-variate relation cannot be represented by a single surface. If the equation of the relation under observation is not known, as is usually the case, adjustment by the method of least squares is also impossible. In such case, when a nomogram is to be constructed, the data should be adjusted by the following tentative method.

Let the observed values of r corresponding to different values of t and u be denoted according to the scheme shown in Table I. Plot the values of r in the first row against the values of t as shown in Fig. 8 with a vertical scale as large as practicable; draw a smooth curve passing through most of the points and leaving only a few of them on either side; and take the values of r from the curve. These values will be de-

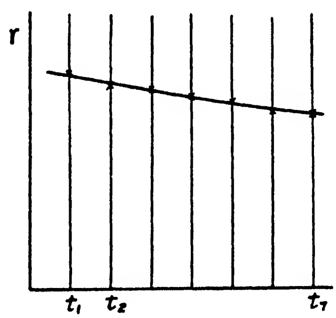


FIG. 8.

noted as a_1', a_2', . . . a_7'. The results of the method applied to the values in the other rows will be denoted in a similar manner.

The second course of treatment consists of plots with the adjusted values of r against the values of u; and from the curve determined by the values of a_2', b_2', c_2' . . ., g_2', readjusted values a_2'', b_2'', . . ., g_2'' may be obtained. Similarly determine the values a_6'', b_6'', . . ., g_6''.

Finally plot b_1, b_2'', b_3, b_4, b_5, b_6'' and b_7 against t again; draw smooth curve through the points of b_2'' and b_6'' and as many other points as possible; and read from the curve the corresponding values of r. These values will be denoted as b_1'', b_2'', . . ., b_7''. Similarly, determine the values of f_1'', f_2'', . . ., f_7''. The adjusted values marked with double apostrophes are to be used in the construction of the nomogram.

EXAMPLES.

The method of approximation proposed in this paper is unexpectedly simple. If its accuracy is sufficient for practical problems, it will greatly extend the scope of nomography. As it is impossible to formulate a general rule about the magnitude of the error of approximation,

examples will be worked out to indicate what it is capable of achieving in various actual cases.

Example 1. The construction of the nomogram for the relation between the density D of $CdCl_2$, the temperature t and its percentage of concentration per cent. as defined by the observed data given in Fig. 7.

The natural scale of D on which the values of the density are plotted with unit distance of 50 cm. is drawn first. Then the graduation points for $t = 0°$ C. and $t = 100°$ C. are assumed at 15 cm. from the D scale as shown, and are used to determine the graduation points of 25 and 45 per cent. by method of intersection. The coordinates of these two points with respect to the D scale as the Y-axis and the X-axis passing through the graduation point 1.400 on the D scale are found by measurements to be $X = -1.99$, $Y = -9.45$, $X = -2.93$ and $Y = 8.38$.

To avoid the enlargement of the error of plotting by the prolongation of short lines, two lines L_{25} and L_{45} are drawn parallel to the D scale and at distances 19.9 cm. and 14.65 cm. from the points of 25 and 45 per cent. respectively. The position of the point of 0° C. on L_{45} by projection from point 45 per cent. through the point at $D = 1.5893$ may be calculated as follows:

$$1.5893 - 1.4000 = 0.1893, \qquad 50 \times 0.1893 = 9.465,$$
$$9.465 - 8.38 = 1.085, \qquad 5 \times 1.085 = 5.425,$$

and

$$5.425 + 8.38 = 13.805.$$

The distance of the point 0° C. on L_{45} is therefore 13.805 cms. from the X-axis; and its position is accurately defined by this distance. Similarly, other points may be located on L_{45}; and the same set of points on L_{25} may be located in the same manner. The intersections of the corresponding lines give the graduation points of the t scale as shown.

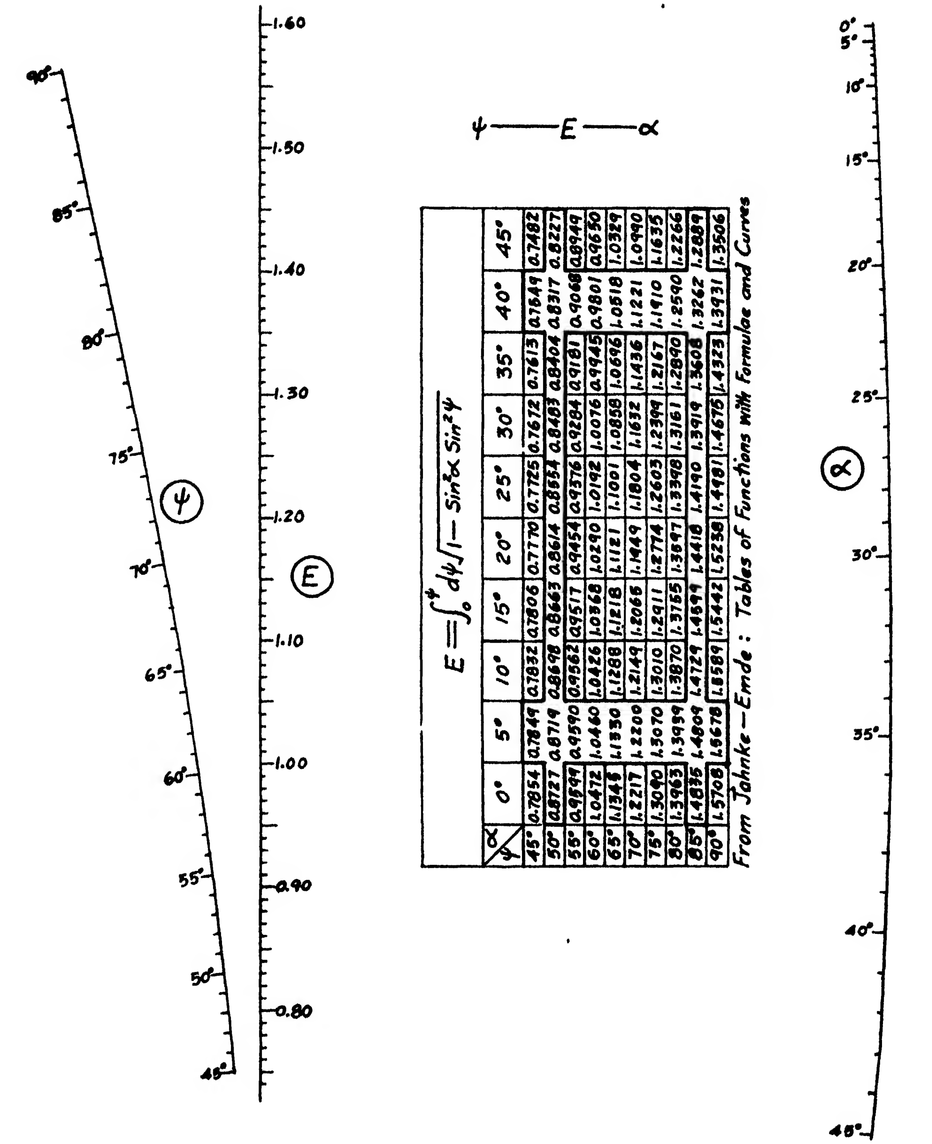
The points 20° C. and 80° C. of the t scale are then used as the centers of projection in constructing the per cent. scale. The data actually used in the construction are those enclosed in the heavy lines of the table. The accuracy of the diagram may be checked by the extra data provided.

Example 2. The construction of the alignment nomogram for the elliptic integral

$$E = \int_0^{\psi} \sqrt{1 - \sin^2 \alpha \sin^2 \psi}\, d\psi$$

from the data given in the table of Fig. 9.

The same method of construction has been used in this case; and the accuracy of the result has reached the level where the theoretical error is in the neighborhood of the error of construction.



$$E = \int_0^{\psi} d\psi \sqrt{1 - \sin^2\alpha \sin^2\psi}$$

ψ \ α	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
45°	0.7854	0.7849	0.7832	0.7806	0.7770	0.7725	0.7672	0.7613	0.7549	0.7482
50°	0.8727	0.8719	0.8698	0.8663	0.8614	0.8551	0.8483	0.8404	0.8317	0.8227
55°	0.9599	0.9590	0.9562	0.9517	0.9454	0.9376	0.9284	0.9181	0.9068	0.8949
60°	1.0472	1.0460	1.0426	1.0368	1.0290	1.0192	1.0076	0.9945	0.9801	0.9650
65°	1.1345	1.1330	1.1288	1.1218	1.1121	1.1001	1.0858	1.0696	1.0518	1.0329
70°	1.2217	1.2200	1.2149	1.2066	1.1949	1.1804	1.1632	1.1436	1.1221	1.0990
75°	1.3090	1.3070	1.3010	1.2911	1.2774	1.2603	1.2399	1.2167	1.1910	1.1635
80°	1.3963	1.3939	1.3870	1.3755	1.3597	1.3398	1.3161	1.2890	1.2590	1.2266
85°	1.4835	1.4809	1.4729	1.4594	1.4418	1.4190	1.3919	1.3608	1.3262	1.2889
90°	1.5708	1.5678	1.5589	1.5442	1.5238	1.4981	1.4675	1.4323	1.3931	1.3506

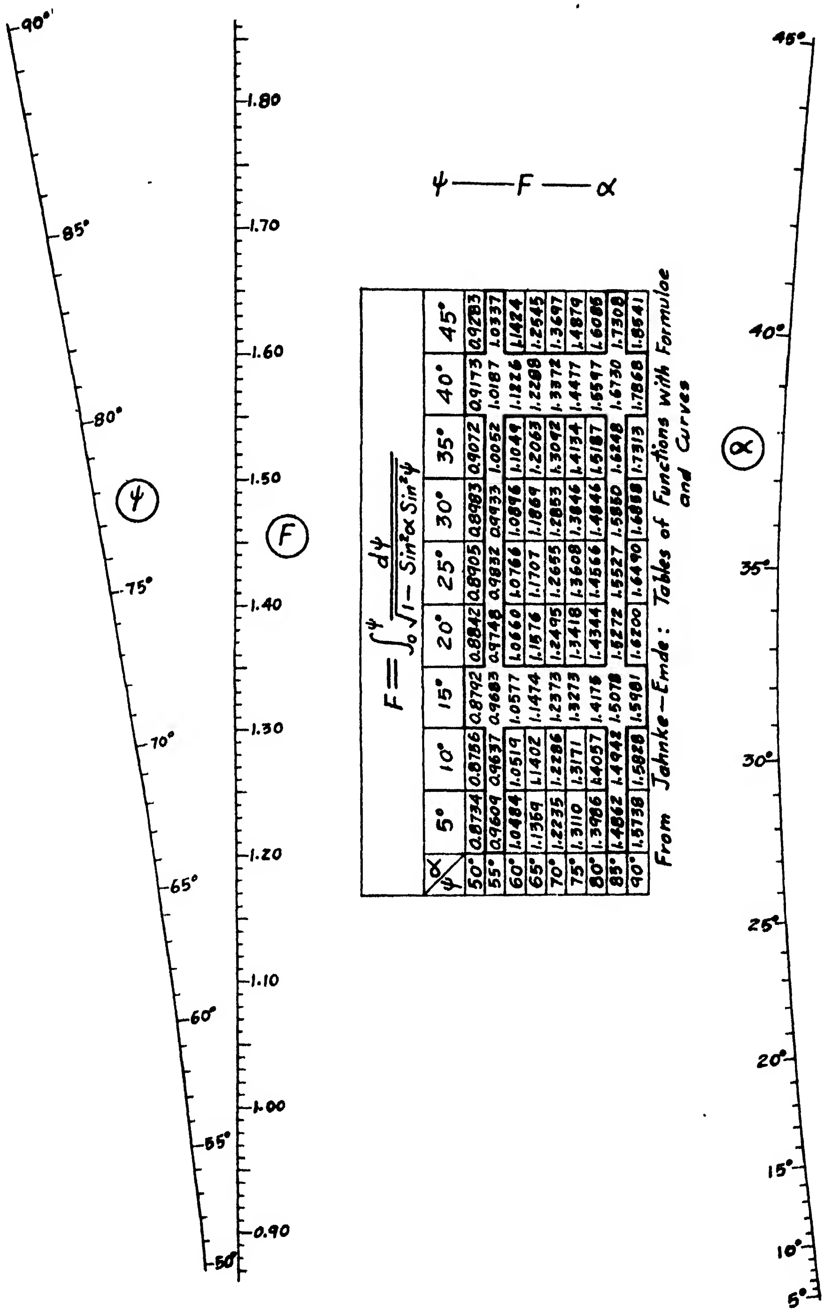
From Jahnke–Emde: Tables of Functions with Formulae and Curves

Fig. 9.

Example 3.—The construction of the alignment nomogram for the elliptic integral

$$F = \int_0^{\psi} \frac{d\psi}{\sqrt{1 - \sin^2\alpha \sin^2\psi}}$$

from the data given in the table of Fig. 10, with an investigation of its error.

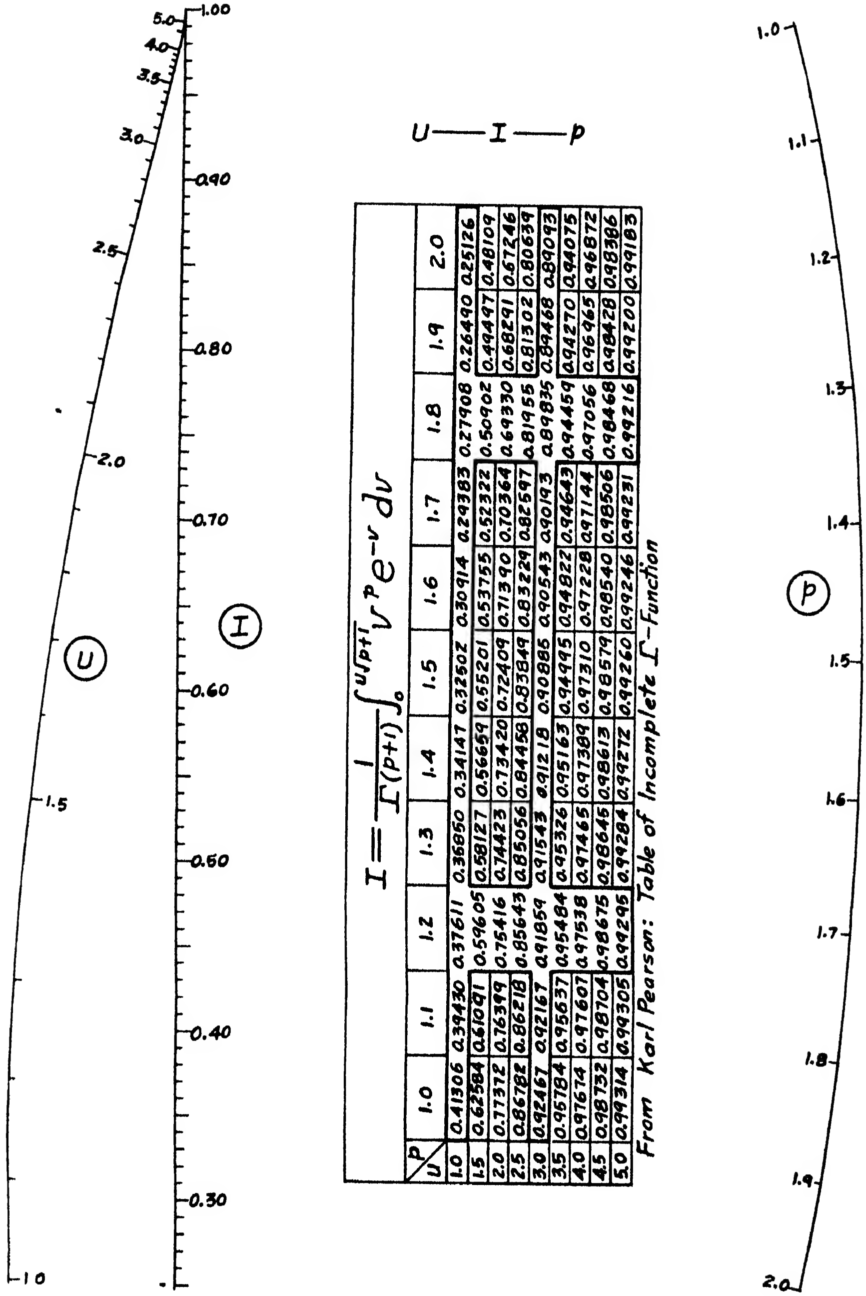


$$F = \int_0^{\psi} \frac{d\psi}{\sqrt{1 - \sin^2\alpha \sin^2\psi}}$$

ψ \ α	5°	10°	15°	20°	25°	30°	35°	40°	45°
50°	0.8734	0.8756	0.8792	0.8842	0.8905	0.8983	0.9072	0.9173	0.9283
55°	0.9609	0.9637	0.9683	0.9748	0.9832	0.9933	1.0052	1.0187	1.0337
60°	1.0484	1.0519	1.0577	1.0660	1.0766	1.0896	1.1049	1.1226	1.1424
65°	1.1369	1.1402	1.1474	1.1576	1.1707	1.1869	1.2063	1.2288	1.2545
70°	1.2235	1.2286	1.2373	1.2495	1.2655	1.2853	1.3092	1.3372	1.3697
75°	1.3110	1.3171	1.3273	1.3418	1.3608	1.3846	1.4134	1.4477	1.4879
80°	1.3986	1.4057	1.4176	1.4344	1.4566	1.4846	1.5187	1.5597	1.6085
85°	1.4862	1.4942	1.5078	1.5272	1.5527	1.5850	1.6248	1.6730	1.7308
90°	1.5738	1.5828	1.5981	1.6200	1.6490	1.6888	1.7313	1.7868	1.8541

From Jahnke—Emde: Tables of Functions with Formulae and Curves

FIG. 10.



$$I = \frac{1}{\Gamma(p+1)} \int_0^{u\sqrt{p+1}} v^p e^{-v}\, dv$$

U \ p	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
1.0	0.41306	0.39430	0.37611	0.35850	0.34147	0.32502	0.30914	0.29383	0.27908	0.26490	0.25126
1.5	0.62384	0.61091	0.59605	0.58127	0.56659	0.55201	0.53755	0.52322	0.50902	0.49497	0.48109
2.0	0.77372	0.76399	0.75416	0.74423	0.73420	0.72409	0.71390	0.70364	0.69330	0.68291	0.67246
2.5	0.86782	0.86218	0.85643	0.85056	0.84458	0.83849	0.83229	0.82597	0.81955	0.81302	0.80639
3.0	0.92467	0.92167	0.91859	0.91543	0.91218	0.90885	0.90543	0.90193	0.89835	0.89468	0.89093
3.5	0.95784	0.95637	0.95484	0.95326	0.95163	0.94995	0.94822	0.94643	0.94459	0.94270	0.94075
4.0	0.97674	0.97607	0.97538	0.97465	0.97389	0.97310	0.97228	0.97144	0.97056	0.96965	0.96872
4.5	0.98732	0.98704	0.98675	0.98645	0.98613	0.98579	0.98540	0.98506	0.98468	0.98428	0.98386
5.0	0.99314	0.99305	0.99295	0.99284	0.99272	0.99260	0.99246	0.99231	0.99216	0.99200	0.99183

From Karl Pearson: Table of Incomplete Γ-Function

FIG. 11.

The construction of the diagram is clearly indicated by the data used. Equation (9) based on $K\alpha$ = infinity gives the following values of errors:

ψ \ α	5°	30°	45°
50°	0.00070	−0.00033	0.00076
70°	−0.00016	0.00069	−0.00062
90°	0.00013	−0.00041	0.00046

Example 4. The construction of the alignment nomogram for the gamma function

$$I = \frac{1}{\Gamma(p+1)} \int_0^{u\sqrt{p+1}} v^p e^{-v} dv$$

from the data given in the table of Fig. 11, with an investigation of its error.

The construction of the diagram needs no explanation. The errors calculated with an equation based upon K_u = infinity and lines of coincidence at u = 1.0, u = 3.0, p = 1.2 and p = 2.0 are given in the following table:

u \ p	1.0	1.6	1.7	1.8
2.0	−0.00090	0.00071		0.00040
4.0		−0.00011		
5.0			−0.00001	

The same set of data with the same lines of coincidence but with K_u = 1.00797 give the errors as follows:

	1.0	1.6	1.7	1.8
2.0	0.00012	0.00010		0.00008
4.0		0.00004		
5.0			0.00003	

Example 5. The construction of the alignment nomogram for the relation between the density D of H_2SO_4, the temperature t and its percentage of concentration per cent. as defined by the data given in the International Critical Tables. Vol. III, pages 56–57.

Fig. 5 illustrates an arrangement whereby important data may be shown with greater accuracy. The diagram covering the relation from 1 to 50 per cent. is based on the lines of coincidence at 5 per cent., 45 per cent., 5° C. and 95° C. with K_t = 0.9000; and the other one covering the relation from 50 to 100 per cent. is based on the lines at 55 per cent., 90 per cent., 5° C. and 95° C. with K_t = infinity.

Example 6. Change of error of approximation due to change of scale.

The error of surface fitting given by the following data of the density of H_2SO_4

t \ %	5%	45%	70%	90%
5° C.	1.03545	1.35912	1.62453	1.83064
50° C.	1.01920	1.32506	1.58380	1.78290
95° C.	0.96789	1.29222	1.54582	1.73787

at (70 per cent., 50° C.) with lines of coincidence at 5 per cent., 90 per cent., 5° C. and 95° C. and a point of contact at (45 per cent., 50° C.) may be found to be 0.00094 which is equal to 1/918 of the total variation of the density (1.83064 − 0.96789).

The data may be transformed by any functional relation, and $\log_{10}(D + 5)$ gives

t \ %	5%	45%	70%	90%
5° C.	0.7807097	0.8033971	0.8211551	0.8344615
50° C.	0.7795388	0.8010646	0.8184766	0.8314154
95° C.	0.7758201	0.7988039	0.8159640	0.8285227

The error of the logarithm at (70 per cent, 50° C.) calculated under the same condition as before is 0.0000270 for which the corresponding error in D is 0.00041 which is equal to 1/2100 of the total variation of the density. In this case the accuracy is much improved by using the functional scale $\log_{10}(D + 5)$. Logarithmic transformation of the data sometimes reduces the error of fitting.

SIGNIFICANCE OF THE EXAMPLES.

The magnitude of the error of surface fitting cannot be predicted without going into actual calculations. The above examples serve the purpose of giving a general indication of the degree of accuracy and point to the following probabilities:

(a) A single alignment nomogram covering the ordinary ranges of variation of the variables of practical problems may be expected to be accurate to about 99.5 per cent.

(b) A suitable point of contact will generally reduce the error of approximation.

(c) A higher degree of accuracy may be obtained by dividing the region into several smaller areas to be represented by separate diagrams.

(d) Equation (8) may be used to compute many of the required values which could otherwise be obtained only by direct observation.

SYMMETRICAL BI-MODAL FREQUENCY CURVES.

BY

V. H. GOTTSCHALK, Ph.D.

Fifty years ago Karl Pearson, after futile attempts to split frequency curves into two component normal (Gauss) curves, declared the problem unsolvable and proceeded to develop the famous system of formulas now known as Pearson's Curves.

The splitting problem seems not to have been taken up since and may indeed be impossible, as experience with other theoretical solutions indicates, but it is surprising to find that some symmetrical distributions may be represented as the sum or difference of two normal components,

$$y = \frac{N_1}{\sqrt{2\pi\nu_{21}}} e^{-\frac{(x-s)^2}{2\nu_{21}}} + \frac{N_2}{\sqrt{2\pi\nu_{22}}} e^{-\frac{(x-t)^2}{2\nu_{22}}}$$

to give a fair approximation to the original data.

The above equation contains the six unknowns N_1, N_2, ν_{21}, ν_{22}, s and t, which Pearson determined by setting up equations obtained on integrating $\int_{-\infty}^{\infty} ydx$, $\int_{-\infty}^{\infty} yxdx$, up to $\int_{-\infty}^{\infty} yx^5dx$. This gives the six equations:

$$N_1 + N_2 = N, \tag{1}$$

$$N_1 s + N_2 t = N\nu_1, \tag{2}$$

$$N_1(\nu_{21} + s^2) + N_2(\nu_{22} + t^2) = N\nu_2, \tag{3}$$

$$N_1(3s\nu_{21} + s^3) + N_2(3t\nu_{22} + t^3) = N\nu_3, \tag{4}$$

$$N_1(3\nu_{21}^2 + 6s^2\nu_{21} + s^4) + N_2(3\nu_{22}^2 + 6t^2\nu_{22} + t^4) = N\nu_4, \tag{5}$$

$$N_1(15s\nu_{21}^2 + 10s^3\nu_{21} + s^5) + N_2(15t\nu_{22}^2 + 10t^3\nu_{22} + t^5) = N\nu_5. \tag{6}$$

From (1) and (2) we get:

$$N_1 = N\left(\frac{\nu_1 - t}{s - t}\right), \qquad \text{and} \qquad N_2 = N\left(\frac{s - \nu_1}{s - t}\right)$$

and these substituted in (3) and (4) give:

$$\nu_1(\nu_{21} - \nu_{22} + s^2 - t^2) - (t\nu_{21} - s\nu_{22} + s^2t - st^2) = \nu_2(s - t)$$

and

$$\nu_1(3s\nu_1 - 3t\nu_{22} + s^3 - t^3) - st(3\nu_{21} - 3\nu_{22} + s^2 - t^2) = \nu_3(s - t)$$

which may be simplified by choosing the origin at the mean ordinate,

thus making $\nu_1 = 0$, so that

$$t\nu_{21} - s\nu_{22} = -(\nu_2 + st)(s - t)$$

and

$$\nu_{21} - \nu_{22} = -\left\{\frac{\nu_3 + st(s + t)}{3st}\right\}(s - t)$$

from which a second pair of unknowns may be determined:

$$\nu_{21} = \nu_2 + st - \left\{\frac{\nu_3 + st(s + t)}{3t}\right\},$$

$$\nu_{22} = \nu_2 + st - \left\{\frac{\nu_3 + st(s + t)}{3s}\right\}.$$

When the above values of N_1, N_2, ν_{21}, and ν_{22}, together with $\nu_1 = 0$, are substituted in (5) and (6), the equations for the last pair of unknowns are found to be:

$$2s^2t^2(s + t)^2 + 4\nu_3 st(s + t) - \nu_3^2 - 3(\nu_4 - 3\nu_2^2)st - 6s^3t^3 = 0$$

and

$$2s^2t^2(s + t)^3 - (s + t)(5\nu_3^2 + 4s^3t^3) - 3(\nu_5 - 10\nu_2\nu_3)st + 20\nu_3 s^2t^2 = 0.$$

Eliminating $(s + t)$ between these two equations and using M and P to represent $(\nu_4 - 3\nu_2^2)$ and $(\nu_5 - 10\nu_2\nu_3)$, respectively, gives Pearson's nonic:

$$\begin{aligned} s^9t^9 &+ 3.5Ms^7t^7 + 1.5\nu_3^2s^6t^6 + s^5t^5(3.75M^2 + 3P\nu_3) \\ &+ s^4t^4(18.5M\nu_3^2 - 0.75P^2) + s^3t^3(12\nu_3^4 + 1.125M^3 - 4.5MP\nu_3) \\ &+ s^2t^2(-2.625M^2\nu_3^2 - 3P\nu_3^3) - 4M\nu_3^4 st - \nu_3^6 = 0. \end{aligned}$$

In reconsidering the problem, it seemed that a lower degree final equation might be obtained by finding a suitable substitute for the fifth moment, and a likely substitute appeared to be the first "half moment," which determines the position of the mean ordinate

$$\begin{aligned} \sum_{-\infty}^{0} yx = Ln_{1L} &= \frac{N_1}{\sqrt{2\pi\nu_{21}}}\int_{-\infty}^{0} e^{\frac{-(x-s)^2}{2\nu_{21}}} xdx + \frac{N_2}{\sqrt{2\pi\nu_{22}}}\int_{-\infty}^{0} e^{\frac{-(x-t)^2}{2\nu_{22}}} xdx \\ &= -\frac{N_1}{\sqrt{2\pi\nu_{21}}}\int_{0}^{\infty} e^{\frac{-(x-s)^2}{2\nu_{21}}} xdx - \frac{N_2}{\sqrt{2\pi\nu_{22}}}\int_{0}^{\infty} e^{\frac{-(x-t)^2}{2\nu_{22}}} xdx \\ &= -\sum_{0}^{\infty} yx = -Rn_{1R} \end{aligned}$$

but which does not appear in the above equations.

Carrying through this idea confirms Pearson's dictum that the problem is unsolvable, but it is precisely this integration over half the range that first showed the possibility of separating the data for a symmetrical distribution into two groups, each represented by a normal curve.

If we write:

$$L_S = \frac{N_1}{\sqrt{2\pi\nu_{21}}} \int_{-\infty}^{0} e^{\frac{-(x-s)^2}{2\nu_{21}}} dx, \qquad L_T = \frac{N_2}{\sqrt{2\pi\nu_{22}}} \int_{-\infty}^{0} e^{\frac{-(x-t)^2}{2\nu_{22}}} dx,$$

$$L_S l_S = \frac{N_1}{\sqrt{2\pi\nu_{21}}} \int_{-\infty}^{0} e^{\frac{-(x-s)^2}{2\nu_{21}}} x dx, \qquad L_T l_T = \frac{N_2}{\sqrt{2\pi\nu_{22}}} \int_{-\infty}^{0} e^{\frac{-(x-t)^2}{2\nu_{22}}} x dx$$

and $L_S l_{2S}$, $L_T l_{3T}$, etc., for higher moments, the following relations are obtained:

$$sL_S - L_S l_S = \nu_{21} \left\{ \frac{N_1}{\sqrt{2\pi\nu_{21}}} e^{-\frac{s^2}{2\nu_{21}}} \right\}, \tag{7}$$

$$tL_T - L_T l_T = \nu_{22} \left\{ \frac{N_2}{\sqrt{2\pi\nu_{22}}} e^{-\frac{t^2}{2\nu_{22}}} \right\}, \tag{8}$$

$$\begin{aligned} \nu_{21}L_S + sL_S l_S &= L_S l_{2S}, & \nu_{22}L_T + tL_T l_T &= L_T l_{2T},\\ 2\nu_{21}L_S l_S + sL_S l_{2S} &= L_S l_{3S}, & 2\nu_{22}L_T l_T + tL_T l_{2T} &= L_T l_{3T},\\ 3\nu_{21}L_S l_{2S} + sL_S l_{3S} &= L_S l_{4S}, & 3\nu_{22}L_T l_{2T} + tL_T l_{3T} &= L_T l_{4T}, \end{aligned}$$

and so on. The expressions in parentheses are the intercepts on the ordinate at the origin, for which the symbols y_{01} and y_{02} will be used.

L_S, L_T and $L_S l_S$, $L_T l_T$ are not known, but their sums are obtained in the regular process of computing the full moments if $\sum y$, $\sum yx$, $\sum yx^2$, etc., are summed separately to the left of the origin. That is, we have:

$$L_S + L_T = \sum_{-\infty}^{0} y = L, \tag{9}$$

$$L_S l_S + L_T l_T = \sum_{-\infty}^{0} yx = Ln_{1L}, \tag{10}$$

which with (7) and (8) give expressions for L_S, L_T, $L_S l_S$, and $L_T l_T$ which when substituted in

$$L_S l_{2S} + L_T l_{2T} = \sum_{-\infty}^{0} yx^2 = Ln_{2L}$$

and

$$L_S l_{3S} + L_T l_{3T} = \sum_{-\infty}^{0} yx^3 = Ln_{3L}$$

give the two generally-valid equations:

$$\begin{aligned} &\nu_{21}y_{01}(\nu_{21} - \nu_{22} + st - t^2) + \nu_{22}y_{02}(\nu_{21} - \nu_{22} + s^2 - st)\\ &\qquad = L(n_{2L} - \nu_2)(s - t) - L(n_{1L} - \nu_1)(\nu_{21} - \nu_{22} + s^2 - t^2), \end{aligned}$$

and

$$\nu_{21}y_{01}(s\nu_{21} + 2t\nu_{21} - 3t\nu_{22} + s^2t - t^3) \\ + \nu_{22}y_{02}(3s\nu_{21} - t\nu_{22} - 2s\nu_{22} + s^3 - st^2) \\ = L(n_{3L} - \nu_3)(s - t) - L(n_{1L} - \nu_1)(3s\nu_{21} - 3t\nu_{22} + s^3 - t^3),$$

from which we get $y_{01} + y_{02} = Y_0$, yielding the following equation for the ordinate at the origin:

$$\begin{aligned} Y_0\nu_{21}\nu_{22}\{2(\nu_{21} - \nu_{22})^2 + (t\nu_{21} - s\nu_{22})(s - t)\} \\ = L(n_{3L} - \nu_3)\{(\nu_{21} - \nu_{22})^2 + (t\nu_{21} - s\nu_{22})(s - t)\} \\ + L(n_{2L} - \nu_2)\{-(\nu_{21} - \nu_{22})(s\nu_{21} - t\nu_{22}) \\ - 2(\nu_{21} - \nu_{22})(t\nu_{21} - s\nu_{22}) - (t\nu_{21} - s\nu_{22})(s^2 - t^2)\} \\ + L(n_{1L} - \nu_1)\{-2(\nu_{21}^2 - \nu_{22}^2)(\nu_{21} - \nu_{22}) + (s - t)^2\nu_{21}\nu_{22} \\ + 2(t\nu_{21} - s\nu_{22})^2 + st(t\nu_{21} - s\nu_{22})(s - t)\}. \end{aligned} \tag{11}$$

Although equation (11) and other "partial moment" equations do not graduate skew curves satisfactorily, the following considerations and computations show that acceptable results are obtained, simply and directly, when applied to symmetrical distributions.

A symmetrical bi-modal curve can arise in only two ways: (1) as the sum of two identical Gauss curves at equal distances on opposite sides of the mean ordinate, and (2) as two different Gauss curves added or subtracted with their maximum ordinates coinciding.

An astonishingly simple solution for the first case is obtained by putting $N_1 = N_2$, $\nu_{21} = \nu_{22}$, $s = -t$, in equations (1) to (4); since $\nu_1 = \nu_3 = 0$ for symmetrical curves, we get at once:

$$N_1 = \frac{N}{2}, \qquad \nu_{21} = \nu_2 - s^2, \qquad s^4 = \frac{3\nu_2^2 - \nu_4}{2}.$$

Another solution for s^2 may be obtained by substituting $\nu_{21} = \nu_{22}$, $s = -t$ in equation (11) and then eliminating ν_{21} by use of its equivalent, $\nu_2 - s^2$, as found above. The resulting expression is a quadratic in s^2, whose solution for the purposes of computation may be written:

$$s^2 = \frac{Ln_{1L} + Y_0\nu_2 \pm \sqrt{(Ln_{1L})^2 + Y_0(Ln_{3L} - \nu_2 Ln_{1L})}}{Y_0}.$$

The following three examples of symmetrical distributions used to test these two formulas and one later one are taken from the second edition, 1927, of Elderton's "Frequency Curves and Correlation."

The first case is illustrated by two examples: the first one, a synthetic double hump curve given on page 132, and the second, an actual distribution characterized by Elderton as "a rather interesting example of a symmetrical distribution" (page 85) "not capable of satisfactory graduation by the normal curve of error" (page 129).

For the double-hump curve, $N = N_1 + N_2 = 2031$; $\nu_2 = 4.610,537$; $\nu_4 = 46.016,741$, and hence $3\nu_2^2 - \nu_4 = 17.7544$, corresponding to $-s = t = 1.727$ and $\nu_{21} = \nu_{22} = 1.631$, with $N_1 = N_2 = 1015.5$, according to the simple formula. To use the "half moment" formula requires also $Y_o = 291$; $Ln_{1L} = -1,816$; $Ln_{3L} = -14,044$, and hence

$$s^2 = -1.630,010 \pm 4.410,848 = +2.780,838$$

or

$$s = -1.6676 \qquad \text{and} \qquad \nu_{21} = 1.8297.$$

In other words, the two formulas do not give identical results and it may be interesting to list both sets.

In Table I, the second column gives the y's computed by the equation

$$y = 794.3 \left\{ \frac{1}{\sqrt{2\pi}} e^{\frac{-(0.782x+1.351)^2}{2}} + \frac{1}{\sqrt{2\pi}} e^{\frac{-(0.782x-1.351)^2}{2}} \right.$$

corresponding to the values of s and ν_{21} from the simple formula involving ν_4; the next three columns list y_1, y_2, and $y = y_1 + y_2$ computed from the equation

$$y = 750.7 \left\{ \frac{1}{\sqrt{2\pi}} e^{\frac{-(0.739x+1.233)^2}{2}} + \frac{1}{\sqrt{2\pi}} e^{\frac{-(0.739x-1.233)^2}{2}} \right.$$

corresponding to the values given by the "half moment" formula. The last column lists y as given by Elderton.

TABLE I

Double-Hump Curve (page 132)

x	y (calculated by 1st method)	(Calculated by 2nd method)			y (Elderton)
		y_1	y_2	$y = y_1 + y_2$	
0	254.4	140.0	140.0	280.0	291
1	302.1	42.8	265.1	307.9	303
2	314.2	7.6	290.6	298.2	286
3	193.4	0.8	184.4	185.2	193
4	65.4	0.04	67.8	67.8	78
5	13.0	0	14.4	14.4	10
6	1.2		1.8	1.8	-
7	—		0.13	0.1	—

This reproduces the double-hump nicely, especially by the "half moment" formula which fits the data the better of the two.

The second example of the first type of symmetrical curves is adapted for computation by the simple formula only, because the maximum ordinate is not given, and because the curve has $\nu_3 = 0.120,452$ instead of $\nu_3 = 0$ as is characteristic of symmetrical curves; it furthermore shows its skewness in that L is not equal to R, and Ln_{3L} is not equal to $-Rn_{3R}$. Since, however, further comparison of the two for-

mulas for the first case is desirable, the data for the "half moment" formula were obtained as follows: (a) Y_0 was estimated from the graph of the curve (page 87) to be 463; (b) Ln_{3L} was replaced by $L(n_{3L} - \nu_3)$ as in equation (11) and thus $-3,258.177 - (852 \times 0.120,452) = -3,360.802$ which is practically identical with $R(\nu_3 - n_{3R}) = 831 \times 0.120,452) - 3,460.891$, as it should be, since $Ln_{3L} + Rn_{3R} = N\nu_3 = L\nu_3 + R\nu_3$.

The second column in Table II gives y as computed from the equation

$$y = 824\left\{\frac{1}{\sqrt{2\pi}}e^{-\frac{(0.9792x+0.9131)^2}{2}} + \frac{1}{\sqrt{2\pi}}e^{-\frac{(0.9792x-0.9131)^2}{2}}\right.$$

obtained from the simple formula with

$$N_1 = N_2 = \frac{N}{2} = 841.5, \qquad s^4 = t^4 = \frac{3\nu_2^2 - \nu_4}{2} = \frac{1.51238}{2},$$

or $s = 0.9131$, and $\nu_{21} = \nu_2 - s^2 = 1.912,505 - 0.869,592 = 1.042,913$. The third, fourth, and fifth columns list y_1, y_2, and $y = y_1 + y_2$, respectively, as obtained from the equation

$$y = 734\left\{\frac{1}{\sqrt{2\pi}}e^{-\frac{(0.8722x+0.6746)^2}{2}} + \frac{1}{\sqrt{2\pi}}e^{-\frac{(0.8722x-0.6746)^2}{2}}\right.$$

to correspond with the value of $s^2 = 0.597,993$ obtained from the quadratic formula with $Y_0 = 463$; $Ln_{1L} = -Rn_{1R} = -963.734,780$; $L(n_{3L} - \nu_3) = R(\nu_3 - n_{3R}) = -3,360.802$, when L is taken as 852 and R as 831, with $\nu_3 = +0.120,452$.

The last column of Table II lists y as given by Elderton.

TABLE II.

"*Symmetrical*" *Distribution* (pages 85 and 129)

x	y (calculated by 1st method)	(Calculated by 2nd method) y_1	y_2	$y = y_1 + y_2$	y (Elderton)
−5.498,515	0	0.06	0	0	—
−4.498,515	0.7	1.5	0	1.5	—
−3.498,515	14.2	17.4	0.3	17.7	11
−2.498,515	102.7	94.4	5.0	99.4	116
−1.498,515	301.3	239.7	41.1	280.8	274
−0.498,515	423.5	284.5	158.3	442.8	451
0.501,485	423.4	157.8	284.7	442.4	432
1.501,485	300.7	40.9	239.4	280.3	267
2.501,485	102.2	5.0	94.1	99.1	116
3.501,485	14.1	0.3	17.3	17.6	16
4.501,485	0.7	0	1.5	1.5	
5.501,485	0		0.06	0	

These results of the new formula compare favorably with the three graduations that Elderton considers excellent.

The second case for symmetrical curves requires $s = t = 0$, with, of course, $\nu_1 = \nu_3 = \nu_5 = 0$. These values substituted in equations (1) to (4) give only

$$N_1 = N\left(\frac{\nu_2 - \nu_{22}}{\nu_{21} - \nu_{22}}\right) = N\left(\frac{\nu_4 - 3\nu_{22}^2}{3\nu_{21}^2 - 3\nu_{22}^2}\right).$$

The first equation for determining ν_{21} and ν_{22} is, therefore,

$$3\nu_2(\nu_{21} + \nu_{22}) - 3\nu_{21}\nu_{22} = \nu_4$$

obtained by equating the two values of N_1, and the second equation is:

$$2Ln_{1L}(\nu_{21} + \nu_{22}) + 2Y_o\nu_{21}\nu_{22} = Ln_{3l}$$

obtained from equation (11) by the substitution of $s = t = 0$.

Solving for ν_{21} leads to the quadratic

$$\nu_{21}^2 - \left\{\frac{2Y_o\nu_4 + 3Ln_{3l}}{6(Y_o\nu_2 + Ln_{1L})}\right\}\nu_{21} + \left\{\frac{3\nu_2Ln_{3L} - 2\nu_4Ln_{1L}}{6(Y_o\nu_2 + Ln_{1L})}\right\} = 0,$$

whose two solutions represent ν_{21} and ν_{22}, which, together with the consequent values of N_1 and N_2, solves the problem.

Applying this to the example cited on page 89, by Elderton, as a "reminder that the 'normal curve' does not give entirely satisfactory results even with symmetrical distributions," gives the figures shown in Table III and computed by the equation

$$y = 924\left\{\frac{1}{\sqrt{2\pi}}e^{\frac{-(0.335x)^2}{2}}\right\} + 4{,}846\left\{\frac{1}{\sqrt{2\pi}}e^{\frac{-(0.669x)^2}{2}}\right\}$$

derived from the solution of the quadratic

$$\nu_{21}^2 - 11.124\nu_{21} + 20.186 = 0,$$

TABLE III.

Symmetrical Distribution (page 89)

x	y_1	y_2	$y = y_1 + y_2$	y (Elderton)
0	368.6	1,933.3	2,301.9	2,244
1	348.5	1,545.8	1,894.1	1,856
2	294.5	798.8	1,084.3	1,106
3	224.6	258.0	482.6	527
4	150.2	53.9	204.1	225
5	90.6	7.2	97.8	93
6	48.9	0.6	49.5	38
7	23.6	0.03	23.6	16
8	10.2	0	10.2	7
9	3.9	—	3.9	4
10	1.3		1.3	2
11	0.4		0.4	1
12	0.1		0.1	1

corresponding to the curve constants: $N = 10{,}000$; $\nu_2 = 4.0678$; $\nu_4 = 75.1846$; $Y_0 = 2{,}244$; $Ln_{1L} = -7{,}491$; $Ln_{3L} = -76{,}055$. The two superimposed Gauss curves have $N_1 = 2{,}754.8$, $\nu_{21} = 8.8895$, and $N_2 = 7{,}245.2$, $\nu_{22} = 2.2345$, respectively.

In connection with this actual example of a second case symmetrical distribution, it may be stated that the algebraic sum of two Gauss curves is not another Gauss curve, but their product is:

$$\frac{N_1}{\sqrt{2\pi\nu_{21}}} e^{\frac{-(x-s)^2}{2\nu_{21}}} \times \frac{N_2}{\sqrt{2\pi\nu_{22}}} e^{\frac{-(x-t)^2}{2\nu_{12}}} = \left[\frac{N_1N_2}{2\pi\sqrt{\nu_{21}\nu_{22}}} e^{\frac{-(s+t)^2}{2(\nu_{21}+\nu_{22})}}\right] e^{\frac{-(x-b)^2}{2a}},$$

where

$$a = \left(\frac{t\nu_{21} + s\nu_{22}}{\nu_{21} + \nu_{22}}\right) \quad \text{and} \quad b = \left(\frac{\nu_{21}\nu_{22}}{\nu_{21} + \nu_{22}}\right).$$

In the absence of an example of a symmetrical distribution of the second kind in which two Gauss curves are subtracted, such a curve was synthesized and, to add interest, in such a way as to produce a double hump. The equation for this synthetic double-humped curve is:

$$y = \frac{20{,}000}{\sqrt{2\pi \times 4}} e^{-\frac{x^2}{2\times 4}} - \frac{8{,}000}{\sqrt{2\pi \times}} e^{-\frac{x^2}{2\times 1}}$$

for which $N = 12{,}000$; $\nu_2 = 5.9999$; $\nu_4 = 77.9821$ and $Y_0 = 797.9$; $Ln_{1L} = -12{,}710.8$; $Ln_{3L} = -121{,}268.2$, giving $\nu_{21}^2 - 5.0336\nu_{21} + 4.2128 = 0$ by the second case quadratic and leading to

$$N_1 = 20{,}332; \quad \nu_{21} = 3.9750;$$
$$N_2 = -8{,}332; \quad \nu_{22} = 1.0587.$$

The difference between the synthetic curve and its graduation by the "half moment" formula for the second type of symmetrical bi-modal curves is shown in Table IV.

TABLE IV.

Synthetic Double-Humped Curve.

x	As graduated			As synthesized		
	y_1	y_2	$y=y_1-y_2$	y_1	y_2	$y=y_1-y_2$
0	4068.4	3230.6	837.8	3989.4	3191.5	797.6
1	3588.6	2014.0	1574.6	3520.7	1935.8	1584.9
2	2459.8	488.3	1971.5	2419.7	431.9	1987.8
3	1311.4	46.0	1265.4	1295.2	35.4	1259.8
4	543.2	1.7	541.5	539.9	1.0	538.1
5	175.2	0	175.2	175.3	0	175.3
6	44.0	—	44.0	44.3	—	44.3
7	8.6	—	8.6	8.7	—	8.7
8	1.3	—	1.3	1.3	—	1.3
9	0.2	—	0.2	0.2	—	0.2
10	0		0	0		0

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

MEASUREMENT OF TOTAL STRATOSPHERE OZONE.

It is recognized that the variation in the total amount of ozone in the stratosphere has a direct correlation with latitude and season, and that different types of air masses show variations in ozone content that are associated with their origin or movement. However, it is not known at present whether any direct relationship exists between such ozone variations and current weather conditions. The development of a suitable technique for routine measurement of total ozone is of great interest in order to establish possible correlations between ozone concentrations and weather phenomena. If such correlations do exist, ozone data may become very useful in weather forecasting.

At the request of the Naval Research Laboratory, Ralph Stair of the National Bureau of Standards recently made total-ozone measurements of the stratosphere over the Organ Mountains in New Mexico by means of ground-based equipment employing a photocell and selected filters. This method serves as a check upon ozone determinations by V-2 rocket flights from White Sands Proving Ground. The technique developed by the Bureau makes use of phototubes sensitive to ultraviolet radiation of wavelengths under 3400 A and filters with transmittances beginning at 2900 to 3100 A and increasing with wavelength. Since ozone is strongly absorbing within this region, the spectral energy distribution of sunlight reaching the earth's surface is greatly affected by its presence. Hence, when the transmittances of the filters are measured for sunlight (using the phototube as a detector), the observed value depends upon the solar energy distribution, and therefore is a function of the amount of ozone within the beam of sunlight under study.

The spectral energy curve of sunlight outside the earth's atmosphere may be calculated from measurements of filter transmittances of sunlight (for various altitudes of the sun) after correction has been made for Rayleigh scattering at the altitude and location of the observing station. Knowledge of the spectral response curve of the phototube, the spectral transmittances of the filters, and the spectral transmittances of selected amounts of ozone is required for this computation. When the spectral energy distribution of solar radiation outside the earth's atmosphere has been determined (within the spectral region extending from about 3000 to 3400 A) by this method, or has been obtained from data of other observers, the total amount of ozone in the stratosphere may be derived as a function of the observed filter transmittance of a

* Communicated by the Director.

single filter. After this relationship has been established, the total amount of ozone may be quickly determined from one filter transmittance measurement.

In the investigation at San Augustine Pass in the Organ Mountains, two phototubes and four filters were employed, providing eight independent determinations of the ozone value for the period of June 29 to July 4, 1947. The extremes of these eight determinations were about 0.19 and 0.24 cm. of ozone at normal temperature and pressure (that is, if all the ozone in the stratosphere were brought to sea level and reduced to 0 C., the total thickness would be 0.19 to 0.24 cm). During this period the mean value varied from about 0.20 to 0.22 cm. with changes in the type of air mass. A mean value was observed for continental tropical air, the higher value for a polar air mass, and finally the lower value for a tropical maritime air mass. The weighted mean of all determinations was 0.21 cm.

Additional measurements during the week of December 15, 1947 should give a winter value for total stratosphere ozone at the same location. These measurements will provide an independent set of ozone data for the time of the year when the sun is near its maximum declination south, while the mid-summer determinations furnish data corresponding approximately to the maximum northern declination of the sun. Positive conclusions of the ozone-weather relationship can be made only after more comprehensive data have been obtained and analyzed from many different observing stations, a project that is being planned by the Weather Bureau.

While extensive details are involved in the development of calibration curves and data for each phototube and set of filters, the Bureau's method becomes simple for routine operation at a particular observing station, after the original calibration data and curves have been prepared. Furthermore, since the auxiliary equipment consists only of a simple direct-current amplifier, the equipment for this type of work is inexpensive, and easily assembled and transported to suitable observing stations. The method is therefore recommended where routine measurements of ozone concentration are to be made at widely distributed stations.

NEW FLOW CALORIMETER MEASURES SPECIFIC HEATS OF GASES.

A new flow calorimeter for more accurate measurements of the specific heats of gases has been developed by R. B. Scott of the National Bureau of Standards. This apparatus was constructed in connection with a program sponsored jointly by the Bureau and the Office of Rubber Reserve for the determination of the thermodynamic properties of materials involved in the production of synthetic rubber and related substances. The greater precision of the new instrument is chiefly due to

the most complete elimination of heat leaks by various features of design. The small heat capacity of the calorimeter also results in improved accuracy through the easier and more rapid attainment of a constant temperature distribution within the system, since only under conditions of steady temperature is an accurate measurement possible.

Accurate data on specific heats and other thermal properties of gases are of value to chemical industry, where they are used in calculations that predict the results of chemical reactions. With sufficient knowledge of the thermodynamic properties of the compounds involved, it is possible to state the range of temperature within which a given reaction will occur and to estimate closely the relative amounts of the products.

Specific heat is most simply obtained experimentally by adding a measured amount of heat to a fixed mass and recording the temperature rise. However, the heat capacity per unit volume of a gas is so low that small accidental heat leaks are often comparable in size to the heat used in actually raising the temperature of the gas. Reduction of the ratio of heat loss or gain to heat input is thus one of the chief problems of gas calorimetry. In the calorimeter constructed at the National Bureau of Standards, this is accomplished by causing the gas to flow continuously at a uniform rate through an insulated chamber in which it is heated electrically. The heat capacity is then calculated from the heat input, the temperature rise of the gas, and the rate of flow. With gas calorimeters of this general type, a correction is usually made for heat leaks by making measurements at varying flow rates and extrapolating the apparent specific heats to infinite rate of flow, where such errors are negligible. In the new apparatus, however, special precautions have reduced heat leaks to such an extent that a correction for them is unnecessary. This advantage greatly lessens the number of necessary observations and also makes the results more trustworthy, since there are errors in calorimetric measurements that increase with increasing flow rate and cannot be removed by extrapolation to infinite flow.

The entire calorimeter is immersed in a constant-temperature bath. The gas whose specific heat is to be measured enters the system through a copper helix, in which it is brought to the temperature of the bath. It then flows into the calorimeter proper, where it traverses a labyrinth surrounding a constantan heater, and leaves the calorimeter through a throttle valve. The temperature rise within the calorimeter is determined by appropriately located thermocouple junctions.

A uniform rate of gas flow is obtained by maintaining a constant pressure on the inlet side of the calorimeter and a constant lower presure at the outlet. Adjustment of the throttle valve in the outlet tube then provides the desired rate of flow, which is determined by weighing the gas collected in a condensing reservoir within a given time.

In the construction of the calorimeter, resistance to flow was reduced as much as possible, so that gases having low vapor pressures could be passed through the system at satisfactory rates. Since it was necessary to measure the specific heats of vapors that corrode copper and silver, all parts traversed by the gas were made of monel metal.

Heat leaks to or from the interior of the calorimeter are minimized in several ways. Conduction along the inlet and outlet tubes is reduced through the use of tubing of monel, a poor-conducting metal, having a wall thickness of 0.01 inch. Radiation along the tubes is prevented by a series of baffles, which in addition promote thermal equilibrium between the gas and the tube walls. Heat leaks due to either conduction or radiation are also lessened by providing a long section of tubing between the inlet and the heater, and by heating the outlet to the final temperature of the gas. Radiation to or from the inner portion of the calorimeter is prevented by a copper radiation shield kept at the temperature of the inner adjacent calorimeter wall by means of an electric heater. The necessary adjustments in the power to the heater are made in accordance with the readings of a three-junction thermocouple.

The accuracy of the apparatus was checked by measuring the specific heat of oxygen,[1] which had previously been accurately calculated at the Bureau from the latest spectroscopic data for this gas. The experimental values obtained at $-30°$, $40°$, and $90°$ C. differed from the spectroscopic values by 0.06, 0.00, and 0.10 per cent. respectively. From a consideration of the data for oxygen, the probable error of the specific heats obtained with this calorimeter is estimated as 0.07 per cent.

The new calorimeter has been used at the Bureau by Ruth K. Cheney, Jane W. Mellors, R. B. Scott, and P. F. Wacker to determine the specific heats from $-30°$ to $90°$ C. of isobutane, isobutylene, 1-butene, *cis*-2-butene, butadiene, styrene, and ethylbenzene.[1,2] Isobutane is a principal raw material for the production of aviation gasoline and is also converted to isobutene—a substance used in large quantities for the preparation of butyl rubber and isobutanol. 1-Butene, like the 2-butenes, is converted to butadiene, the major component of GR-S rubber. A knowledge of the thermodynamic properties of these materials is expected to aid in the more efficient operation of plants in which they are produced or used.

[1] Paul F. Wacker, Ruth K. Cheney, and Russel B. Scott, "Heat Capacities of Gaseous Oxygen, Isobutane, and 1-Butene from $-30°$ C. to $+90°$ C.," *J. Research*, NBS **38**, 651 (1947), RP 1804.

[2] Russel B. Scott and Jane W. Mellors, "Specific Heats of Gaseous 1,3-Butadiene, Isobutene, Styrene, and Ethylbenzene," *J. Research*, NBS **34**, 243 (1945), RP 1640.

THE FRANKLIN INSTITUTE.

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STATED MONTHLY MEETING, WEDNESDAY, FEBRUARY 18, 1948.

The stated monthly meeting of The Franklin Institute was held on February 18, 1948 at 8:15 p. m. in the Lecture Hall. Mr. Richard T. Nalle, President, presided. There were approximately 200 persons in attendance.

The President announced that the minutes of the stated monthly meeting of December had been printed in full in the January issue of the JOURNAL and if no additions or corrections were offered from the floor the minutes would stand approved as printed. There was no dissent.

The Secretary was then called upon for his report. He announced the following members had been elected to Institute membership in the month of January:

Active	35
Associate	20
Student	43

It was announced that the total membership in The Franklin Institute as of January 31 was 5423.

The Secretary then informed the audience of two fascinating exhibits which are to be installed in the Institute in the early spring and summer. The one on Minting will be coincident with the coining of the new fifty cent piece which will bear the head of Benjamin Franklin on the obverse side and the Liberty Bell on the reverse side. Mrs. Nellie Taylor Ross, Director of the Mint in Washington, D. C., and former Governor of Wyoming, has been cooperating with us on this project.

The second exhibit is a model Railway which will be displayed about the middle of June. This exhibit is being sponsored by the Baltimore and Ohio Railway and is purported to be one of the finest models of its kind in the country.

Other exceptionally fine exhibits which will be featured in the Museum in 1948 include the Nuclear Energy Exhibit and the Bell Telephone Exhibit.

President Nalle then introduced Mr. L. F. Livingston, speaker of the evening. Mr. Livingston is manager of the Extension Division of the E. I. du Pont de Nemours & Company in Wilmington, Delaware. Mr. Livingston gave a very enjoyable talk which he appropriately called "Preview of Progress." He showed many types of products which can be made of Polythene and demonstrated its remarkable properties. Polythene, Mr. Livingston explained, is an inert substance affected by neither heat nor cold and can be readily colored. He told us of new applications made of Lucite, Cellophane, Nylon, Neoprene, together with a number of others and showed us samples of some which are now marketable.

Through Mr. Livingston's talk the audience gained a greater insight into aspects of the latest chemical research, though much is still in its experimental stage—as our speaker so adequately phrased it, "so far" these characteristics have been found.

At the conclusion of Mr. Livingston's excellent lecture the President adjourned the meeting with a rising vote of thanks to the speaker of the evening.

HENRY B. ALLEN,
Secretary

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M., Wednesdays and Thursdays from 2 P.M. until 10 P.M.

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A Survey of the Principles and Practice of Wave Guides, by L. G. H. Huxley. 328 pages, drawings, 14 × 22 cm. Cambridge, University Press; New York, The Macmillan Co.; 1947. Price, $4.75.

Computing Mechanisms and Linkages, by Antonin Svoboda, Ed. by Hubert M. James. 359 pages, tables, drawings and plate, 16 × 23 cm. New York, McGraw-Hill Book Co., 1948. Price, $4.50.

Crystal Rectifiers, by Henry C. Torrey and Charles A. Whitmer. 443 pages, tables, drawings and illustrations, 16 × 23 cm. New York, McGraw-Hill Book Co., 1948. Price, $6.00.

The American Annual of Photography of 1948, Volume 62, edited by F. R. Fraprie and Franklin I. Jordan. 216 pages, illustrations, 19 × 25 cm. Boston, American Photographical Publishing Co.; London, Chapman & Hall, Ltd.; 1947. Price, $2.00 (Paper).

BOOK REVIEWS.

ADULT EDUCATION FOR HOMEMAKING, by L. Belle Pollard. Second edition. 194 pages, 14 × 22 cms. New York, John Wiley & Sons, Inc., 1947. Price $2.75.

The material contained in this edition is a revision of an earlier book entitled "Adult Education in Homemaking."

It is designed for college use, as well as for the use of teachers who are responsible for the directing of the education of older youth and adults.

Written as a summary of experience it deals with the promotion, organization, supervision, and evaluation of educational programs.

HENRY M. MICHAEL.

ARCHITECTURAL CONSTRUCTION, by Theodore Crane. 414 pages, drawings and illustrations, 16 × 25 cms. New York, John Wiley & Sons, Inc., 1947. Price $6.00.

This book deals with the problem of making an appropriate choice for the structural portions of a building, as governed by the geographical location, site conditions, type of occupancy, equipment, and architectural design.

It is written for students in architecture, architectural engineering, and building construction, and portions of the text should prove of value to mechanical and electrical engineers who require a knowledge of modern building assemblies.

In the nine chapters of his book Mr. Crane furnishes the background for successful solution of constructional problems. He discusses building codes and design standards, the choice of framing, flooring and roofing materials, wall assemblies and foundations, as well as widespan designs.

260 pertinent illustrations round out this comprehensive volume.

HENRY M. MICHAEL.

YARN AND CLOTH CALCULATIONS, by Lloyd H. Jackson. First edition. 196 pages, drawing, 13 × 22 cms. New York, Textile Book Publishers, Inc., 1947. Price $6.00 in U. S. and Canada, $7.00 in foreign countries.

In this short volume Mr. Jackson covers the essentials of yarn and cloth calculations, having in mind the needs of the textile student and the technician.

In its ten chapters he explains the yarn numbering systems and their conversions, ply yarns, grey cloth, warp, filling, and cloth calculations, the theory of fabric construction, calculations based on fabric analysis, and fabric analysis itself.

A list of problems has been added to enable the student to adopt the various formulae to practical applications.

HENRY N. MICHAEL.

FLIGHT ENGINEERING AND CRUISE CONTROL, by Harris G. Moe. 209 pages, drawings and illustrations, 15 × 23 cms. New York, John Wiley & Sons, Inc., 1947. Price $4.00.

The material contained in this volume is essentially written for those professional pilots and flight-crew members who, although technically and mechanically minded, do not have the necessary aeronautical engineering training to understand the subject in question, which hitherto had been presented in a highly technical manner. In his book, Mr. Moe treats this basically technical subject in such a manner that the reader need not be highly trained in order to understand it.

The contents of the book cover the following: altimeters and types of altitude, air speed, power and associated variables, power and airplane performance, supercharged engines, propellers, basic cruising techniques, flight planning and control, weight and balance, and jet propulsion.

The forward-looking approach of this book is evidenced by its discussion of air compressibility and jet propulsion.

HENRY N. MICHAEL.

PHOTOGRAPHIC FACTS AND FORMULAS, by E. J. Wall and Franklin I. Jordan. 364 pages, drawings, tables, 16 × 24 cms. Boston, American Photographic Publishing Co., 1947. Price $5.00.

The present edition of this book (the first edition was published some forty years ago) furnishes the photographic technician with a ready reference which contains the fundamental formulae and rules of photography. While not making any pretense of imparting purely scientific information, the descriptive matter is plain and concise.

This latest edition has been improved by the elimination of many pages covering obsolete processes, which have been replaced with material covering the most recent developments.

HENRY N. MICHAEL.

METEOROLOGICAL FACTORS IN RADIO WAVE PROPAGATION. Report on a Conference held on 8 April 1946 at The Royal Institution, London, by The Physical Society and The Royal Meteorological Society. 325 pages, drawings, 17 × 26 cms. London, The Physical Society. Price 24 s. to non-members.

The papers contained in this report have been presented at the conference held in the Royal Institution, London, on April 8, 1946. They describe the various investigations concerned with the effect of the meteorological conditions of the lower atmosphere on the bending of very short radio waves transmitted through it.

The papers, twenty-one in all, deal with many aspects of the refractive index of the atmosphere and in this volume form a compact contribution to the study of radar.

HENRY N. MICHAEL.

IMPROVING SUPERVISION, by Frank Cushman and Robert W. Cushman. 232 pages, 13 × 19 cms. New York, John Wiley & Sons, Inc., 1947. Price $2.50.

Placing special emphasis on human relations problems, the authors have written this book for the purpose of making available to all supervisors ideas which they can apply to carry out their assignments more effectively. It has been especially designed for "follow-up reading" in connection with foreman and supervisor training programs.

The book should also be of interest to employers, managers, and training directors, since it deals with human engineering.

HENRY N. MICHAEL.

SAGA IN STEEL AND CONCRETE, NORWEGIAN ENGINEERS IN AMERICA, by Kenneth Bjork. 504 pages, illustrations, 15 × 23 cms. Northfield, Minn., Norwegian-American Historical Association, 1947. Price $4.00.

In a novel approach to the influence of national elements on the development of America, Dr. Bjork here treats of a particularly limited segment of the immigrants to this country—

Norwegian engineers. Although relatively small numerically in proportion to the whole immigration, this group is especially significant for two reasons: first, those immigrating constituted a very high percentage of the graduates of the technical schools in Norway, practically 25 per cent up to 1910, with some years running 50 per cent or more; and second, these engineers made many vital contributions to the technological building of the United States, a record which is presented in this volume in the lives and works of many of these engineers.

An introductory chapter surveys rapidly the beginnings of the industrial revolution in Norway, the establishment of technical schools there, and the factors which resulted in the emigration of so many of the graduates of those schools to the United States. The pioneers of this movement came in the 1860's, 1870's, and 1880's and found their way into various fields.

The chapter entitled "A Philadelphia Story" will undoubtedly prove most interesting to many JOURNAL readers for it concerns three men who settled in Philadelphia. The three are Tinius Olsen, who produced the first commercial testing machine; Henrik V. von Zernikow Loss, who succeeded in manufacturing an all-steel wheel and put this wheel on the all-steel railway car that had just been produced; and Mauritz C. Indahl, who was mainly responsible for developing the monotype machine into the useful versatile machine it is today. It is interesting to note that all three worked at some time for William Sellers, prominent Philadelphia manufacturer, as did so many others of these Norwegian engineers.

Then Dr. Bjork continues by devoting separate chapters to various types of engineering. In writing about bridges he gives primary attention to Thomas G. Pihlfeldt, who was instrumental in working out the Chicago type bascule bridge. In the field of tunnels the contributions seem even more significant for Olaf Hoff successfully applied the method of laying a tunnel in a prepared underwater trench, and Ole Singstad was responsible for developing a system of ventilation which has made possible the great automobile tunnels of the present.

Engineers who built skyscrapers, did metallurgical work, worked in the electric power field, built ships, and designed machines are among the others included. Of special interest is the chapter dealing with scientific management, which introduces Carl G. L. Barth, an assistant of Frederick W. Taylor, "the father of scientific management." Barth's vital contribution was the development of a slide rule suitable for ascertaining the correct cutting speed and feed for a lathe. After Taylor's death Barth became the leading exponent of the system with which he had been so closely associated during its period of formulation.

The activities of the Norwegian engineers in social and professional organizations among members of their own nationality are discussed. A concluding chapter offers a survey of their social philosophy. This volume is particularly important for the author's treatment of the contribution of one group of immigrants to the technological development of America. It is of interest also for the descriptions it contains of many outstanding engineering accomplishments and it will prove of considerable value as a source of biographical information about many engineers concerning whom such information may not be readily accessible.

G. E. PETTENGILL.

LECTURES DELIVERED AT PENNSYLVANIA SHORT COURSE FOR SEWAGE WORKS OPERATORS, 1945. 109 pages, illustrations, 17 × 25 cms. Pennsylvania Sewage Works Association, 1946.

LABORATORY GUIDE FOR SEWAGE WORKS OPERATORS, by D. Paul Rogers. 92 pages, illustrations, 17 × 25 cms. Pennsylvania Sewage Works Association, 1946.

These two volumes result from a course given in the spring of 1945 in Philadelphia sponsored by the United States Office of Education through the Engineering, Science and Management War Training Program. The content of the course was developed with the cooperation of the Pennsylvania Department of Health, the Pennsylvania Sewage Works Association and the Sewage Works Operators Association of Southeastern Pennsylvania.

The fifteen lectures were given by various individuals; some were state and city engineers, others were men connected with industrial concerns. The subject matter is designed for the sewage plant operator and considers sewage, reasons for its treatment, its characteristics, and the various treatment processes. Lectures were devoted to each of several types of equipment

including Imhoff tanks, intermittent sand filters, trickling filter, sludge digestion units, activated sludge plants, and chlorinators. Other lectures included the matter of sewage plant records, the effect of industrial waste on sewage treatment and the operation and maintenance of mechanical equipment.

The laboratory guide, while based on the original notes and outlines of the various instructors, has been rewritten by Mr. Rogers to obtain a uniform style of presentation. No attempt has been made to include tests not discussed in the course, nor does inclusion mean that every test is necessary at all sewage treatment plants. The method of treatment is clear and concise, comprising a definition of the test, indication of its purpose or use in sewage treatment works operation, specific step by step directions of procedure, and interpretation of the results by means of examples and notes.

As an introductory presentation of the fundamentals of sewage plant operation these volumes should be of value to the sewage works operator.

G. E. PETTENGILL.

TECHNICAL DRAWING PROBLEMS, by Frederick E. Giesecke, Alva Mitchell, and Henry Cecil Spencer. Second edition. 105 pages, drawings, 14 × 28 cms. New York, The Macmillan Co., 1947. Price $2.75 (Paper).

The 105 problems included in this volume are designed to cover the fundamental principles of technical drawing. They do not constitute a complete course in themselves, but are to be used in conjunction with a reference text. The "work plan" which prefaces the problems is designed to coordinate with the authors' text "Technical Drawing."

The individual problems are detachable.

HENRY N. MICHAEL.

ULTRAHIGH FREQUENCY TRANSMISSION AND RADIATION, by Nathan Marchand. 322 pages, drawings, 15 × 24 cms. New York, John Wiley & Sons, Inc., London, Chapman and Hall, Limited, 1947. Price $4.50.

The widespread use of ultrahigh frequencies presents many new problems in transmission and radiation which can be solved only by a thorough familiarization with the fundamentals.

Mr. Marchand has kept in mind the various applications and has written his book to meet the requirements of both the college student and the practicing engineer. The mks system is used throughout.

HENRY N. MICHAEL.

TECHNIQUES OF OBSERVING THE WEATHER, by B. C. Haynes. 272 pages, tables, drawings, and illustrations, 14 × 22 cms. New York, John Wiley & Sons, Inc., 1947. Price $4.00.

This book is written for high-school and college courses in elementary meteorology. It may be also used profitably by laymen who wish to take up the hobby of weather observing. For the guidance of the latter, Mr. Haynes includes a chapter on improvised weather stations. Otherwise, for the sake of uniformity needed in observing, the text follows closely the various United States Weather Bureau instructions on observations.

HENRY N. MICHAEL.

INDUSTRIAL HEALTH ENGINEERING, by Allen D. Brandt. 395 pages, drawings, tables, and illustrations, 15 × 24 cms. New York, John Wiley & Sons, Inc., 1947. Price $6.00.

Excessive concentrations of dusts, fumes, mists, gases, exposures to varying or unhealthy temperatures, to high humidity, harmful rays, noise, and to inadequate illumination, are of vital interest to the industrial health engineer. Dr. Brandt's book makes available to plant engineers, to construction and consulting engineering firms, and to others, in a condensed, practical and useful fashion, a summary of the information and data needed to determine what control measures should be employed and how control equipment should be designed.

Besides offering designs and control measures, Dr. Brandt classifies and evaluates the principal industrial atmospheric contamination.

The appendix contains tables of conversion, composition of some trade name solvents, size and characteristics of particular matters, and other pertinent data.

HENRY N. MICHAEL.

COLLEGE PHYSICS, by John A. Eldridge. Third edition. 720 pages, drawings, and illustrations, 14 × 22 cms. New York, John Wiley & Sons, Inc., London, Chapman & Hall, Limited, 1947. Price $4.50.

The third edition of Professor Eldridge's textbook retains the same approach as the earlier editions. In his own words, he "tried to write a text-book which will appeal to the student with only a general interest in the subject—non-science majors—and a text-book which will at the same time give a solid foundation for a student with a direct professional interest in physics." This resolution has been well carried out.

In this rewritten edition a somewhat greater number of worked-out examples are given and the order of presentation in mechanics is changed. In electricity considerable emphasis is given to electrostatics, fields, and potentials in space.

HENRY N. MICHAEL.

HEAT PUMPS, by Philip Sporn, E. R. Ambrose, and Theodore Baumeister. 188 pages, illustrations, 13 × 21 cms. New York, John Wiley & Sons, Inc., 1947. Price $3.75.

In this pioneer volume the authors have presented the fundamentals concerning the theory, construction, and operation of heat pumps. "The heat pump is the name commonly applied in present commercial practice to a year-round air-conditioning system employing refrigeration equipment in a manner which enables a surface to deliver usable heat to a space during the winter period and to abstract heat from the same space during the summer period." The idea of thus using the heat rejected from a refrigeration system is not new, but previously has found only a limited number of applications. Nor is it limited to heating as there are various potential industrial and manufacturing operations which employ the basic heat-pump principle.

The thermodynamic principles involved are outlined first in the book and then the authors proceed to a discussion of the four basic heat-pump systems, illustrating them by six different designs. The systems are air-to-air, water-to-air, air-to-liquid, and water-to-water, the first-named medium being the source of heat and the second being used to remove the heat from the condenser. Considerable attention is paid to design factors which must be considered, particularly climatic conditions and the source of heat. The use of government weather records in securing the climatological data is explained.

In the matter of equipment design the authors have limited their treatment to those aspects which are essential to the adequate development and choice of apparatus suitable for heat-pump service. This involves the consideration of compressors, heat transmission surfaces and fans. One chapter is devoted to an actual example detailing the working out of the equipment selection for a typical domestic installation of a heat pump.

The necessity for defrosting when air is used as a heat source is emphasized and several methods are outlined for accomplishing this end. Control systems are of various types, the all-electric being recommended as generally best for small systems, while the electric-pneumatic is best for the larger systems. The use of the heat pump for evaporation, distillation, and concentration in industrial applications is noticed and its importance in the last war in the distillation of drinking water from sea water is pointed out. Of especial interest are the comments on the economics of the heat pump and its potential effect on the power-system load curve. Illustrations and operating data of several installations are given in a concluding chapter. Bibliographical references are provided for further study.

This first book on the heat pump is of importance for its presentation of a system which may become of great significance in the heating and industrial development of America.

G. E. PETTENGILL.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Manometric Determination of Phosphomonoesterase (Alkaline Phosphatase).—CHARLES A. ZITTLE. In the hydrolysis of monoesters of phosphoric acid a tertiary acid group is released. This group is not sufficiently acid ($pK_a = 12.4$) to release CO_2 from $NaHCO_3$ and accordingly does not provide the basis for a manometric procedure for following the hydrolysis of monoesters of phosphoric acid. These compounds, however, do have a property that makes it possible to follow their hydrolysis manometrically;—the secondary, as well as the primary, phosphoric acid group in the free acid is a weaker acid group ($pK_a = 6.8$) than is the same group in a great variety of mono-substituted derivatives of phosphoric acid (**1**,**2**,**3**,**4**). When hydrolysis of such compounds takes place the medium becomes more alkaline and in the presence of $NaHCO_3$ and an atmosphere of CO_2 some of the CO_2 is absorbed. The application of the method to the substrates β-glycerophosphate and guanylic acid is described herein. A limitation of the procedure is the narrow range of pH in the neighborhood of pH 6.8 to which it is applicable. This is considerably below the optimum for phosphomonoesterase (alkaline phosphatase) (pH 9 to 10), but it is in the neighborhood of the physiological range which is desirable for some types of studies (**5**).

EXPERIMENTAL.

In the hydrolysis of one equivalent of sodium β-glycerophosphate ($pK_2 = 6.34$) calculations show that about 0.21 equivalent of base would be released at pH 7.0, 0.24 equivalent at pH 6.8, 0.42 equivalent at pH 6.5. In the hydrolysis of one equivalent of guanylic acid ($pK_2 = 5.9$) about 0.39 equivalent of base would be released at pH 6.8. To be closer to the physiological range of pH and also to avoid the range where the base release changes considerably with pH, the experiments to be reported were performed at pH 6.8. As noted above, this is much below the optimum pH for the activity of this phosphatase.

The experiments were performed as follows: 0.5 cc. of 0.025 M $NaHCO_3$, 1.0 cc. of 0.43 M sodium β-glycerophosphate adjusted to pH 6.8 with N HCl, and water were placed in the bottom of Warburg flasks to make the total volume 3.5 cc. including the enzyme which was placed in the side arm. The enzyme, prepared from calf intestinal mucosa and partially purified (**6**), was used in a concentration of about 0.4 per cent in water in amounts of 0.2 to 0.8 cc. The atmosphere was 5 per cent CO_2—95 per cent N_2; the temperature was 37°. The results obtained are shown on the graph in Fig. 1. The absorption of

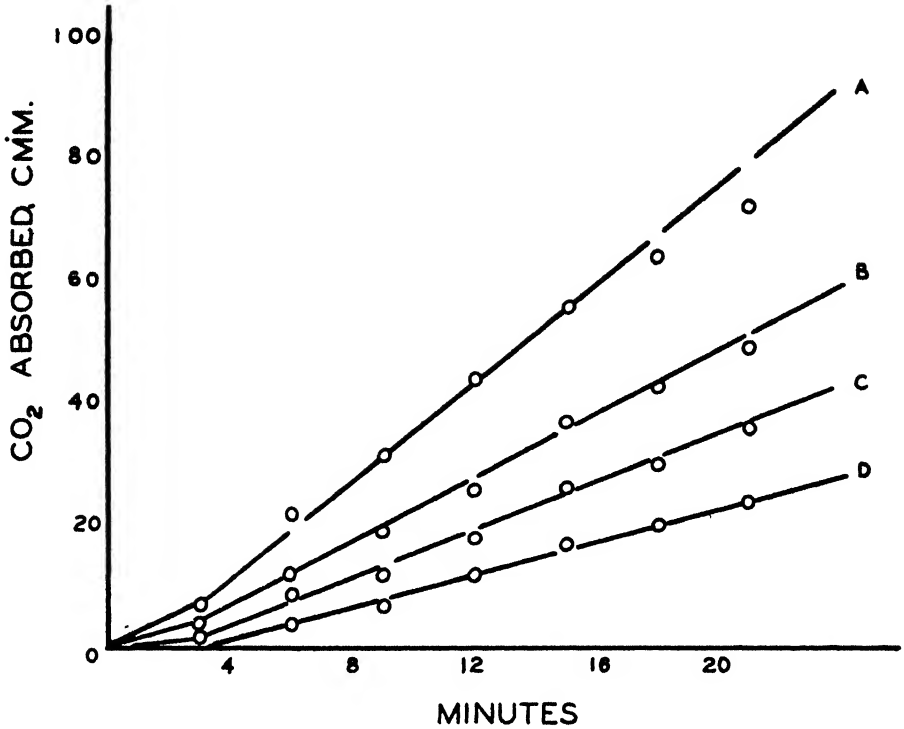

FIG. 1. Activity of phosphomonoesterase with glycerophosphate as substrate.

The rate of CO_2 absorption was measured manometrically with different amounts of the enzyme present. Curve A, 2.4 mg.; curve B, 1.6 mg.; curve C, 1.2 mg.; curve D, 0.8 mg.

CO_2 is proportional to the amount of enzyme used in each flask; the average is 166 cmm. (range 163 to 169) per 10 min. per 10 mg. of enzyme. The results with guanylic acid were similar but the rate of CO_2 absorption was about 20 per cent faster than with glycerophosphate.

Experiments were performed in which the amount of glycerophosphate was varied to determine the amount of substrate required for optimum activity of the enzyme; the amount used above (10 mg. per flask) provided a satisfactory excess.

The required pH was attained in other experiments with 100 per cent CO_2 and 0.6 cc. of 0.5 *M* $NaHCO_3$ with similar results.

Magnesium chloride in a final concentration of 0.004 *M* stimulated the enzyme about 10 per cent; larger concentrations were inhibitory.

Both sodium citrate and histidine in final concentrations of 0.02 *M* exerted more than 80 per cent inhibition in the system employed, which may be due to the ability of these compounds to bind positive ions necessary for activation of the enzyme. The actual amount of inhibition due to this effect is difficult to assess for both of these compounds have groups ionizing (pK of 5.4 and 6.0, respectively) in the neighborhood of the pH employed; hence, some of the base released by the hydrolysis

of glycerophosphate will be neutralized by buffer and consequently will not be available for the absorption of CO_2. An attempt was made to assess the magnitude of this effect by introducing small amounts of base, in the solid form, and measuring the CO_2 absorbed; Na_2CO_3, Na_3PO_4, and LiOH were employed. In an experiment with Na_3PO_4, 0.3 cc. of a 0.01 *M* solution was dried in the side arm of each of several flasks. When this amount of Na_3PO_4 was introduced into the system described previously with only $NaHCO_3$ and water to make the volume 3.5 cc., 65 cmm. of gas were absorbed which is a volume of about the expected magnitude. When, in addition to the $NaHCO_3$, the substrate glycerophosphate was present, only 19 cmm. of gas were absorbed. When with these substances was added 1.0 cc. of 0.1 *M* sodium phosphate, pH 6.8, or 1.0 cc. of 0.1 *M* sodium citrate, then the addition of Na_3PO_4 caused a *release* of gas (21 and 12 cmm., respectively). Comparable results were obtained with Na_2CO_3 and LiOH. This obviously does not represent the state of affairs when base is released by the hydrolysis of phosphate esters in the presence of buffering systems, and accordingly some other procedure must be found for assessing the effect of buffer compounds.

DISCUSSION.

The manometric method described for measuring phosphomonoesterase is convenient and useful where comparative assay data are desired. It has been used for following the fractionation of the phosphoesterase of calf intestinal mucosa which has both monoesterase and diesterase activity. The diesterase activity was also determined manometrically (6). Studies now in progress indicate that the two activities can be separated, that is, that diesterase and monoesterase are two distinct enzymes.

Preparations of phosphomonoesterase were assayed by both the present method and the method of Huggins and Talalay (7) which employs the phenolphthalein ester of phosphoric acid as substrate; an activity of 12,000 units per 1.0 g. of enzyme by the latter method corresponded to 100 cmm. per 10 min. per 10 mg. of enzyme by the manometric procedure.

SUMMARY.

A procedure for the manometric estimation of phosphomonoesterase is described.

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CURRENT TOPICS.

X-Ray Speeds Chemical Comparisons.—Economical and rapid means of making chemical comparisons is provided by a new instrument known as the X-ray photometer. With this device it has been found possible to determine satisfactorily the tetraethyl lead content of gasoline, the concentration of an acid in water, the per cent. chlorination of a plastic, or the per cent. ash in coal. These determinations are made by measuring and comparing the X-ray absorption of a sample and a reference.

While the general principle of utilizing X-ray absorption as a means of making comparisons of chemical compounds has long been known, the particular method employed in the X-ray photometer is an outgrowth of wartime experience. When the Army Ordnance Department sought a rapid and reliable method of checking the explosive charge in hand grenade fuses to prevent premature detonation, engineers of General Electric then devised a method of X-ray testing which made it possible to check fuses accurately at a rate of 4,000 per hour.

Experience with this fuse-testing apparatus led to further experiments, and to the development of the X-ray photometer by the company's General Engineering and Consulting Laboratory. Essentially this device consists of a source of X-rays, a fluorescent screen, and multiplier phototube, an amplifier and an indicating instrument. The X-ray beam is interrupted by a synchronous motor-driven chopper in such a way that half of the beam passes alternately through each of two analyzer cells, one containing the reference and the other containing the sample. In the half of the beam passing through the reference there is placed also an aluminum attenuator disc, the angular position of which corresponds with a particular thickness of metal.

The X-rays from the two halves of the beam are received alternately on the fluorescent screen, from which the fluorescence is transmitted to the multiplier phototube. The output of this tube passes through an amplifier to a peak comparator where it registers on a microammeter as a d-c signal indicative of the difference in intensities of the two halves of the beam.

If the sample and the reference are identical the intensities of the two halves of the beam as received on the fluorescent screen are equal and the ammeter reading is zero without the use of the attenuator. If the sample and the reference are different it becomes necessary to introduce aluminum by means of the attenuator until the two halves of the beam become of equal intensity and the reading returns to zero. From the thickness of aluminum introduced into the reference half of the beam it is possible to determine empirically the proportion of certain elements in the sample as compared with the reference. In general the method is most applicable where there is a considerable difference in the atomic numbers of the main substance and the particular ingredient to be measured.

In actual operation the X-ray photometer has been found to work satisfactorily up to a speed of 6 minus samples per hour. Under most circumstances

the limiting factor has been found to be the time required for the preparation of the sample. Liquids, which must be measured as they are put into the analyzer cell, require more time than solids when the latter are specimens of uniform thickness. Powdered solids such as coal, which must be weighed into the analyzer cell, require more time than liquids. Preparation of the samples, however, can be arranged independently of the operation of the instrument. If this is done there is no difficulty in maintaining the 6 minus per hour rate regardless of the physical form of the material to be tested.

An outstanding advantage of this method of comparison is that it is independent of the physical state of the substance being tested, because the amount of X-ray absorption by a given mass of material is always the same, whether the material is hot or cold, gaseous, liquid, or solid. For example, the absorption per gram is the same in steam, in water and in ice.

Similarly, because X-ray absorption is an atomic property, measurements will be identical when an element is alone and when it is in chemical combination. An oxygen atom, for instance, will register the same whether it is as an element or in any oxygen compound.

Other advantages of the X-ray photometer method of chemical comparison are its speed and convenience. The complete apparatus is housed in a mobile cabinet of moderate size, and can be operated by any reasonably competent technician. Furthermore the analysis is made without loss or alteration of the sample tested.

In its present commercial form the X-ray photometers is available both as a manual type, but recording and self-balancing features can be added when desired.

R. H. O.

The Drain on Fertility.—"Our soil resources have been damaged to an alarming degree," Secretary Anderson warned recently. "And the drain of fertility has been accelerated by production for war and for relief of the postwar world food shortage. As soils lose fertility, farmers are more restricted in the crops they can raise. Also, the dangers of erosion increase. This in turn increases the danger of floods and of filling our streams, lakes and reservoirs with silt. Farming becomes a less profitable occupation and, as the mineral resources of the soil decrease, the hazards of poor quality national food supply increase. . . .

"Farmers want more fertilizer than is now available, despite the fact that they are using about three times as much plant food material as in 1920 and about twice as much as in 1940. The needs of the soil call for much more fertilizer than is now being used. The Department of Agriculture is working with the War Department to find ways of modifying existing facilities which may be used by private industry to meet additional demands for nitrogen fertilizer in this country."

R. H. O.

X-Ray Protection. (*Electrical Engineering*, Vol. 66, No. 10.)—An extensive program for determining the effectiveness of concrete as a protective barrier against million-volt wide-beam X-rays is being conducted by scientists at the National Bureau of Standards. Optimum wall thicknesses and most

desirable types of construction necessary for maximum short wave length X-ray protection are being investigated.

Broad beam X-rays which allow simultaneous examination of wide areas have come into extensive use for detecting flaws in all types of metal, and have created a special personnel protection problem. When a broad X-ray beam enters a thick concrete wall, it is scattered and rescattered many times with the result that a considerable fraction of the beam emerges on the other side, endangering personnel. Quantitative information on the amount of this scattering is part of the present investigation program.

To determine protection standards, a 1.5-million-volt X-ray machine has been converted so that it will produce beams of varying widths. It is mounted so that it can be directed downward into a radiation pit about six feet square and 20 feet below the target of the X-ray tube. Special instruments that can be shifted to various positions by remote control are used to explore the strength of the radiation in the pit. Both the control and the X-ray reading equipment are operated from a central control room surrounded by 18 inches of concrete and located 75 feet from the radiation pit.

Test slabs of concrete, each weighing 2½ tons and measuring six inches in thickness, are placed over the mouth of the pit one at a time, thus enabling investigators to vary the protective barrier from six inches to five feet in thickness. Since space and weight are of critical importance in many installations, arrangements also have been made for producing "sandwiches" of lead and concrete in alternate layers. At present, the best proportions between these two materials are entirely unknown.

Among the problems being investigated is the shielding of such machines as betatrons that operate at 50 million to 300 million volts Because the beams emerging from such machines cover a very small angle (for the 50 million-volt betatron the angle is six or eight degrees), it may be possible to simplify the shielding problem.

For the direct beam, a very great thickness of concrete may be required. On the other hand, rays which are scattered sideways out of this beam are of a very much less penetrating character, and so it may be found that the protection problem for the side of the beam may not be seriously different from that at one or two million volts.

The collection and evaluation of data obtained in this investigation program can bring about sufficient protection at maximum economy. That the economic phase of high-voltage X-ray shielding is important is indicated by the fact that the cost of protection alone in a 50-million-volt medical or industrial installation may represent half to three quarters of the total cost of the equipment.

R. H. O.

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A STUDY OF PATENT POLICIES IN EDUCATIONAL INSTITUTIONS, GIVING SPECIFIC ATTENTION TO THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.*

BY

VINCENT LEE McKUSICK, S.M.[1]

(Continued from March issue.)

TABLE OF CONTENTS, PART II.

* A paper presented in the graduate seminar in electrical engineering at Massachusetts Institute of Technology by the author.

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(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

CHAPTER III

M.I.T. PATENT POLICY[1]

A. *Development of a Patent Policy at the Institute*

Before 1930 the M.I.T. experience with patents was not too different from that of every comparable educational institution. Patentable inventions had been made, and in each case tradition was followed. Either no patent was obtained or else it was considered strictly the property and individual concern of the inventor. In some instances the inventor, as Professor Hardy [2] with his spectrophotometer or color analyzer, applied to the Institute for a decision in regard to ownership of the invention and was given a *carte blanche* to develop the patent right as his own.

In 1932 an all-faculty committee [3] studied the patent problem and drew up a statement of policy regarding inventions. This statement, approved by the Faculty and by the Institute Executive Committee, has remained essentially unchanged as the fundamental patent policy of the Institute. Its context, which deals only in general terms with the property rights in inventions, may be summarized as follows:

1. Inventions from research financed entirely by the Institute shall be the Institute's exclusive property, from which M.I.T. may receive all benefits.
2. The Institute will have no rights in inventions
   (a) toward which the Institute contributes substantially nothing in funds or in time of a staff member, or
   (b) which are made by students whether paying tuition or receiving scholarship aid (The Institute is considered to administer, and not contribute, scholarship funds.), or
   (c) which are made in a program of research financed *in toto* (including overhead) by an outside sponsor.[4]
3. Intermediate cases will be handled on their own merit by special arrangement, in absence of which, title to the patent remains with the Institute.

This policy statement, formulated as it was (and only could be) in generalizations, has been the basis for Patent Committee decisions of equity since 1932. Enhanced by its brevity and clearness it has been voluntarily accepted by the staff members, making unnecessary the

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[1] The writer is indebted to Dean John W. M. Bunker, who, until recently, was Chairman of the M.I.T. Patent Committees, for much of the material presented in this chapter.

[2] Statement by Dr. Arthur C. Hardy to Miss Ahearn, of the M.I.T. patent office, March 5, 1947.

[3] Ref. 48. (The reference numbers of the footnotes correspond with the numbered items of the bibliography at the end of this paper.)

[4] The Institute no longer gives up *all* rights in inventions made in a research program, financed entirely by an *outside* party. (See the section "Patent Policy in Regard to D.I.C. Contracts," later in this chapter.)

compulsory reporting of inventions as was the regulation at the University of California for some years.[5]

The motivation for the 1932 patent statement was probably much the same as that indicated in the preceding chapter for many other universities. The economic winds were blowing toward a better-defined policy. At the Institute research related to Vitamin D was in progress [6] and the early Wisconsin experience with the Steenbock patent was not wasted on the committee. A member of the faculty committee said at that time, "I wish to emphasize that *control* and not *royalties* was the objective." [7] By that statement he evidently meant that the purpose of the new patent policy was primarily to avoid misappropriation of Institute discoveries and to encourage their public use.

Up to 1937 some thirty patents had been assigned directly to the Institute and managed by the Patent Committee. In that year an arrangement was made with Research Corporation to handle all the legal and commercial aspects of such Institute inventions as might be turned over to it. [8] Research Corporation opened a Boston office directed by Carroll L. Wilson, former assistant to President Compton and at present General Manager of the U. S. Atomic Energy Commission. The office was moved to M.I.T. during the war; about a year ago the files were moved to the New York office of Research Corporation to coordinate all patent management in a separate division there.

The original ten-year agreement between the Institute and Research Corporation was renewed recently. During 1946 the earlier agreement was considered in the light of experience and minor modifications were made, the chief change being a simplification of the method for dividing income between the Institute and Research Corporation.

Research Corporation cannot relieve the Patent Committee of many administrative decisions. At the request of the faculty Patent Committee, a new administrative Committee on Patent Management assumed responsibility in 1942 for making policy for patent management.

During the war a patent office with two patent attorneys working on a part-time basis was set up in the Division of Industrial Cooperation to handle patent problems arising out of the government research contracts. This office, while finishing up government patent matters, is continuing to give the D.I.C. assistance in its industrial patent problems and to expedite the work of the faculty Patent Committee and the Patent Management Committee.

B. *The M.I.T. Patent Policy*

A patent policy by its nature cannot be written down as an immutable law to be applied without reservation in every case. Whether

[5] Ref. 40, p. 122.
[6] Ref. 45, p. 142 ff.
[7] Prof. William F. Ryan in ref. 32, p. 209. Italics are supplied by this present writer.
[8] Ref. 20.

the problem involves decisions of ownership, licensing, or patent litigation, the case must be weighed on its own merits. Generalizations here are dangerous, and any 'policy' reported herein should be interpreted only as the trend of the 15 years' Institute experience and as the current approach to the problems.

Quite as much a part of the Institute patent policy as the specific treatment of the problems to be considered below in the general attitude of the Patent Committees toward patents.[9] First, it is their hope that successful patent protection and management can be effected without the Institute and its staff becoming excessively patent conscious. They realize the dangers to university research that might result from careless management of patents or too great emphasis placed upon patenting by staff members. At the same time they believe that a healthy faculty feeling is created in the knowledge that patentable inventions will not be thrown to the patent-sharks or be allowed to go unused. The Institute patent machinery should be considered, they feel, a major institutional aid available to the individual inventor. Finally, they intend that patent delays should not hold up publication through scientific journals. Publication anywhere of information in regard to an invention usually bars obtaining a patent in foreign countries; however, once application is filed in this country publication may be made without losing the foreign patent rights. No case where patenting has seriously delayed publication is known to have occurred at M.I.T.

1. *Ownership*

The formal 1932 statement, already summarized, forms the basis for the question that must be answered in each case; namely, who owns the property rights to an invention or other development at the Institute? Since no formal agreement in regard to patents exists between staff members and the Institute (except in some cooperative research with outside sponsorship) each situation must be treated individually. Some special cases have developed further refinements in the stated policy. For instance, the Institute is considered to have no equity in developments resulting from consulting activities by staff members. As a 1937 statement of the Patent Committee [10] put it, these inventions should be handled as "provided for in the usual code of ethics of consulting engineers." The Institute, believing better teaching results, encourages outside consulting by the faculty, but it can take no responsibility for the success or failure of staff members as consultants. A patent-conscious staff member could reserve for himself the benefits from his patents by making prior arrangements for their commerciali-

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[9] Gleaned from talks with members of the Institute Patent Committees.

[10] Ref. 20, p. 350.

zation with outside concerns in the course of his consulting activities. Such a circumventing of Institute policy is better combated by building faculty good will than by prescribing arbitrary rules of procedure.

The government research contracts, totaling $93,000,000 in the war period,[11] have posed a new type of ownership problem. The Institute has had little to say in its solution except in bargaining for the form of the original contract. The government used two standard patent contract forms: the "long form" and the "short form." The short form states in a few words that all patents developed under the contract shall belong to the government. The long form gives prior rights to all patents to the contractor, i.e. the Institute, with the provision that the government may receive a royalty-free, unrestricted license, and provides that all inventions on which the contractor does not elect to patent be reported to the government. It then may patent them. The Radiation Laboratory operated under a contract provision of the short form, but in most of the four hundred other contracts the long form was used. The Institute, believing that a university makes a sizable contribution to government-sponsored research through the accumulated experience of its faculty and in overhead expenses [12] that cannot be adequately covered by contract provisions, preferred the long form. The first contribution was made less exclusively by M.I.T. in the Radiation Laboratory, the staff of which represented a great many universities. All these considerations are wrapped up with the total problem of government sponsorship of research, and are outside the scope of this study. The interested reader will find a more complete statement of Institute policy in President Compton's report of 1945.

Graduate students who now do theses while employed under a government contract are treated as all other employees rather than as students receiving scholarship aid. By the provisions of the government contract, the Institute is required to be in a position to give to the government licenses on all patentable discoveries developed under the contract.

The rights to inventions from industry-sponsored research will be specifically considered later in this chapter.

2. *Attitude toward Income from Patents*

This attitude is at the heart of any patent policy. The Institute Patent Committee apparently believes that income is secondary to control and development of patentable inventions in the public interest. However, it is realized at the same time that an Institute administrator

[11] Ref. 35, p. 7.

[12] Ref. 35. The M.I.T. Special Bulletin, "Policy Relating to Governmental Contracts for Research at the Institute" (November, 1944), explains the philosophy behind Institute policy re government contracts.

has responsibilities to manage patent property in the same business-like manner as he does other property. Thus, the policy is to receive all possible income that is consistent with the best public relations. Drawing the border-line is the difficulty.

In this connection portions of President Compton's talk to the Alumni Association [13] on January 30, 1943 are pertinent:

We have found how to handle patents in an educational institution of our type in a manner which is clearly in the public benefit and which cannot endanger the institution from a *public relations* or a *legal** point-of-view.... The indications at the present time are that income from this source may be substantial enough to strengthen in a significant way our facilities and opportunities.

The Research Corporation agreement with the Institute provides that 50 per cent. of gross income, after certain special expenses are deducted, is returned on Institute patents. The "special" expenses include costs of foreign patents, of developmental research which the Institute may authorze in writing, and of certain litigation expenses.

Pharmaceutical or public health patents, considered by a few universities as improper sources of income, are not differentiated by the Institute in its patent policy. In practical terms the borderline between a public health patent and an engineering patent is often a tenuous one. Except that public opinion may be especially susceptible to irritation by seeming profiteering on the public's ills, it is difficult to see any theoretical consideration that allows an educational institution to concern itself any *less* with the general welfare in regard to engineering patents than in regard to pharmaceutical discoveries.

3. *Reward to the Inventor*

The standard return to the inventor, established by the M.I.T. Patent Committee, is 7 per cent of the gross returns from patents. Seven per cent follows the pattern set by Research Corporation. The preceding chapter indicated the wide variation among universities in this regard. One patent administrator of long practical experience has said that the 7 per cent gross is, for the average invention, about as good as an independent research foundation's 15 per cent grant. Even for a successful invention 15 to 25 per cent of the receipts, it is estimated, is by necessity expended in completing, developing and protecting the invention.

The Patent Committees [14] intend that the reward to the inventor should serve two purposes:

a. Patentable invention should have its main reward in salary and staff promotions in the same way that unpatentable scientific discoveries are rewarded and in "that support of staff research,

[13] Ref. 22.

* Italics are supplied by this present writer.

[14] Ref. 20, p. 348; ref. 34, pages unnumbered.

which [is] due to those who contribute to the welfare of the Institute and to the advancement of science."

b. The inventor should also participate directly in the proceeds of his invention.

4. *Licensing Policy*

Research Corporation has a policy of giving only non-exclusive and unrestricted licenses. It has been highly successful in inducing industry to use its patents on this non-preferential basis.[15] In view of the fact that Research Corporation has generally limited its acceptance of patents to those of a fundamental or basic nature, just the type of patent that usually demands further developmental expenditures and thus requires exclusive licensing, the success of its non-exclusive licensing policy is especially significant. The Research Corporation's "servicing" program, particularly for the Cottrell electrostatic-precipitation patents, bears part of the developmental costs which business concerns usually believe can be recouped only through an exclusive license. The licensee also can hope to obtain compensation for his own efforts in developing the invention from ownership of later improvement patents.

Cases sometimes arise in which the unrestricted licensing policy is clearly failing to bring a valuable patent into public use. One such case, that of the Van de Graaff patent group, will be discussed in the succeeding chapter. Research Corporation, after conference with the Patent Management Committee of the Institute, may give an exclusive license for a limited period or for a particular use. Although the non-exclusive licensing policy is preferred, the public welfare in the development of a useful invention may demand modification of that policy.

5. *Patent Policy in Regard to D.I.C. Contracts* [16]

Before research sponsored at the Institute by an industrial concern through the Division of Industrial Cooperation is undertaken, a contract specifying the disposition of patent rights is accepted by all parties concerned. Although the original Institute policy statement gave full patent rights to the industry financing a research program *in toto*, even by 1937 "such closed or confidential research projects" were rare. [17] The tendency is to avoid contracts with industry which sign away all Institute rights to resulting inventions. The philosophy behind this stand is apparently twofold.

In the first place, the Division of Industrial Cooperation feels a responsibility to the public and private contributors to the Institute endowment quite as strongly as any momentary responsibility to a particular sponsor. Secondly, the Division of Industrial Cooperation realizes that inventions and discoveries are not spawned simply by the

[15] Ref. 44, p. 9.

[16] From this writer's talk with *D.I.C.* Director, N. M. Sage, March 21, 1947.

[17] Ref. 20, p. 350.

expenditure of funds, but usually result only from accumulated experience in the art, experience toward which the Institute has contributed substantially. Whether a valuable invention is made under a particular contract is highly accidental.

This philosophy is tempered by the practical considerations that industry should be offered some attraction to sponsor worth-while research, and that once an invention is made, at least a limited protection for the manufacturer is often necessary to bring it to public use.

The standard agreement with a cooperating industry which pays complete cost for the research has these patent provisions:

1. The sponsor has the right (or responsibility) to file and prosecute at its own expense all patent applications with assignment to the sponsor.
2. The sponsor has full rights to the invention, with the exceptions that
   (a) to such uses as the invention may have outside the sponsored "field of research" the Institute reserves full rights, the sponsor continuing to hold only the free right to employ the invention in his own business, and
   (b) after 10 years from the date of filing the patent application the Institute has the free right to grant licenses to others.

In effect the sponsor is given free use of any invention resulting from the cooperative research for the life of the patent and a monopoly use of the invention in the restricted "field of research" for about half the patent duration. Ten years was chosen as a sufficient time to recoup the sponsor's expenses and to bring the invention into public use. D.I.C. contracts with industry, although they have totaled about $800,000 annually in recent years,[18] have not been the prolific patent-producers they might be expected to be under these arrangements. Only three groups of patents are now assigned to the industries sponsoring the research. One of these patent groups applies to the improvements on high-frequency capacitors made by Professor Balsbaugh and is assigned to the original sponsor, Aircraft-Marine Products, Inc.

Before the above method for giving preferential treatment to the industrial sponsor was instituted, industry commonly received the patents by assignment, but the Institute reserved the right to license others, in which case the royalty fees were divided equally between the Institute and the sponsor.

6. *Litigation Policy*

Research Corporation, while avoiding aggressive litigation, does everything possible to protect its licensees in the use of its patents. The fundamental nature of the patents it accepts, as noted before, has

[18] Ref. 35.

probably aided in the legal successes of Research Corporation. Another factor is its concern with public interest in licensing. In the case of several Institute patents interference proceedings in the Patent Office and infringement suits have occurred. There appears to be a tendency to settle these disputes outside of court on the basis of the non-legal equities of each situation. For instance, Research Corporation was involved in an interference with Bell Laboratories in regard to the Barrow inventions for a hollow-tube transmission system. The parties to the interferences came to terms before any decision by the Patent Office, with the result that Bell Laboratories paid a sizable lump sum for a non-exclusive license in the communications field. In the case of the Cartwright-Turner patents for reducing light reflections from surfaces of light transmitting articles, such as lenses, investigation in the course of infringement suits against Eastman-Kodak indicated that there was considerable question as to the priority of invention. As a result, the several licensees under the patents were notified that their rights under certain claims could not be protected and further royalties would be collected only on a much narrower basis.

Their semi-public nature is probably an element of both strength and weakness in the position of the Institute and its agent, Research Corporation, in patent litigation. Since public relations must be kept in mind constantly and since the Institute can never really dissociate itself from its agent, litigation cannot be pushed with the legal ferocity which is characteristic of suits between competitive industries. On the other hand, respect for the educational institution, if properly maintained, can keep legal disputes involving a university on a higher plane than industrial conflicts. Evidence indicates that business concerns are willing to bargain (and in many cases to make generous concessions to universities) rather than fight to the bitter end.

CHAPTER IV

HOW THE M.I.T. PATENT POLICY IS CARRIED OUT

Policies are effective only if procedures carry them out as intended by their authors. Patent policies in particular may fail or succeed by the procedures affecting the university's relations with its staff, with the public, and with industry.

The procedures used at M.I.T. in obtaining a patent and in its management will first be outlined briefly. Then a procedural history of representative groups of patents will be traced in detail.

A. *Patent Procedures* [1]

The first step in the patent procedure is taken voluntarily by the individual staff member. Having made a discovery which he believes

[1] Ref. 34, pages not numbered. The illustrative material used in this section comes chiefly from talks with Miss Catherine Ahearn of the M.I.T. patent office.

should be patented, he submits a complete description to the Patent Committee on a standard disclosure form provided by the committee. The Patent Committee then determines probable invention, the division of equity, and the proper inventor reward and refers the case to the Committee on Patent Management. The Patent Management Committee may decide either to prosecute patent application or to waive all Institute rights to the invention in favor of the inventor.

1. *The Patent Committees*

The committees under a single chairman [2] direct M.I.T. patents. The Patent Committee, consisting chiefly of faculty members, has the the three duties already mentioned.

a. It determines inventorship and dates of conception, of disclosure, and of reduction-to-practice from the disclosure statement of the staff member.
b. It weighs the equities of the Institute, of the inventor, and of other parties in the invention.
c. It recommends to the Management Committee the extent of inventor participation in financial returns.

In short, the Patent Committee interprets the formal 1932 policy statement in regard to each case brought to it.

The Patent Management Committee, with membership including the D.I.C. Director, the Treasurer of the Institute, and the Executive Vice-President, handles all the management problems connected with obtaining the patent and putting it to use. The Management Committee may apply any of three alternative methods for the administration of an Institute patent. The Committee may manage the patent directly in the name of the Institute, it may offer it to Research Corporation, or it may turn it over to some other agency or corporation for administration.

Although a few patents, as that for the Brown-Forrester remote-control system, are administered directly by the Institute as a temporary expedient, an outside agency for administration is generally preferred. A standing arrangement is kept with Research Corporation and a very great majority of the Institute patents are assigned to that Corporation. [3] However, the Institute is free to offer only such patents as it wishes and the Research Corporation may refuse to accept an assignment.

In some cases where some company other than Research Corporation

[2] Prof. John E. Burchard was recently appointed chairman of the patent committees.

[3] Some 88 patents and 41 patent applications on Institute inventions are now under Research Corporation administration. These patents fall into about twenty-four groups such as the Van de Graaff group of eight patents and two applications. Only four issued patents are under other administration than Research Corporation.

is better prepared to commercialize the patent, that company may be chosen to act as agent in the same capacity as Research Corporation. For instance, the nine patent applications pending on a low-temperature refrigeration system and liquid-oxygen-making equipment, of which Collins, McMahon, and Keyes are inventors, have been assigned to Arthur D. Little, Inc. The group of inventions was made partly under an O.S.R.D. contract. The Little company is specifically given the right to license other firms; the Institute receives two-thirds of all net proceeds. This assignment, although apparently equivalent to an exclusive license, must be interpreted in light of two facts. A. D. Little Company, as a research and consulting firm, naturally would be interested in licensing manufacturers and, secondly, a controlling share in A. D. Little is owned by the Institute. Before 1936 the Institute gained some of the advantages of the present arrangement with Research Corporation by agreements similar to this one with A. D. Little. The Milas patents for preparation of anti-rachitic substances (Vitamin D) were originally assigned to duPont Company which paid royalties to the Institute and assumed responsibility for all litigation.

2. *Research Corporation*

Research Corporation is a three-headed organization. It is a manufacturing and engineering concern, a foundation dispersing research funds, and a patent-managing agency. With a separate division for handling patents assigned to it by universities, Research Corporation is putting increased emphasis on the last function. It now holds patents in some 49 major technical groups, such as vitamins, textiles, radio communications, and automotive engineering.

Research Corporation, as contrasted with a university research foundation, enjoys two major procedural advantages.[4] It exercises, in the first place, a greater bargaining power by virtue of the large groups of patents which it holds in allied fields. It can drive better bargains with industry than a university can in the case of an isolated patent. Secondly, patent management is its specialty. Research Corporation has standing contacts with industrial users of patents, it has the manpower that universities lack, and it has sufficient patents to gain the efficiency incident to specialization within the patent-management division.

In procedure Research Corporation takes responsibility for all management problems from prosecution of the patent application on. It does no manufacturing or developmental engineering, as it has performed for the Cottrell patents, although its officers are on the lookout for a product which would complement the electrostatic-precipitation equipment which is a cyclical capital good. Further developmental re-

[4] This writer's talk with Dr. Joseph W. Barker, President of Research Corporation, New York City, April 10, 1947.

search on a patented invention, if necessary, is encouraged in two ways:

a. A company, or group of companies, may be found to sponsor further research at a university for which the sponsors receive preferential treatment in licensing the original patent. A group of pharmaceutical companies was organized to finance continued research on Vitamin A by Dr. Milas at the Institute.
b. A company may be encouraged to perform the developmental research by an exclusive license until the company has recouped twice the developmental cost.

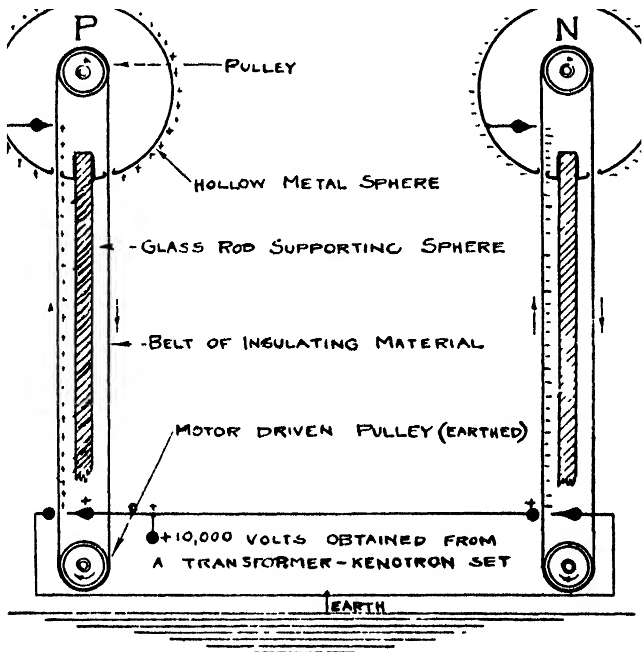

FIG. 1. Essential parts of the Van de Graaff generator. (From ref. 31, p. 150.)

Both methods are frowned upon by Research Corporation; it would much prefer to hold to a policy of non-exclusive licensing.

B. *Policy and Procedure for Certain Patent Groups*

Two patent groups have been selected for a thorough investigation to show how the M.I.T. patent policy is managed. One group involves engineering inventions; the other, public health. Neither is strictly representative of M.I.T. patent procedure as it is now carried out; each is a special case. However, since both groups date from the early thirties, their developments demonstrate the vicissitudes through which patent management at the Institute has passed.

1. *The Van de Graaff Group* [5]

The Van de Graaff electrostatic generator[6] makes possible production of a direct current potential of several million volts and a steady current of a few milliamperes. It has been used in high-voltage X-ray equipment, in investigation of high-voltage insulation and other phenomena, and in nuclear research for accelerating charged particles. The generator consists essentially of a high-voltage terminal resting upon an insulating column and of a rapidly-moving non-conducting belt which transfers electric charge between ground and the terminal to build up a high voltage on the latter. The charge is sprayed on the belt from corona points supplied with a direct current potential from a transformer-rectifier set; it may be removed by similar corona points at the terminal or by a self-induction arrangement.

Application for the basic patent on the Van de Graaff electrostatic generator was filed soon after Prof. Van de Graaff came to the Institute from Princeton in 1931. A bit later he applied for a patent on a system for generation, long-distance transmission, and utilization of high-voltage direct current. The transmission system, shown in Fig. 2, comprised a large-diameter tubular ground conductor and a smaller-diameter high-potential conductor spaced coaxially with the ground conductor. A high vacuum was maintained for insulating purposes within the hollow ground conductor. The high-voltage electrostatic generator and electrostatic motor were enclosed in high-vacuum cases with direct access to the connecting high-vacuum cable. The generator was similar in principle to the belt generator; it substituted a rotating phenolic disk for the belt as the charge-carrying element and had improved methods for depositing as well as removing the charge from the disk. The electrostatic motor applied the principles of the generator in reverse The system was designed to exceed greatly the 250,000-volt and 300-mile limitations on A.C. transmission.

A Federal agency about 1933 indicated interest in the direct current transmission system and hopes were raised that it might be made commercially practicable.[7] Since a clear title to the patent was necessary, an examination of the property rights involved was started. Van de Graaff worked on his electrostatic generator while at Princeton as a Fellow of the National Research Council, but he had conceived the invention earlier. After coming to M.I.T., he continued development of his electrostatic generator for work in nuclear physics with a grant from Research Corporation and, at the same time, worked out the direct-current transmission system outlined above. To the latter development, which of the two first seemed to have commercial value,

[5] Eight patents have been issued and two additional applications have been filed.

[6] Figure 1 shows the essential parts of the Van de Graaff generator.

[7] Ref. 39, p. 41 ff.

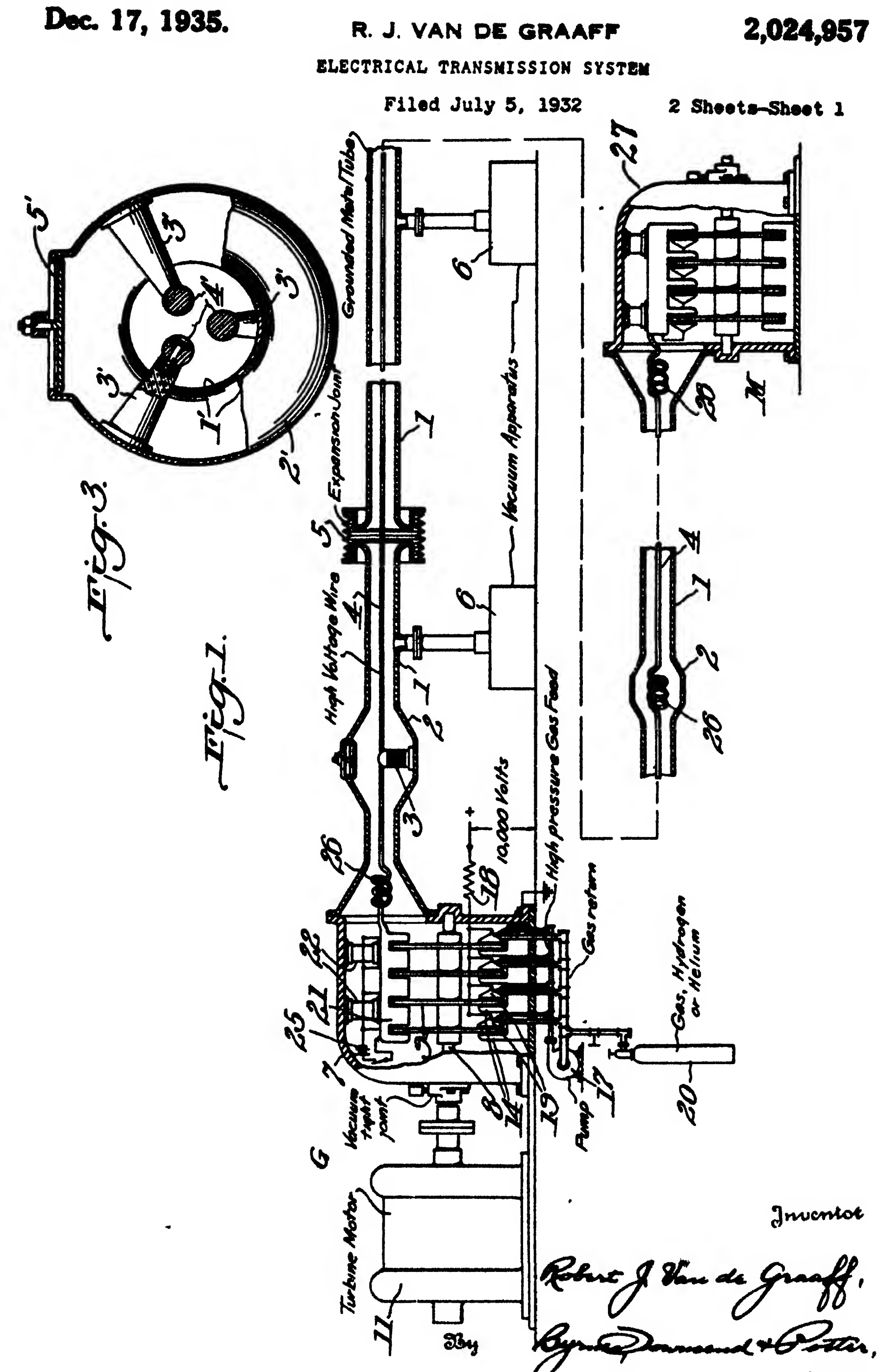

FIG. 2. Patent drawing for the Van de Graaff direct-current transmission system. (From U. S. patent 2,024,957.)

the National Research Council, Princeton, M.I.T., and Research Corporation as well as the inventor had some claim. A contract was prepared to provide for division of proceeds in proportion to the contributions of each party. While it was pending, a difference of opinion arising over whether to give exclusive control to the government in competition with private utilities stopped further commercial development by the government agency. The patent for the transmission system was eventually assigned to Research Corporation, but it has never been used commercially. This case illustrates well the confused equities that may arise in the ownership of university patents.

The development of the high-voltage generator started perhaps less spectacularly, but certainly more steadily, than the development of the transmission system. Dr. John Trump had early become associated with Dr. Van de Graaff at the Institute, and had turned his attention to developing high-voltage X-ray apparatus using the Van de Graaff principle of high-voltage generation. Under Dr. Trump's direction three or more X-ray machines of over one million volts were built for hospitals for cancer therapy.[8] By taking advantage of the high dielectric strength of compressed gases a compact machine with an operating potential of a million volts was built. During the War Van de Graaff directed building of five X-ray outfits of several million volts for examination of heavy munitions castings.

The property rights in the later patents belonged more clearly to the inventors and the Institute than did the rights to the basic patent. The Research Corporation found a fairly great number of licensees for the Van de Graaff-Trump patents. However, in spite of the many licensees, commercialization was slow and royalty returns were insignificant. The few returns were chiefly in the form of flat license fees rather than royalties. Moreover, the royalty fee of 10 per cent of net sales was thought by some to be too high to encourage development. The intervention of the war period soon after this group of patents become available made impossible an adequate trial of the earlier license method. The General Electric Company developed an electromagnetic high-voltage generator of compact size that reputedly could compete with the electrostatic generator for X-ray purposes.[9]

To speed up the commercial development of the high voltage machines, the High Voltage Engineering Corporation, of which Dr. Trump is an officer, was organized within the past two years. The new corporation intends to conduct further developmental research and to manufacture high-voltage equipment. Because considerable expense in development, engineering, and production was necessary for an extended period before High Voltage Engineering Corporation could realize any profits, and because the original inventors held a sizable stock interest

[8] Ref. 36 and ref. 37.

[9] Ref. 37.

in the corporation, all licenses, except a few granted to universities, for the Van de Graaff-Trump patent group were recalled and High Voltage Engineering Corporation received an exclusive license for the limited period of ten years and a non-exclusive license after the termination of that period. The royalty fee is 3 per cent of net sales during the exclusive ten-year period and 2 per cent thereafter.

A further question in this case arises: who shall own further development patents on the Van de Graaff-Trump apparatus that might be made by the High Voltage Engineering Corporation. These developmental patents may be of controlling importance to the commercial value of the high-voltage apparatus. It apparently is felt that the Institute has appreciable property right to these future patents, particularly if an M.I.T. staff member participates in the developments. All future patents for high-voltage equipment assigned to the Institute and Research Corporation go to High Voltage Corporation under the same provisions as those by which present patents are licensed.

The license agreement between Research Corporation and High Voltage Engineering Corporation contains commendable provisions for revocation of the exclusive license in the event the latter corporation participates in any activities which are officially condemned as illegal by the Attorney-General of the United States or the Federal Trade Commission. The agreement also specifies several other practices on the part of the licensee which would automatically invalidate the license:

1. Illegal tie-in sales or tie-in contracts.
2. Illegal blacklisting of potential customers.
3. Combination or agreement with others to fix prices or restrain trade.
4. Failure to use a licensed process if more efficient or less expensive.
5. Failure to serve a geographical area to the best of its ability.

The license agreement is obviously attempting to protect the Research Corporation (and the Institute indirectly) from unfavorable legal action and publicity similar to that which the Wisconsin Foundation received from its exclusive licensing practices. Perhaps a license agreement like this is the solution to the dilemma between maximum patent income and the best interests of the public. The results of this agreement will indicate whether substantial income can be obtained and the public interest preserved by an exclusive license bounded by protective restrictions.

2. *The Milas Vitamin D Patents* [10]

These patents for the preparation of antirachitic substances demonstrate in their administration two special characteristics:

a. These patents in the public health field are often believed to be in a different category from engineering patents. Of course this

[10] Ref. 45.

distinction can be over-emphasized for many engineering inventions, such as the Van de Graaff generator which has been used for X-ray therapeutic purposes, have some application in public health. In a similar fashion, public health discoveries, as the Milas peroxides, may find their chief use in other than the therapeutic applications.

b. These patents now are assigned to and are directly administered by the Institute. They antedate the arrangement with Research Corporation and have thus been handled separately. In their administration they represent early experiments in patent management and, as will be obvious, do not exemplify present Institute patent procedure.

As contrasted with the earlier Steenbock method of irradiating the base substance (perhaps yeast, ergosterol, or milk) with ultra-violet light from an electric arc discharge, Dr. Nicholas A. Milas in 1932 discovered that the activatible substance could be equally well treated by an electrodeless discharge.[11] The ergosterol was introduced into a partially evacuated chamber situated in the field of a high-frequency electric oscillator. The principle by which the Vitamin D activation occurs is not well understood.

In the absence, in the early thirties, of any Institute machinery for patent development, an offer from duPont Company to assume responsibility for the patent administration and to pay royalties was accepted. The patent rights were assigned to duPont, which financed prosecution of the patent application involving interference proceedings.[12] Some royalties were paid to the Institute.

To get the full picture of the situation in regard to Vitamin D, remember that the Steenbock patent controlled a simple and fairly inexpensive process. The Wisconsin Foundation had already, that is before 1932, developed a commercial use for the Steenbock process, and by its licensing policy the Foundation made it both unattractive and legally impossible for the licensee to use a competing patented process, as the Sperti or the Milas process. A memorandum, bearing a 1937 date and written by a duPont executive, is quoted in a published reference cited elsewhere in this paper [13] as stating:

> He [another duPont executive] felt that it would be preferable to deal with the Wisconsin Alumni Foundation since they could be of considerable value to the industry in policing and regulating matters. If, for any reason such arrangements become unnecessary, it would then be possible to consider alternative procedures not involving the use of Foundation patents, that is, we might commercialize the Milas Process.

Although this statement indicates that the Wisconsin Foundation's licensing policy was responsible for the small use of the Milas process, the probable difficulty in producing Vitamin D on an inexpensive, in-

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[11] Ref. 45.

[12] *Loc. cit.*

[13] Ref. 1, p. 88

dustrial scale was apparently a more important deterrent to its use. After the Steenbock process became public property following the court invalidation of the patent, the Milas patent was assigned once more to the Institute. Although one company has taken a license, the Milas process is no longer in extensive commercial use.

This experience points up, in the mind of this writer, the value of Research Corporation in administering university patents in the public interest. Research Corporation could have "insulated" the educational institution from any possible legal or public relations difficulties, while pushing the commercialization of the patented invention as fast as economically possible.

C. *Evaluation of the M.I.T. Patent Procedure*

Research Corporation relieves the Institute of the major part of its procedural difficulties with patents. The advantages of Research Corporation, procedure-wise, have already been noted.

In patent procedure the feelings of the inventor must always be kept in mind. At the Institute both patent committees frequently ask the inventor to express his views in controversial decisions. It has been suggested to this writer that the inventor serve as an *ad hoc* member of both committees while they are considering his invention. The presence of the inventor in each committee while it discussed hotly-contested issues would, however, impair, rather than improve, staff harmony; committee decisions would lose a desirable impersonality. It is better that the committees work in closest touch with the inventor, while remaining impartial juries for controversial questions.

CHAPTER V

AN OVERALL EVALUATION OF PATENT POLICY IN EDUCATIONAL INSTITUTIONS

Two major questions of university research policy, the external problem and the internal problem, were raised in Chapter I. If any single answer to these questions is suggested by the data of the intervening chapters, it is that no patent policy will suit the special needs of all universities, nor perhaps of all departments within a single large university. Patent policy must grow within the climate of each campus; it must adjust itself to the conditions peculiar to each university.

In spite of the natural limitations upon any generalizations, an overall evaluation of university patent policy is in order. Such an evaluation will attempt to judge the merits of patent policy trends as solutions of the external and internal problems of university research.

Two criteria of university patent policies have been applied implicitly throughout this paper and now should be expressly applied. One criterion concerns the public interest which must be preserved and advanced. Society has a stake in the results of scientific research, first, by virtue of the sweeping consequences of invention and, secondly,

by virtue of the social or collective, rather than exclusively individual, nature of the inventive process. "In the realm of organisms all life comes from life, and in social change, including invention, all culture comes from culture; there is no spontaneous generation in either area." So runs the theory.[1] Although this theory of the nature of invention has in recent years been carried to a dangerous absurdity by some jurists to prove that there is no individual invention, and therefore no patent right, it does reinforce the concern of the general public in policy relative to patents.

A second criterion of university patent policy—and it is in reality part of the first—is this: the objectives of the educational institution must be advanced. Because it is a public institution, because it is partially supported either directly by legislative appropriations or indirectly by tax-exemption, a university ultimately shares the objectives of the general public. However, since the end purposes of higher education are more immediate and specific than the indefinite "public interest," this second criterion is a useful and important one.

A. *The Internal Problem*

The relations between the university and its staff in patent matters involves the basic question of division of equities in staff inventions. Deciding the reward to be given the inventor is secondary to that fundamental problem. The right of a university to all patents from research done under its auspices is legally as indisputable as the similar right asserted by the industrial employer. Other factors may make inadvisable complete assertion of this right however.

There are advantages in leaving patent rights to the inventor.[2] He may have more personal interest in pushing aggressively the commercialization of his "brainchild" than would the administrator behind the desk distant from the source of invention. Furthermore, the technical experience of the inventor is frequently important in giving consulting aid to the industrial patent licensees; thus the inventor asks, "why take from me the reins of patent management?" Finally, staff members apparently feel that the freedom from excessive supervision which their research in universities enjoys is a major inducement to lead the academic life. The right to the results of their research, whether patentable or not, is, they argue, part of the academic freedom that differentiates their position in the university from that of the scientist in industrial research.

Offsetting these arguments for letting the staff inventor do as he

[1] Ref. 13, pp. 544–545; ref. 8, p. 421.

[2] The United States Government, which is in many respects in an even poorer position than universities to commercialize patents (ref. 5, pp. 25–28), leaves patent rights to the inventor in many of its departments doing scientific research, reserving to itself only a free, non-exclusive license. The National Advisory Committee for Aeronautics is one such department.

likes with his patents are several disadvantages of such a "hands-off" policy. First, the university inventor seldom has the time or special abilities to be a successful business man. Both the legal protection of a patent and its commercialization are time-consuming and specialized operations. Division of labor is a sound principle in patent management. The individual inventor with a few patents would have difficulty in duplicating the smoothly-working machinery of Research Corporation, or a similar group which can offer an experienced, specialized staff for patent management. Secondly, universities are in a better position to bargain with Big Industry than is the individual inventor. In the third place, and this is the major consideration, it is unfair to place upon a single staff member the responsibility for developing his patent in keeping with public interest and the objectives of the university. Whatever methods he might use in patent management would reflect upon the university with which he was associated. The experience with the Drinker respirator patent at Harvard illustrates a university's involvement in what initially may appear a private matter. It is a university responsibility to decide its research policy.

Although sentiment may argue for complete inventor rights in university patents, practical considerations weigh heavily toward institutional ownership of patents. Some flexibility in dividing equities may permit the best decision in each case. If an inventor demonstrates particular interest in commercializing his invention and if he has the abilities and means at hand for doing so, a patent committee might well consider giving him the "green light" to go ahead on his own. The Institute has, indeed, been the source of small businesses growing from staff discoveries developed by their inventors. The Ruge-DeForest strain gauge may be cited as an example of a staff-commercialized invention. The difficulty in deviating from a fixed policy in dividing patent equities is that inter-faculty relations may be harmed by charges of personal favoritism in committee decisions. For this reason it can logically be argued that a patent committee should attempt no subtle distinctions in equity-division and that all patents made by men on the university payroll should become university property. Such a rigid policy, approaching that applied in industry, but modified to the extent of giving more generous direct reward to the inventor than is customarily given by industry, may best contribute to satisfactory staff relations.

What constitutes "a generous reward" for the inventor is a difficult question. Universities have hit upon no common answer. This writer suggests that, in keeping with an inflexible policy that all patents on inventions developed by staff members belong to the university, a larger percentage of gross returns than the common 7 per cent should be given. The exact figure must be decided upon in light of faculty salaries and policy relative to consulting practices. Probably quite as

important as his financial participation in the patent income is the inventor's feeling of participation in managing "his" patent. To that end the patent committees in doing business in regard to his patent should constantly seek the advice of the inventor.

B. *The External Problem*

The external patent problem, how shall research findings be put to public use, is more difficult to solve than the internal problem. That universities must see that the results of their research come into public use cannot be doubted; that patents are a proper way to encourage such public use is also generally accepted, even by most of those universities which refuse to profit from patents. Furthermore, universities need no longer experiment to find the best mechanism for patent management. The legal and administrative advantages of a non-profit and socially-conscious agency, such as Research Corporation, are firmly established.

A troublesome question remains. To what extent should university patents be pushed as a source of income? The key administrators of each university must answer this question, weighing its financial needs against subtle considerations of public relations. In general in seeking patent income the university is limited by public opinion to giving only non-exclusive licenses. Exclusive licenses may be necessary for social reasons, that is, in order to bring inventions into the widest possible use. However, this device cannot be used to maximize income from patents. Consequently, the only way for a university to push patents for income is by strengthening the administrative methods for patent management. Research Corporation's success in inducing companies to accept non-exclusive licenses and its efficiency in patent management should particularly appeal to a university wishing to earn sizable patent income without legal or public-relations difficulties.

This paper has admittedly proposed no significant solution to the external problem. However, if the alternatives available to the patent administrator are made a bit clearer, if the issues are more clearly drawn by the preceding discussion of actual university experiences, each university may be aided in its individual decision in regard to patent income.

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APPENDIX

SURVEY OF PATENT POLICY IN AMERICAN UNIVERSITIES AND COLLEGES

University or College	Policy Toward Patent Income	Patent Ownership*	Agency Used for Patent Management	Policy on Patents from Outside-Sponsored Research	Reward to Inventors	Remarks
University of Alabama[1]	Affirmative	I	A faculty patent committee		"Some percentage" of net profits	$200 spent by Univ. in developing an invention gives patent rights to the Univ.
Alabama Polytechnic Institute[2]	Affirmative	U	Auburn Research Foundation	Flexible policy; up to exclusive license may be given.		
University of Arizona[3]	Affirmative	I	Faculty patent committee and Research Corp.	Written agreement specifies disposition of patents.	7% gross	If inventor chooses not to use Research Corp., he must pay 10% of gross earnings to the University.
University of California[4]	Affirmative	I	Research Corp.		7% gross	From 1926–1931 reports of patentable inventions by faculty members was required; since then, inventions have been left to the inventor, but the Univ. will accept assignment.
California Institute of Tech.[5]		I_e	University Board of Trustees		Most of net proceeds	
Carnegie Institute of Tech.[6]	Affirmative	U	Board of Trustees	License fee remitted to contributing companies until such remissions equal total contributions.		Up to 1943 no control over patents except those from the Coal and Metallurgical Laboratories.
Case School of Applied Science[7]	Affirmative	U	Research Corp.	All patent right reserved to the sponsor.		

N.B. Blanks mean that no information is available.

* *Code for "Patent Ownership."*

I All patent rights expressly left to the *inventor* (subscript "e" indicates that assignment to the university or associated agency is encouraged).

U Rights to patents that are the direct results of the staff member's "regular duties on University time and at its expense" are expressly the property of the *university*.

This appendix is admittedly an oversimplified compilation of many complex points-of-view upon the university patent problem. The inherent limitations of a chart of this type have been pointed out earlier. (*Supra* Chap. II, Sec. C.)

University or College	Policy Toward Patent Income	Patent Ownership*	Agency Used for Patent Management	Policy on Patents from Outside—Sponsored Research	Reward to Inventors	Remarks
University of Chicago[8]	Negative—Neither the University or the individual staff inventor may profit from patents.					
University of Cincinnati[9]	Affirm-ative	I	U. of C. Research Foundation	Patentable results of fellow-ships go to the donor.		The General Development Laboratory was owned jointly by a food-distrib-uting company and the Univ. Sub-stantial patent profits have accrued to the Univ.
Colorado School of Mines[10]	Neutral (only the Colorado territorial rights are reserved by the School)					
University of Colorado[11]	Affirm-ative	I		Sponsor may elect to receive all patent rights by paying patents charge of 20% of total costs and inventor fee of 100 dollars.		
Columbia University[13]	Affirm-ative	I_e	University Patents, Inc. and Research Corporation			Original Administrative board was organized to receive Vitamin D pat-ents from a staff member. The staff of the Medical School is forbidden to patent.
University of Connecticut[12]	Afirm-ative	U	Research Founda-tion (1945)		At least 20% net	
Cornell University[14]	Affirm-ative	I_e	Cornell Research Foundation, Inc. (1931)	No previous agreement exc. sponsor is assured of "preferen-tial treatment." Exclusive li-cense for short term may be given, exc. for patents from the experiment station.	15% net	Net revenue of "3 or 4000 dollars a year" is reported; exclusive license for the patent life will not be given. The Medical College and its staff do not take patents for profit. Net Income from royalties is returned to depart-ment.
University of Florida[15]	Affirm-ative	U	Faculty Research Council and the Univ. Board of Control	Sponsor may choose among: 1. Free, non-exclusive license. 2. Joint ownership with the University. 3. Full ownership on payment of a patents fee.	At least 25%	Inventor proceeds from thesis patents are divided ⅔ to the faculty member directing the research and ⅓ to the student. Reports of inventions re-quired.

University or College	Policy Toward Patent Income	Patent Ownership*	Agency Used for Patent Management	Policy on Patents from Outside—Sponsored Research	Reward to Inventors	Remarks
Fordham University[16]	Affirmative	See remarks				For estimated proceeds: a. Less than $6000—left to staff member, who is assessed for overhead plus 10% net fees. b. More than $6000—equity decided by an *ad hoc* committee. Inventor and Univ. *may* divide costs and proceeds 50–50.
Georgia School of Technology[17]	Affirmative	I_e	Industrial Development Corporation, Inc. (Private non-profit)	All rights to sponsor who pays all costs. Fixed agreement with sponsor before research is undertaken.	15 to 33% net proceeds	
Harvard University[18]	Negative	I		Research directors may make their own arrangements with industry.		Neither Univ. nor staff members may profit from patents on therapeutics. Other patents are the private concern of the inventor. Research Corporation administers some public health patents for control, but on a non-profit basis.
University of Illinois[19]	Affirmative	U	Board of Trustees for policy; Faculty Patent Committee for management	Preferential treatment; title kept by the Univ.	"Liberal share"	Flexible management and policy for patents.
Illinois Inst. of Tech.[20] (Armour Research Foundation)	Affirmative			All rights go to the sponsor.		Large amount of industrial research is done by special research staffs.
Iowa State College[21]	Affirmative	U	Iowa State College Research Foundation, Inc.	Title given to the sponsor; only shop rights kept by the College.	15% net (50% before 1938)	Reserve fund for litigation not to exceed 5% gross receipts.
Johns Hopkins University[22]	Negative (Patent-holding by the university or associated agency considered undesirable.)					The Medical School faculty may not patent; the engineering staff may.

University or College	Policy Toward Patent Income	Patent Ownership*	Agency Used for Patent Management	Policy on Patents from Outside—Sponsored Research	Reward to Inventors	Remarks
Univerity of Kansas[23]	Affirmative	I_e	Univ. of Kansas Research Foundation	"Preferential consideration" for sponsor—all patent rights in some cases.	At least 15% net	Staff members are under no obligation to assign patents to the Research Foundation. (Kansas State College follows similar policy in all regards.)
LaFayette University[24]		I_e	LaFayette Alumni Research Foundation (1943)			
Lehigh University[25]	Affirmative	U	Board of Trustees	Preferential treatment through prior opportunity to buy or license.	50%	Lehigh Research Foundation does not manage patents.
University of Maine[26]	Affirmative	U		Explicit contract usually gives all rights to sponsor.	"Just compensation"	Policy formed in 1938 in emulation of Penn. State policy.
MIT.	Affirmative	U	Patent Committees and Research Corporation	10-yr. exclusive license to sponsor.	7% gross	
University of Michigan[27]	Affirmative	U	The University	Sponsor may purchase patent for additional patent fee of 10% of research costs.		
Michigan State College[28]	Affirmative	U	Faculty Patent Committee and State Board of Agriculture		20% gross	
University of Minnesota[29]	Affirmative	I_e	Univ. of Minn. Research Foundation	In some cases exclusive license given sponsor with royalty for the University.	About 10% net†	Profits from thyroxin patents were used for research in allied fields; such is general policy of the Univ.
Ohio State University[30]	Affirmative		Ohio State Research Foundation (1936)		Adjusted in each case	No fixed policy; each patent and the circumstances leading to it are separately considered.

† The inventor of the thyroxin patent received 10% net; the total net profits were divided equally between the manufacturer and the university.

University or College	Policy Toward Patent Income	Patent Ownership*	Agency Used for Patent Management	Policy on Patents from Outside-Sponsored Research	Reward to Inventors	Remarks
University of Pennsylvania[31]	Negative (particularly toward medical patents)			Free, nonexclusive license only.		A 1932 policy statement forbade either Univ. or staff member to profit from patents. Later statements narrowed the restriction to public health patents.
Princeton University[32]	Affirmative	U	Patent Committee and Research	Maximum consideration is free, exclusive license in the technical field of the research. ($100 Inventor fee paid by the sponsor.)	7% gross	Special consideration in awarding patent income given to the dept. producing the patentable discovery.
Purdue University[33]	Affirmative		Purdue Research Foundation	Patents rights kept by the Univ.; "small" royalties paid by sponsor.	"Adjusted equitably" (up to ⅓ gross, in cases)	Flexibility in policy and procedures to meet each situation.
Rhode Island State College[34]	Affirmative	U	College research comm.	College and investigator participate in profits, if patent is assigned to sponsor.	Not fixed	
University of Rochester[35]	Affirmative	U	A patent comm. and a patent managing corporation	Variable—up to a free-exclusive license may be given.	"Some share"	
Rutgers University[36]	Affirmative		Endowment Foundation, Inc.		"A portion of net proceeds"	
St. Louis University[37]	Affirmative		Committee on Grants for Research (in School of Medicine)		0 (from theelin patents)	Dr. E. A. Doisy's theelin patents (1930) developed basis for later patent management methods. Comm. on grants administers testing lab.
Stanford University[38]	Affirmative	I	Faculty-administration patent comm.			Drs. Hanzlik and Melutens (1932) assigned Univ. rights to Iodobismitol, a therapeutic agent.

University or College	Policy Toward Patent Income	Patent Ownership*	Agency Used for Patent Management	Policy on Patents from Outside—Sponsored Research	Reward to Inventors	Remarks
University of Tenn.[39]	Affirmative	I_e	Univ. of Tenn. Research Corp.		"Liberal reward"	Univ. officials directly control Research Corp. of the Univ.
University of Texas[40]	Indirect profiting	I	Formerly Research Foundation; now Board of Trustees	No special favor given sponsor.	See Remarks	Inventor owns all patent rights, but must pay following portions of net royalties to Univ.: Under 1,000 dollars, net royalties, 0%; 1,000–5,000 dollars, net royalties, 10%; 5,000 up dollars, net royalties, 20%.
Texas A. & M. College[41]	Affirmative	U	Texas A. & M. Research Foundation	Sponsor gets all patents on payment of total costs, plus 20% and $50 inventor fee.	"At least 20% net after paying costs of the research.	
University of Toronto[42]	Neutral	I	Special committees within university	"Preferential treatment as prospective licensee or assignee."	0	No pressure on staff member to assign patent to the University.
Virginia Polytechnic Institute[43]	Affirmative		V.P.I. Research Foundation	Patent rights go to the sponsor paying all costs.	50% net	Virginia Tech. and the inventor are considered to have equal rights in patents from institutional research.
State College of Washington[44]	Affirmative		State College of Washington Research Foundation			
University of Wisconsin[45]	Affirmative	I_e	Wisconsin Alumni Research Foundation [separate from (1925) university]		15% net	
Wittenburg College[46]	Affirmative		Wittenburg Research Institute		50% net	Research workers receive no income except from fees from sponsor and from patent profits.
Yale University[47]	Negative		A faculty patent committee and the Yale Corporation (the general governing board)	No standard form. Dep't heads may make own arrangements.		Each patent case is considered individually by the patent committee. Research Corp. used at times.

REFERENCES FOR THE APPENDIX

N.B. p.n.—pages not numbered. "NRC files" will refer to pamphlets and unpublished material in the National Research Council files for the Palmer survey.

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21. Ref. 44, App., pp. 10-12; ref. 40, p. 111.
22. Ref. 40, p. 106; ref. 8, pp. 63-66.
23. Ref. 44, App., p. 13; ref. 34.
24. Ref. 44, App., p. 13.
25. Ref. 40, pp. 106-5.
26. Ref. 44, App., p. 15.
27. Ref. 27, p. 5.
28. Ref. 34, p.n.
29. Ref. 40, pp. 113-115; ref. 34, p.n.
30. Ref. 27, App., p. 18.
31. Ref. 40, pp. 124-5; ref. 38, p. 55.
32. Ref. 27, p. 4; ref. 34, p.n.
33. Ref. 44, App., pp. 19-20.
34. Ref. 34, n.p.
35. NRC files.
36. Ref. 44, App., p. 21.
37. Ref. 40, p. 101.
38. Ref. 40, p. 125; ref. 27, p. 5.
39. Ref. 34, p.n.
40. Ref. 34, p.n.
41. Ref. 34, p.n.
42. Ref. 40, p. 100.
43. Ref. 44, App., pp. 23-6.
44. Ref. 44, App., p. 26.
45. Ref 47 and 48; ref. 44, App., p. 27.
46. Ref. 44, App., p. 28.
47. Ref. 27, p. 5; NRC files.

THERMODYNAMICS, PART I: THE SECOND LAW FROM THE STANDPOINT OF THE EQUATION OF STATE.

BY

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ABSTRACT.

In this paper the concepts "equilibrium" and "reversibility" are examined from the standpoint of the number of independent variables which define a thermodynamic system. The endeavor is made to show that the number of independent variables which are selected is crucial to the basic concepts as well as to the second law of thermodynamics. If this number is less than the minimum required to define the system completely, the equation of state for the internal energy will be incomplete. It is our practice to employ incomplete equations of state, and this results in processes which we consider to be irreversible. The Kelvin-Planck and Clausius principles are shown to be somewhat related to the first law of thermodynamics, and their validity arises from the fact that the equations of state which are used are incomplete. If these equations were complete, that is, if the number of independent variables were not less than the minimum required to describe the systems completely, we should have complete conversion of heat into work, as well as work into heat. The difficulties in realizing complete equations are considered.

The entropy concept is considered in detail, and it is shown that there are serious limitations in its application.

1. INTRODUCTION.

The following are two fundamental principles which are at the basis of, and sometimes considered as statements of the second law of thermodynamics:

1. Kelvin-Planck principle: It is impossible to devise a system that, operating in a cycle, will produce no effect other than the extraction of heat from a reservoir and the performance of an equivalent amount of work.

2. Clausius principle: It is impossible to devise a system that, operating in a cycle, will produce no effect other than the transfer of heat from a cooler to a hotter body.

In the classic treatment, a Carnot cycle is employed with one or both of the above principles to develop the entropy concept, and it is shown that $TdS \geqq dq$, where dS is an infinitesimal increase in entropy, T the absolute temperature, and dq an infinitesimal quantity of heat added to a system. This expression is usually taken as the mathematical form of the second law. Actual processes taking place in nature are considered irreversible, and the form $TdS > dq$ applies to

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them, whereas for ideal reversible processes the equality $TdS = dq$ will hold. True reversibility, it is considered, is never realized in practice, although under carefully controlled conditions such a process may be approximated very closely.

The second law of thermodynamics presents a view of nature which is unique, for, according to this law, natural processes are asymmetric, entropy always increases, and the available supply of energy for useful work is constantly decreasing. The validity of this law rests on the negation of certain processes, which as a proof is quite different in character from most other basic physical laws. It is for this reason that one cannot help but contemplate its origin and meaning. We may also examine the basis for the validity of the second law, and ask what notion has been introduced, consciously or by implication, in the development which has led to this law. Such are the questions we wish to consider in this paper. We shall limit ourselves to macroscopic states.

2. EQUILIBRIUM AND REVERSIBILITY.

Two terms basic in classic thermodynamics, and which enter even before the introduction of Carnot or any other cycle or process, are "equilibrium" and "reversibility."

The process involved in a Carnot cycle is quasistatic; that is, the system is assumed to be in equilibrium at each stage of the process. We must, therefore, be able to recognize a state of equilibrium before we can consider the performance of a Carnot cycle or any other thermodynamic process. Carnot, himself, must have judged equilibrium by phenomenological criteria. That is, after deciding on the necessary controls or independent variables for the system, equilibrium must have been considered attained when those variables were constant over a period of time. The criteria which we now generally employ for macroscopic systems, such as that state where S is a maximum, E (internal energy) and $F = E - TS$ (free energy), etc., are minima, cannot be considered at this stage because such criteria are *consequences* of our preliminary considerations, and it would certainly be illogical to invoke them at this early stage. We must, therefore, first be able to define our system completely; mathematically, we must have a set of n independent variables, $\alpha_1, \cdots, \alpha_n$ for the system, and this number must be not less than the *minimum* required to completely define all its properties (including internal energy and temperature). *Equilibrium may then be defined as that state where $\alpha_1, \cdots, \alpha_n$ are constant with time.*

A "reversible" process, as applied to a thermodynamic system, may be defined as one that is performed in such a way that at each stage of the process any infinitesimal displacement in one direction may be restored by an infinitesimal displacement of the same magnitude in the opposite direction, and in so doing the system and its surroundings are

restored to their original states. This implies, of course, that we know enough about the controls of the system and the surroundings to be able to recognize when we have restored them to their initial states. For example, if the system serves as a medium for transforming heat into work, it is obvious that if we cannot identify the system completely, the latter may retain some of the energy, or lose some of its own energy in the process, and therefore the system and its local surroundings will not necessarily be restored to their initial energy states. Before we can undertake the study of cyclic reversible processes, we must be able to define our system completely. As above, we must have a set of n independent variables, and this must be not less than the *minimum* number required to define completely all its properties. If only $(n - 1)$ variables, say, are accounted for, we can never be certain that the system has been carried through a complete cycle. On examining the $(n - 1)$ variables it may appear to us, for example, that the energy content of the system has been restored to its initial state; but, because of a possible change in the nth variable (which, let us say, has been ignored or of which we are unaware), the system may have retained some energy and the process will not be reversible. Therefore, before we are able to judge whether in a process the internal energy and its other properties have been restored to their original value, we must be able to define the energy for our system by an equation of state

$$E = \phi(\alpha_1, \cdots, \alpha_n) \tag{1}$$

and similar equations for its other properties. The explicit form of ϕ need not be known, but it must be complete as far as the number of independent variables is concerned.

Suppose we are aware of the existence, or use only $(n - 1)$ of these variables in our experiments. In holding these fast, one of two possibilities may arise: (*a*) the extra variable may remain constant when the others are constant; or (*b*) it may be changing constantly or after some time interval. In the first case, even though α_n will have a fixed value when $\alpha_1, \cdots, \alpha_{n-1}$ are fixed, we are not certain that the value of α_n will be the same for different experiments, and E will not be single-valued under such conditions. We will have difficulty in developing an equation of state built on $\alpha_1, \cdots, \alpha_{n-1}$ where E is not single-valued. The problem becomes simple once we recognize and use the extra variable α_n. In previous papers (**1**)[2] the writer has shown how this may be applied to specific substances, and he has shown that the states resulting from different constant values in the additional variables may be considered like those we usually call "metastable."

The second case may be considered as representing constant instability or instability at a time when α_n changed. At the instant when

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[2] The boldface numbers in parentheses refer to the list of references appended to this paper.

α_n did change, there will result a change in E, which is very much like that referred to in thermodynamics as a "spontaneous" change.

3. THE ASSUMPTION OF A TWO-VARIABLE SYSTEM.

Realizing the fundamental part which the number of independent variables in the equation of state plays in thermodynamics, let us examine the classic development of the second law. For simplicity we shall consider a closed system (the sum of the component masses to be constant) of a single gaseous or liquid phase. It is commonly assumed that such a system in equilibrium is defined completely by two independent variables, and the equation of state is written in the form $p = f(v, T)$, and $E = g(v, T) = h(p, T)$, etc., where E, p, v, and T are the internal energy, pressure, volume, and temperature, respectively. In other words, a closed single-phase system is said to be in equilibrium if two independent variables are fixed, and this system may be carried through a complete Carnot or any other cycle, provided that these variables are restored to their initial values. Thus we proceed to develop the mathematical form of the second law along classic lines.

It has, however, been generally recognized that certain states are excluded by this assumption. Examples of such states are: (*a*) a stoichiometric mixture of H_2 and N_2 gases free from a catalyst where little or no NH_3 is formed; (*b*) supercooled liquid; etc. Because of the unusual behavior of these states (the writer classes them all as "metastable" states), they are excluded from the thermodynamic considerations at the very outset (2). Later developments in thermodynamics show that such states are not those of stable equilibrium, because they do not satisfy the required conditions that the thermodynamic potential functions be at minimum (or maximum) values. *But the weakness in this argument is that these criteria for stability also rest on the assumption of a two-variable system and therefore cannot rule out states that have been excluded from the very beginning.* We thus see that the classic approach to thermodynamics has been restricted from the very outset.

The basis for the two-variable equation of state would appear to be historical. The early developments in thermodynamics were concerned largely with gases, and we have the work of Boyle, Mariotte, Gay-Lussac, and others who developed the perfect gas law based on an equation with two independent variables. Van der Waals had later developed an improved equation of state, but still with only two independent variables. All subsequent equations retained the basic idea of a two-variable system. It may be safely stated that of the many equations that have been proposed to date, all are of the two-variable type and none has a degree of generality that would permit us to use it for many systems over a large range of pressure, volume, and temperature. Our experimental precision nowadays is far greater than can be accommodated completely by any known rational equation of state based on two variables.

At this point we must not fail to mention the work of De Donder (3) and his co-workers. He did, in a sense, enlarge his equation of state and wrote, for example, for the internal energy of a closed system

$$E = \psi(T, p, \xi) \tag{2}$$

where ξ is the degree of advancement of the chemical reaction taking place within the system. This makes it a three-variable system, similar to those considered by the writer in his previous papers. He has, however, not carried out the full implication of this assumption, but has made the usual fallacy, as we shall illustrate.

De Donder introduces the concept of uncompensated heat, dQ', of the reaction (first used by Clausius) and which is defined by the equation

$$TdS - dQ \equiv dQ'. \tag{3}$$

For an irreversible process, he assumes that $dQ' > 0$; for a reversible process he assumes $dQ' = 0$. He then uses the term A, which he calls affinity and defines it by the equation

$$\frac{dQ'}{dt} = \frac{dQ'}{d\xi} \cdot \frac{d\xi}{dt} = A \frac{d\xi}{dt} \tag{4}$$

where t is the time. De Donder deduces from Eqs. 3 and 4 the following: (1) if $d\xi/dt$ and A are both zero, the system is said to be in a state of true equilibrium; (2) if $A \neq 0$, while $d\xi/dt = 0$, the system is said to be in a state of false or metastable equilibrium. For both types of equilibrium $\xi =$ constant. Obviously his criterion of equilibrium is so designed as to be consistent with the ordinary practice of making S a maximum. In the section of this paper entitled "The Entropy Concept" we shall examine the role of S in the enlarged equation of state, and show the incompatibility in De Donder's assumption.

4. AN ENLARGED EQUATION OF STATE.

In a previous paper (1) the writer considered the reasons why a two-variable equation of state for a closed system is not satisfactory. Consider, first, the perfect gas law. None of the permanent gases satisfies this law to the accuracy of the experimental data over an appreciable range of p, T. There is one characteristic of true gases which is not usually ascribed to a perfect gas, and that is the ability to undergo transformations (liquefaction, ionization, etc.). If such characteristics are to be taken into account, the writer has shown that the equation of state would have to employ more than two independent variables. Any simple system which occurs in nature will have at least one type of transformation. Take the case of helium, to which the perfect gas law applies best. For low enough temperatures this gas undergoes a liquefaction, and for high enough temperatures it may be ionized. A closed system which includes a liquid and its saturated vapor, although

complex from many standpoints, has a single type of transformation and may be treated thermodynamically like dissociation within a single gaseous phase. This has been considered by the writer, and he has shown that in order to satisfy a closed system with a single type of transformation we must employ three independent variables and write the equation in the form

$$E = f(p, x, T) = F(p, v, T) \tag{5}$$

where E and v are the internal energy and volume, respectively, of the entire system, and x the degree of transformation. For a closed liquid-vapor system, x may be defined as the ratio of the mass of the vapor to the total mass of the system. For a single closed gaseous system subject to dissociation, x may be the ratio of the mass of the dissociated components to the total mass. The writer points out that this equation may include "metastable" as well as the ordinary states.

For systems with several types of transformation taking place simultaneously, an equation involving more than three independent variables would be required. Let x_i be the degree of transformation for the ith type within a given closed system; for n distinct types of transformation the equation of state would be

$$E = f(p, v, x_1, \cdots, x_n) = \phi\,(p, T, x_1, \cdots, x_n) \tag{6}$$

and

$$T = \psi(p, v, x_1, \cdots, x_n) \tag{7}$$

where v is the total volume of the system.

The simplest system is one with a single type of transformation, for in nature even the so-called permanent gases have at least one such type. For such a system, write $E = f(p, v, x) = \phi(p, T, x)$ and $T = \psi(p, v, x)$. How will this affect the development of the second law as based on Carnot's cycle? An isothermal will be a surface in a three-dimensional space, and the projection of this surface on the (p, v)-plane will, in general, be an area rather than a single line. Again, the conditions for an adiabatic will be

$$dq = dE + pdv = (\partial E/\partial p)_{v,x}dp + [(\partial E/\partial v)_{x,p} + p]dv + (\partial E/\partial x)_{p,v}dx = 0 \tag{8}$$

which, in combination with $T = \psi(p, v, x)$, reduces the adiabatic to the form $T = f(p, v)$. In the (p, v)-plane this will be an area, and not a single line. We see, then, that to draw a Carnot cycle in the (p, v)-plane of finite area which is to represent a fixed amount of work would have no meaning. Also, Carnot's theorem which states that no engine is more efficient than a Carnot engine when operating between the same temperatures, and its corollary which states that all Carnot engines are equally efficient when operating between the same temperatures, would likewise have no meaning.

5. THE ENTROPY CONCEPT.

We shall consider Carathéodory's work below in the section entitled "Carathéodory's Development of the Second Law." Here we shall examine the formal meaning of entropy and its part in the enlarged system.

The gist of the mathematical form of the second law is that, for a reversible process the quantity dq, although in itself not a perfect differential, has an integrating factor T (or some function of T) such that $dS = dq/T$ is a perfect differential and S becomes a characteristic thermodynamic potential function for the system.

A differential of a function of two independent variables will always have an integrating factor. For a function with more than two independent variables, the existence of an integrating factor is the exception. Considering the difficulties (at least from the classic standpoint) that arise in developing the second law when more than two independent variables are required to define each phase, it is important that we inquire into the potential character of S for such cases. We shall limit ourselves to fluid systems, either a single gaseous phase or a liquid-vapor system. In the latter case we shall neglect the differences in pressure that may exist between the top and bottom of the containing vessel because of the added weight of the liquid. In both cases, we shall assume that the system is in equilibrium, that is, the independent variables which define the system to be constant with respect to time, and we shall assume that the pressure is uniform throughout the system. If within each system the transformation is of a single type, the writer has shown that each may be completely described by three independent variables, such as (p, T, x) or (p, v, T), where x is the degree of transformation, and $v = V/M$, where V is the volume and M the mass of the *entire* system. We may write $E = f(p, T, x) = F(p, v, T)$; and therefore $v = \phi(p, T, x)$ and $x = \psi(p, v, T)$. x may change with a change in v only, when $p, T =$ constant; or, x may change while $v =$ constant, but where p, or T, or both change. Therefore, changes in x need not always be associated with mechanical work; this work must ultimately be accounted for by a change in v only, and for a quasistatic process the mechanical work done by each system will be $W = \int pdv$.

Consider, first, the (p, T, x) space. The first law of thermodynamics requires that E form an exact differential; therefore,

$$
\begin{aligned}
dq &= dE + pdv \\
&= \left[\left(\frac{\partial E}{\partial p}\right)_{T,x} + p\left(\frac{\partial v}{\partial p}\right)_{T,x}\right]dp \\
&\qquad + \left[\left(\frac{\partial E}{\partial T}\right)_{x,p} + p\left(\frac{\partial v}{\partial T}\right)_{x,p}\right]dT + \left[\left(\frac{\partial E}{\partial x}\right)_{p,T} + p\left(\frac{\partial v}{\partial x}\right)_{p,T}\right]dx. \qquad (9)
\end{aligned}
$$

Defining entropy in the usual manner, we have

$$dS \equiv \frac{dq}{T} = Pdp + QdT + Rdx \tag{10}$$

where

$$P = \frac{1}{T}\left[\left(\frac{\partial E}{\partial p}\right)_{T,x} + p\left(\frac{\partial v}{\partial p}\right)_{T,x}\right] \tag{11}$$

$$Q = \frac{1}{T}\left[\left(\frac{\partial E}{\partial T}\right)_{x,p} + p\left(\frac{\partial v}{\partial T}\right)_{x,p}\right] \tag{12}$$

$$R = \frac{1}{T}\left[\left(\frac{\partial E}{\partial x}\right)_{p,T} + p\left(\frac{\partial v}{\partial x}\right)_{p,T}\right]. \tag{13}$$

The following conditions, both necessary and sufficient for dS to be a perfect differential, are that P, Q, R have derivatives, and that

$$\left(\frac{\partial P}{\partial T}\right)_{x,p} = \left(\frac{\partial Q}{\partial p}\right)_{T,x}; \qquad \left(\frac{\partial Q}{\partial x}\right)_{p,T} = \left(\frac{\partial R}{\partial T}\right)_{x,p};$$
$$\left(\frac{\partial R}{\partial p}\right)_{T,x} = \left(\frac{\partial P}{\partial x}\right)_{p,T}. \tag{14}$$

Applying these conditions to Eqs. 11, 12, and 13, we have, on simplification:

$$\left(\frac{\partial E}{\partial p}\right)_{T,x} + p\left(\frac{\partial v}{\partial p}\right)_{T,x} + T\left(\frac{\partial v}{\partial T}\right)_{x,p} = 0 \tag{15}$$

$$\left(\frac{\partial E}{\partial x}\right)_{p,T} + p\left(\frac{\partial v}{\partial x}\right)_{p,T} = 0 \tag{16}$$

$$\left(\frac{\partial v}{\partial x}\right)_{p,T} = 0. \tag{17}$$

Integrating Eq. 17, we find that $v = f(p, T)$, but independent of x, which is a contradiction of our hypothesis that v is a function of p, T, x. Substituting Eq. 17 in Eq. 16 we have $(\partial E/\partial x)_{p,T} = 0$ or $E = g(p, T)$, but independent of x, which is a contradiction of our hypothesis that E is a function of p, T, x. We conclude, then, that dS is not a perfect differential in the (p, T, x) space.

We find also that dS is not a perfect differential in the (p, v, T) space, as follows. Write

$$dS \equiv \frac{dE + pdv}{T} = Pdp + Qdv + RdT$$

where

$$P = \frac{1}{T}\left(\frac{\partial E}{\partial p}\right)_{v,T}; \qquad Q = \frac{1}{T}\left[\left(\frac{\partial E}{\partial v}\right)_{T,p} + p\right];$$
$$R = \frac{1}{T}\left(\frac{\partial E}{\partial T}\right)_{p,v}. \tag{18}$$

The necessary and sufficient conditions for dS to be a perfect differential are that P, Q, R have derivatives, and that

$$\left(\frac{\partial P}{\partial v}\right)_{T,p} = \left(\frac{\partial Q}{\partial p}\right)_{v,T}; \quad \left(\frac{\partial Q}{\partial T}\right)_{p,v} = \left(\frac{\partial R}{\partial v}\right)_{T,p};$$

$$\left(\frac{\partial R}{\partial p}\right)_{v,T} = \left(\frac{\partial P}{\partial T}\right)_{p,v}. \tag{19}$$

Applying these to Eq. 18, we have on simplification

$$\frac{1}{T} = 0 \tag{20}$$

$$\left(\frac{\partial E}{\partial v}\right)_{T,p} + p = 0 \tag{21}$$

$$\left(\frac{\partial E}{\partial p}\right)_{v,T} = 0. \tag{22}$$

Equation 20 has no meaning. Integrating Eq. 22, we have $E = f(v, T)$ but independent of p which is a contradiction of our hypothesis that E is a function of p, v, T. Integrating Eq. 21, we have $E = -pv + g(T, p)$, which is a contradiction to the integrated form of Eq. 22. Thus, for the (p, v, T) space dS is not a perfect differential.

In a two-dimensional space there is always an integrating factor and T is a particular factor for dq. In the three-dimensional spaces which we are considering here, is there an integrating factor M such that Mdq becomes a perfect differential? Suppose we write for the (p, v, T) space

$$dq = dE + pdv = Pdp + Qdv + RdT$$

where

$$P = \left(\frac{\partial E}{\partial p}\right)_{v,T}; \quad Q = \left(\frac{\partial E}{\partial v}\right)_{T,p} + p; \quad R = \left(\frac{\partial E}{\partial T}\right)_{p,v}. \tag{23}$$

The necessary and sufficient condition that an integrating factor exists is that

$$P\left(\frac{\partial R}{\partial v} - \frac{\partial Q}{\partial T}\right) + Q\left(\frac{\partial P}{\partial T} - \frac{\partial R}{\partial p}\right) + R\left(\frac{\partial Q}{\partial p} - \frac{\partial P}{\partial v}\right) = 0. \tag{24}$$

On substituting into Eq. 24 the values of P, Q, R and their partial derivatives as obtained from Eq. 23 and reducing terms, we have

$$\left(\frac{\partial E}{\partial T}\right)_{p,v} = 0 \quad \text{and} \quad E = f(p, v) \tag{25}$$

which is a contradiction of our basic hypothesis.

For the (p, T, x) space consider Eq. 9 and write

$$P = \left(\frac{\partial E}{\partial p}\right)_{T,x} + p\left(\frac{\partial v}{\partial p}\right)_{T,x}; \quad Q = \left(\frac{\partial E}{\partial T}\right)_{x,p} + p\left(\frac{\partial v}{\partial T}\right)_{x,p};$$

$$R = \left(\frac{\partial E}{\partial x}\right)_{p,T} + p\left(\frac{\partial v}{\partial x}\right)_{p,T}. \tag{26}$$

The necessary and sufficient condition that an integrating factor exists is that

$$P\left(\frac{\partial R}{\partial T} - \frac{\partial Q}{\partial x}\right) + Q\left(\frac{\partial P}{\partial x} - \frac{\partial R}{\partial p}\right) + R\left(\frac{\partial Q}{\partial p} - \frac{\partial P}{\partial T}\right) = 0. \tag{27}$$

Substituting Eq. 26 in Eq. 27, we have after reduction, the relation that

$$\left(\frac{\partial E}{\partial v}\right)_{p,T} = \left(\frac{\partial E}{\partial v}\right)_{x,p} \tag{28}$$

which is the required condition for integrability.

To examine this equation write $E = f(p, T, x)$ and $v = \phi(p, T, x)$. If p, T = constant, both E and v will be functions of x only, and therefore the left side of Eq. 28 will be a function of x only. If x, p = constant, both E and v will be functions of T only, and therefore the right side of Eq. 28 will be a function of T only. Since we have assumed p, T, x as three independent variables, equating of a function of only x to one of only T is a contradiction.

We may then conclude that there is no integrating factor M which will make Mdq integrable in the (p, v, T) or (p, T, x) spaces.

This answers the point raised above in section entitled "The Assumption of a Two-Variable System" with regard to De Donder's work. He employs Eq. 3 together with S, which we now see are incompatible.

This does not, of course, mean that we have no thermodynamic potential functions in these three-dimensional spaces. In a previous paper (1) the writer has shown the following to meet all requirements:

$$\Phi_\alpha = pvT, \quad (29a) \qquad \Phi_\beta = pxT, \quad (29b)$$

$$\mathrm{H}_\alpha = E + pv, \quad (30a) \qquad \mathrm{H}_\beta = E + px, \quad (30b)$$

$$\Omega_\alpha = \frac{E + pv}{T}, \quad (31a) \qquad \Omega_\beta = \frac{E + px}{T}. \quad (31b)$$

6. THE KELVIN-PLANCK PRINCIPLE.

In the light of the preceding sections, what can be said of this principle? Consider the system as any type of machine, or chemical or biological system, which receives heat at certain intervals from one or several sources and converts it into work (mechanical, electrical, etc.), and at other intervals receives work and converts it into heat.

In order to define completely the internal energy E of the system at each instant during its operation, we must know all its independent variables, say n in number. Let us then write

$$E = f(\alpha_1, \cdots, \alpha_n). \tag{32}$$

If dq is the heat received by the system during an infinitesimal time, and dW the work done on the surroundings by the system during this same time, from the first law we have

$$dq - dW = dE. \tag{33}$$

If the system is carried through a complete cycle, as may be judged by the fact that $\alpha_1, \cdots, \alpha_n$, return to their original values, we have from Eq. 33

$$\oint dq - \oint dW = \oint dE = 0. \tag{34}$$

This states that the net heat received by the system in this cycle is completely transformed into work. If the cycle is reversed, we would have a complete transformation of work into heat. These two processes are permitted by the first law, regardless of whether the cyclic process is isothermal or otherwise, *provided that the cycle is complete. This latter condition is possible only under the condition that Eq. 32 is a complete equation of state, for only then can Eq. 34 be true.* For a fuller significance of these statements let us consider the following.

Let us divide our independent variables into two groups, the first being $\alpha_1, \cdots, \alpha_i$, and the second group being $\alpha_j, \cdots, \alpha_n$. Write

$$\int dE = \int\left[\frac{\partial E}{\partial \alpha_1} d\alpha_1 + \cdots + \frac{\partial E}{\partial \alpha_i} d\alpha_i\right] + \int\left[\frac{\partial E}{\partial \alpha_j} d\alpha_j + \cdots + \frac{\partial E}{\partial \alpha_n} d\alpha_n\right] = \int dE_{\text{I}} + \int dE_{\text{II}} \tag{35}$$

for the sake of brevity. Then

$$\int dq - \int dW - \int dE_{\text{II}} = \int dE_{\text{I}}. \tag{36}$$

Suppose that for some reason we are unaware of, or consciously neglect, the second group of variables, and assume erroneously that $E = \psi(\alpha_1, \cdots, \alpha_i)$ defines the internal energy of the system completely. If we carry through a process where the first group of variables return to their original values, we should conclude that the system has gone through a complete cycle and that $\oint dE_{\text{I}} = 0$. In general, the second group of variables will not return to their original values, so that $\int dE_{\text{II}} \neq 0$. For this false cyclic process we would write from Eq. 36

$$\oint dq - \oint dW - \int dE_{\text{II}} = \oint dE_{\text{I}} = 0 \tag{37}$$

or

$$\oint dq = \oint dW + \int dE_{\text{II}}. \tag{38}$$

Let us assume that the temperature T belongs to the first group of variables. Now, if this cyclic process is isothermal, and such that the net heat added to the system is positive, and the net work derived from the system also positive, the Kelvin-Planck principle (which is purely empirical) states that

$$\oint dq > \oint dW. \tag{39}$$

We, therefore, conclude that

$$\int_{q \to W} dE_{\text{II}} > 0 \tag{40}$$

where $q \to W$ refers to a process where the net result is that q transforms into W.

Suppose we reverse the path of integration in Eq. 38. Write this as

$$\oint_{W \to q} dW = \oint_{W \to q} dq - \int_{W \to q} dE_{\text{II}} \tag{41}$$

where $W \to q$ refers to a process where the net result is a conversion of work into heat. The first integral on the right is the net heat transmitted by the system. All forms of potential energy—electric, magnetic, elastic, chemical, etc.,—which may be set up in the system for a short time but which during the process are ultimately realized as work on the surroundings, will be included in the integral on the left. All other energies which are not recovered as work will be retained by the system and included in the second integral on the right. Some of this unrecovered energy will result in changes in the electric, magnetic, elastic, chemical and other properties of the system, whereas the remainder will be converted into heat. We may write

$$-\int_{W \to q} dE_{\text{II}} = \int_{q \to W} dE_{\text{II}} > 0. \tag{42}$$

For cyclic processes involving differences in temperature, we must note that the first law permits a complete transformation of heat into work, or work into heat, *provided the equation of state is complete.*

We see then that the Kelvin-Planck principle is not unrelated to the first law of thermodynamics. It is merely an indirect statement of the fact that the equations of state which we use in our thermodynamic considerations are incomplete.

A few illustrative examples may clarify the position that we have taken. Consider helium gas at approximately room temperature and

atmospheric pressure. There will be no transformations (in the macroscopic sense), and the equation of state may be written as $E = f(p, T) = h(p, v)$ or $p = \phi(v, T)$. An isothermal will be a single line in the (p, v)-plane; in going slowly in one direction heat converts completely into work, and in the opposite direction work converts completely into heat. In an isothermal cyclic process

$$\oint dq = \oint dW = 0.$$

If, however, a gas such as NH_3 were used, which is subject to a transformation ($2NH_3 \rightleftharpoons N_2 + 3H_2$), and we restricted its behavior to an

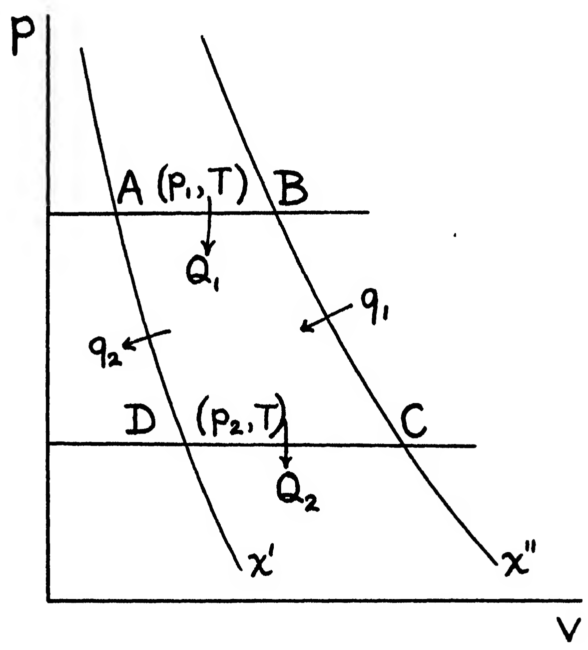

FIG. 1. Family of lines of constant (x, T) in the (p, v)-plane.

equation of state of only two independent variables, we would find that isothermal expansions and compressions at one temperature T would not necessarily lie on the same line in the (p, v)-plane. This is because the dissociation and association processes are not necessarily functions of only p, v. In classic thermodynamics the mass-action law is introduced to make these processes functions of only two variables, but in doing this certain (metastable) states have to be left out. This leaves us in the dilemma considered above in the section on "The Assumption of a Two-Variable System." To overcome this difficulty we should write, for this system, $E = f(p, v, T) = \phi(p, x, T)$ or $x = \psi(p, v, T)$. With T = constant, x will be a function of p, v. Thus we can carry out

an isothermal, cyclic process for this system where $\oint dq = \oint dW \neq 0$. The area $\int pdv$ may, however, be arbitrarily chosen.

Let us next consider a system consisting of the combination of liquid water and its saturated vapor. Such a system may be described by an equation with three independent variables such as given above for NH_3. Referring to Fig. 1, we may draw a family of lines of constant (x, T) in the (p, v)-plane. Starting at A, we may allow the system to expand to B with p_1, T = constant, and the amount of heat received by the system from outside will be $Q_1 = \lambda_{p_1, T}(x'' - x')$, where $\lambda_{p_1, T}$ is the latent heat of vaporization at p_1, T. The system may then be allowed to expand at constant (x'', T) until C is reached. Heat q_1 will be received from the surroundings during this expansion. At C we may compress the system to D with p_2, T = constant, and the heat delivered to the surroundings will be $Q_2 = \lambda_{p_2, T}(x'' - x')$. Finally, we may compress the system to A at constant (x', T), and deliver heat q_2 to the surroundings. The net work performed in this cycle will be represented by the area $ABCD$, which need not be zero, and the net heat received by the system will be

$$(\lambda_{p_1, T} - \lambda_{p_2, T})(x'' - x') + q_1 - q_2.$$

By the first law this work and heat are equal, and we have complete conversion.

In the cases of NH_3 and liquid-vapor water above, if the equation of state were not completely defined by three independent variables, if, for example, there were actually two types of transformation taking place simultaneously, we should require x_1 and x_2, say, in addition to p, T to define the internal energy. A simple process like that along AB where x_1 varied, would not be possible unless we had means for controlling x_2, p, T. Actual cases, particularly those where catalytic reactions may be involved and the number of x's may be much greater than 2, certainly are difficult systems for study, and one cannot state when a cycle has been completed unless one knows all the details of the chemical reactions and the thermodynamics which are involved.

7. CLAUSIUS' PRINCIPLE.

What light may be shed on this principle from the above point of view?

Consider a system operating between two reservoirs A and B at temperatures T_1 and T_2, respectively. Equation 38 applies to any case where the equation of state is incomplete; let us assume that it applies to the present system. If there is a net flow of heat from A to B with the system acting as the medium for this process, then $\int_{q \to W} dE_{II}$ will be positive, which implies that some of the heat of A remains with the system and is not transmitted to B. The work done on the surround-

ings will be given by

$$\oint dq - \int_{q\to W} dE_{II}$$

and will be positive if this difference is positive. If this difference is zero, no work is done, and the process may be considered as pure conduction (including radiation if the radiation pressure is not accompanied by work). Loosely, we should describe this process as taking place "by itself," because no work is involved. From this case, where $\oint dq > 0$ with $\oint dW = 0$, it follows by *definition* that $T_1 > T_2$.

We may, if we desire, add work to the system, that is, make $\oint dW < 0$. In doing so we have from Eqs. 41 and 42

$$\oint_{W\to q} dq - \int_{W\to q} dE_{II} = \oint_{W\to q} dq + \int_{q\to W} dE_{II} < 0 \tag{43}$$

where the last integral is positive. Therefore, $\oint_{W\to q} dq$ must be negative in this case, meaning that there must be a net flow of heat from reservoir B at the lower temperature T_2 to the reservoir A at the higher temperature T_1. This process of heat flow from a lower to a higher temperature does not take place "by itself," for it requires work to be done on the system from outside. This is the essence of Clausius' principle.

8. CARATHÉODORY'S DEVELOPMENT OF THE SECOND LAW.

We may now examine Carathéodory's work (4) from the light of the above considerations. He has been able to derive the second law in the mathematical form ($TdS \geq dq$) without the use of Carnot cycle. He employs, however, the classic thermodynamic views, as we see from his Axiom I, in which he states that for each phase of a system, when in equilibrium, there is a function ϵ_i (the internal energy) of the quantities V_i (volume), p_i (pressure), and m_{ji} (the masses of the components). In his general treatment he considers a system of $(n + 1)$ independent variables, subject to the condition that n of these variables are configuration coordinates (v or the like, one volume for each phase). The extra coordinate may be p, which at equilibrium is uniform throughout the system.

Carathéodory rests his entire development on a lemma concerning Pfaffians which is as follows: given a Pfaffian equation (his Eq. 16)

$$dx_0 + X_1dx_1 + X_2dx_2 + \cdots + X_ndx_n = 0 \tag{44}$$

where X_i is a finite, continuous, differentiable function of x_i; if we know that within each region surrounding any arbitrary point P of the space x_i there are points which are inaccessible along the length of the curves which satisfy this equation, then expression 44 above possesses a factor which will make it completely differentiable.

The variables $x_1, \cdots, x_n$ are the configuration coordinates, so that $\sum_{i=1}^{n} X_i dx_i$ represents the work done by the system, leaving the single non-configuration coordinate x_0, which may be a quantity such as pressure. Suppose we find variables in addition to pressure, such as the degree of transformation, to be necessary to define our system completely. The degree of transformation is not, in general, a configuration coordinate, for changes in this coordinate may take place where no work is involved. The system will then be defined by more than one variable of the non-configuration type and the Pfaffian form would be somewhat different from expression 44. We should then have difficulties in developing the second law along the lines of Carathéodory.

A second difficulty exists. In Carathéodory's lemma, the Pfaffian (44) applies to a quasistatic adiabatic change of state for a "simple" system (see his definition on p. 368), which physically is recognized as one which may be subjected to a reversible process. The inaccessible points called for in his lemma lie on paths which are irreversible and cannot be reached from the adiabatic unless work is done. In other words, the lemma calls for the existence of irreversible processes for a system defined completely by the variables $x_0, x_1, \cdots, x_n$. As we have seen above in the section on the Kelvin-Planck principle, if this set of variables defines our system *completely*, we shall have no irreversible processes, and therefore no inaccessible points. Such processes arise, physically, only from incomplete descriptions of a system. Therefore, there is an incompatibility in the basic assumptions of the lemma.

We must, however, grant this fact. If we employ incompletely defined systems, as is always done and if we do not use quantities such as the degree of transformation, an integrating factor for *dq*, and therefore an entropy function, exists. This, of course, will restrict the application of the second law, and for the very simple system of a single gaseous phase in which dissociation occurs, the entropy function will not be applicable.

9. CONCLUSIONS.

In conclusion, what does the above analysis tell us about the second law of thermodynamics?

First, both the Kelvin-Planck and Clausius principles are essentially statements of the fact that we always use incomplete equations of state, and therefore find certain energy-conversion processes which would be permitted by the first law of thermodynàmics to be impossible.

Second, as we learn more about the "controls" of systems, we may expect to realize processes which are more reversible. But it is hardly likely that we shall realize complete reversibility because material systems are highly complex and require very many variables to describe the system completely.

Third, the two principles emanate from the same source, that is, from the fact that $\int_{q \to W} dE_{II} > 0$. This integral, therefore, becomes of very great interest, for it is through this integral that irreversible processes arise.

Fourth, it is the writer's view that the concept of entropy is more restrictive than helpful in giving us an enlarged view of thermodynamics.

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Spores by the Billion.—The "astronomical numbers" of the disease-causing spores that some fungi produce is one reason why plant scientists advise orchard owners to apply protective sprays and thus prevent spores that lodge on the blossoms from developing further and causing infection, according to the U. S. Department of Agriculture.

For example the disease known as brown-rot causes "mummy" peaches: Rot attacks the fruits; they shrivel; some cling; but many drop to the ground. By spring a half-buried mummy peach may develop to five or six cup-shaped fruiting bodies of the fungus. These cups are filled with spores, ready for discharge into the air at about the time the peach blooms in the spring.

A single mummied peach with only three of these cups—called apothecia—may supply spores enough to allow 50 spores to each square inch in an acre of orchard, according to the calculation of John C. Dunegan of the Bureau of Plant Industry, Soils, and Agricultural Engineering.

Dunegan's figuring runs like this: At a recent meeting of Pennsylvania fruit growers he displayed a magnified view of a section of one of these cups. This bit was 1/50 of an inch long and 1/3000 of an inch thick. It contained 275 spores. The spores are too small to be seen without a magnifying glass, and placed end to end it would take more than 2000 of these spores to fill an inch. His specimen came from a typical cup about 2 inches in diameter. Such a cup would contain 150 million spores—more or less. The three cups on his mummy specimen might produce roughly half a billion spores. Infections started by these early spores develop a different type of summer spore and these in turn spread the infection.

These "facts of life" suggest control procedures. One obvious step is to remove as many as possible of the mummies from the trees and from the ground under the trees. In orchards which have had a serious rot problem in preceding years sulfur sprays applied during the blooming period will help reduce the number of blighted blossoms and thus indirectly aid in controlling the disease on the fruit at harvest time. The blossom sprays are, of course, in addition to the regular schedule of spray applications recommended for the control of brown rot, scab and the plum curculio.

R. H. O.

Assembly Line Produces House an Hour. (*Civil Engineering*, Vol. 17, No. 16.)—Automotive mass production methods are now being copied in the manufacture of single- or double-bedroom portable houses equipped with bath and kitchen fixtures, built-in partitions, beds, cabinets and closets, ready for delivery by truck or rail to a home site. Delivered house (priced at $2650 at Wingfoot Homes, Inc. plant, Washington Park, Ill.) can be erected in four hours ready for immediate occupancy.

The quarter-mile-long production line traverses three separate buildings of a reconverted wartime valve factory. The technique employed was developed after two years of pilot operations in the original Wingfoot factory located at Litchfield Park, Ariz.

There is no backtracking of materials. Partial or subassembly sections move forward on dollies along the production-line rails. Overhead units are utilized for lifting floor units and side assemblies and for setting roofs in place. Heavy fork-lift trucks move the finished 3-ton houses to flatcars or trucks. More than 52 homes are in various stages of erection at one time.

R. H. O.

IMPACT BETWEEN CHAIN ROLLER AND SPROCKET IN A CHAIN DRIVE.

BY

R. C. BINDER [1] AND W. V. COVERT [2]

SUMMARY.

This paper presents some analytical relations which may help in developing a better understanding of the mechanics of roller-sprocket impact. Various relative impact velocities are derived. These velocities may be used in impact energy relations for determining limiting sprocket speeds on the basis of roller breakage, noise, heating, and sprocket wear. Some experimental data on roller breakage are presented and used to illustrate a method of organizing data. Impact energy using a certain velocity correlates to some extent with roller breakage.

It is known that the impact between a chain roller and the sprocket tooth has some influence on the performance of a roller chain drive. For example, excessive roller impact may lead to such undesirable conditions as roller breakage, noise, heating, and wear of sprocket teeth. Many successful drives have been developed, and are in operation. There are, however, questions as to just what happens during impact. What are the relations between the variables, and what is the mechanics of the action? Analytical relations might be helpful in correlating experience and in meeting unusual or special cases.

Some analytical work has been presented by Bartlett (**1**),[3] Clevely (**2**), and Archibald (**3**). This previous work, however, does not cover completely the mechanics of impact. The following paper presents some analytical relations not found in the previous literature. These relations might be helpful in developing a better understanding of the mechanics of roller-sprocket impact. Some laboratory results are also given to illustrate portions of the analytical study. There is this major question: what is the relative velocity of impact? This velocity is of concern in setting up an expression for impact kinetic energy. It may be possible to make accurate predictions of roller breakage, noise, heating, and sprocket wear if means are established for accurately determining the impact energy for different operating conditions, as sprocket speed, sprocket size, load on drive, chain tension, and pitch.

Figure 1 illustrates diagrammatically one sprocket (the driver) of a drive, and the chain meshing with the sprocket. After roller A has

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[3] The boldface numbers in parentheses refer to the list of references appended to this paper.

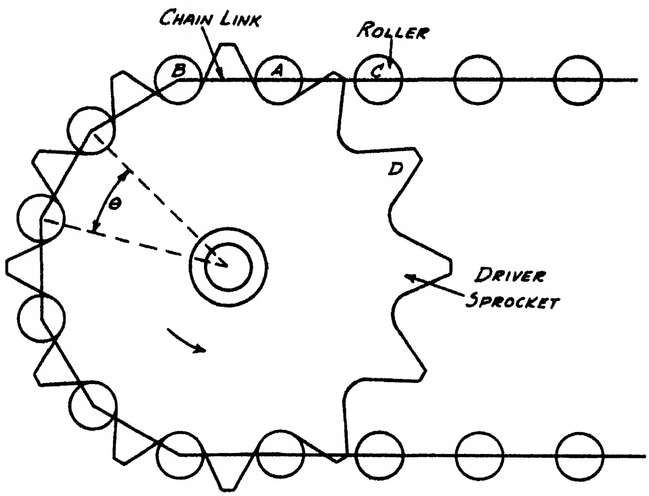

FIG. 1. Diagrammatic sketch of sprocket and chain.

seated on the sprocket, the link connecting rollers A and C starts to turn or articulate, and roller C approaches tooth D. After the sprocket has turned through the angle θ, there has been an impact between roller and sprocket, roller C has seated, and the sequence of events repeats.

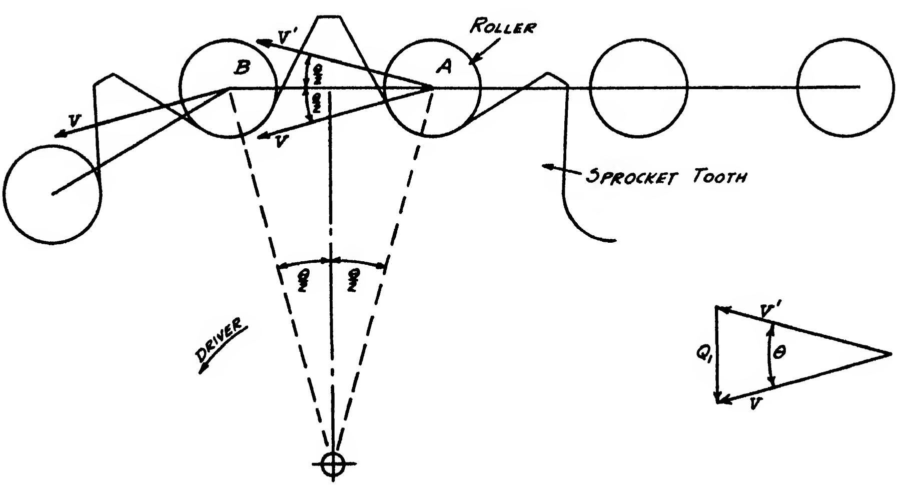

FIG. 2. Positions of rollers for velocity determination.

It will be assumed that the driver and driven sprockets are the same size, that sprocket pitch exactly equals new chain pitch, that the driver rotates at a constant angular speed, and that the strand between sprockets is always parallel to the line of sprocket centers.

The following notation will be used:

N = angular speed of sprocket, revolutions per unit time,
P = chain pitch,
R = radius of sprocket pitch circle,
T = number of teeth on sprocket,
V' = linear velocity of point on pitch line of sprocket, $V' = 2\pi RN$,
θ = $360/T$, angle illustrated in Fig. 1, and
ϕ = theoretical pressure angle, illustrated diagrammatically in Fig. 7.

RELATIVE IMPACT VELOCITY AND IMPACT ENERGY.

Figure 2 illustrates the roller positions for calculating one relative velocity which will be designated as Q_1. This velocity is that between

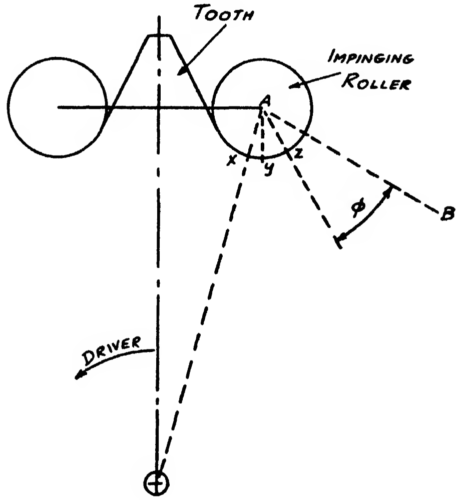

FIG. 3. Points of contact between roller and sprocket

a roller center and the point on the sprocket pitch circle which will be coincident with the roller center after seating. Consider the state an infinitesimal time interval before roller A seats. The velocity of the center of the roller A has a velocity equal to that of the seated roller B, namely $V = 2\pi RN$. The corresponding point on the sprocket pitch circle has the velocity $V' = 2\pi RN$. The two velocities are equal in magnitude but are different in direction. The velocity triangle in Fig. 2 shows that the vector difference is

$$Q_1 = 2V \sin \theta/2 = 4\pi RN \sin 180/T,$$
$$Q_1 = 2\pi NP. \tag{1}$$

The relative velocity is directly proportional to pitch and the angular speed of the sprocket; the relative velocity is independent of the number of sprocket teeth. It can be shown that this relative velocity is constant in magnitude during the entire articulation.

The velocity Q_1 is very simple to calculate, and it is very convenient

for purposes of analysis of the impact problem. It is the one which has been generally used as indicated in previous papers on impact. There is a question, however, as to whether or not this velocity applies for all conditions of operation. The following discusses other possible relative velocities, which are velocities between the roller surface and the tooth surface at the point of contact.

There is a problem as to the exact location of the point of contact. The point of contact may be different in various applications because of different conditions of operation. Figure 3 shows three possibilities: point x is on a radial line joining the roller center and the sprocket center; point y is on a line through the roller center which is parallel to the radial centerline of the tooth; and point z is on the line of theo-

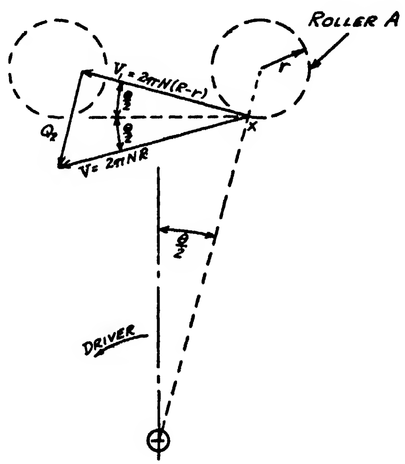

FIG. 4. Relative velocity for contact at point x.

retical tooth pressure. The line AB in Fig. 3 is the polygon chord between centers of the tooth spaces; these centers are on the pitch circle of the sprocket. The measurement of the theoretical pressure angle ϕ is further illustrated in Fig. 7.

It is assumed that, just before seating, the roller does not turn about its own center. For the conditions taken, with the driving strand always parallel to the line of sprocket centers, the resultant velocities of all points on the roller are the same.

Figure 4 shows the velocity vectors for contact at point x. Imagine the state an infinitesimal time interval before roller A contacts. The velocity of any point on the roller before contact is $V = 2\pi RN$. The velocity of point x on the sprocket is $V_1 = 2\pi(R - r)N$ where r is the radius of the roller. Setting up the velocity relations and simplifying gives the final result for the relative velocity Q_2:

$$Q_2 = 2\pi N\sqrt{r^2 + 4R(R - r)\sin^2 180/T}. \qquad (2)$$

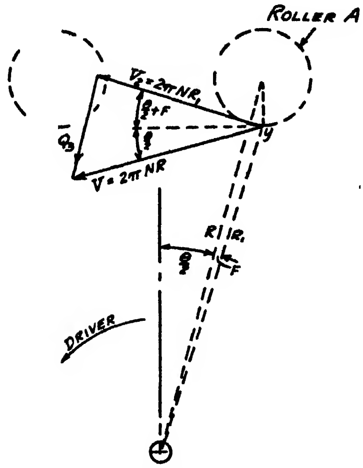

FIG. 5. Relative velocity for contact at point y.

Figure 5 shows the vectors for contact at point y. Imagine the state an infinitesimal time interval before roller A contacts. The velocity of a point on the roller before contact is $V = 2\pi RN$. The velocity V_2 of point y on the sprocket is $V_2 = 2\pi R_1 N$ where

$$R_1 = \sqrt{R^2 + r^2 - 2Rr \cos 180/T}. \tag{3}$$

Setting up the velocity relations and simplifying gives the final result for the relative velocity Q_3:

$$\begin{aligned} Q_3 &= 2\pi N\sqrt{R^2 + R_1^2 - 2RR_1 \cos (360/T + F)}, \\ Q_3 &= 2\pi N\sqrt{P^2 + r^2}. \end{aligned} \tag{4}$$

Figure 6 shows the velocity vectors for contact at point z. The velocity of a point on the roller A before contact is $V = 2\pi RN$. The

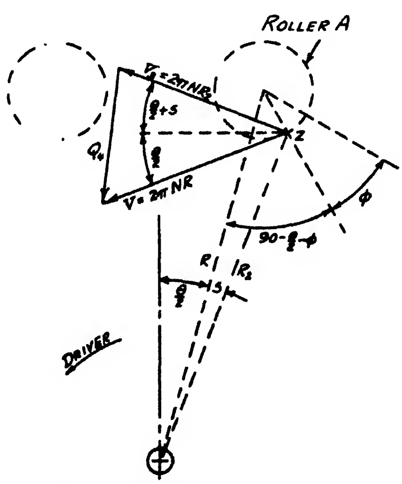

FIG. 6. Relative velocity for contact at point z.

velocity of point z on the sprocket is $V_3 = 2\pi R_2 N$ where

$$R_2 = \sqrt{R^2 + r^2 - 2Rr \sin (180/T + \phi)}. \quad (5)$$

Setting up the velocity relations and simplifying gives the final result for the relative velocity Q_4:

$$Q_4 = 2\pi N \sqrt{R_2^2 + R^2 - 2R_2 R \cos (360/T + S)}. \quad (6)$$

The American Standards Association gives the following value for the theoretical pressure angle for its tooth form:

$$\phi = 35 - 120/T \qquad \text{deg.} \quad (7)$$

For this theoretical pressure angle the velocity Q_4 has the value

$$Q_4 = 2\pi N \sqrt{P^2 + r^2 + 2Pr \sin (55 - 240/T)}. \quad (8)$$

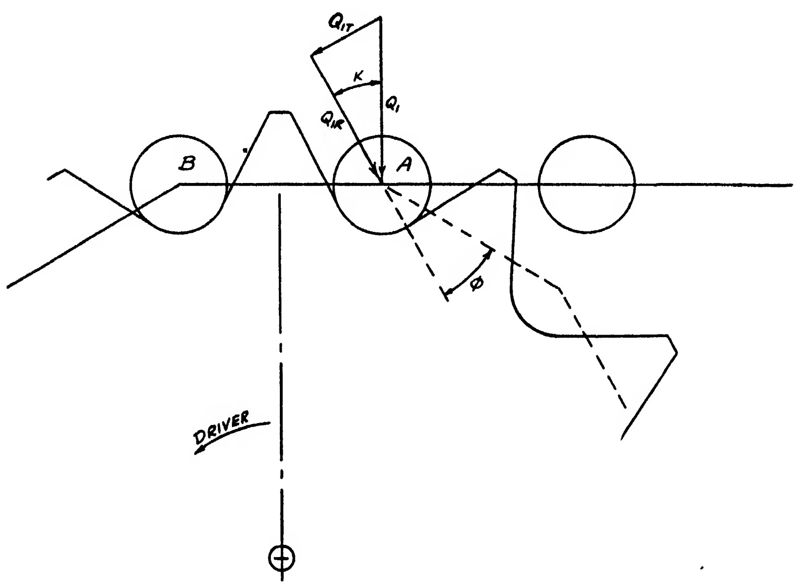

FIG. 7. Components of relative velocity Q_1 parallel and perpendicular to tooth face.

A further possibility is illustrated in Fig. 7. The roller A is just about to seat. The relative velocity Q_1 is resolved into a component Q_{1R} perpendicular to the tooth face and another component Q_{1T} parallel to the tooth face. The angle between Q_1 and the link connecting rollers A and B is 90 deg. The angle K for the American Standards Association tooth form is

$$K = 55 - 240/T \qquad \text{deg.} \quad (9)$$

Then

$$Q_{1R} = Q_1 \cos (55 - 240/T). \quad (10)$$

$$Q_{1T'} = Q_1 \sin (55 - 240/T). \tag{11}$$

The Q_{1R} component may be called a "destructive" component because it involves a velocity change normal to the surfaces at contact.

Each of the other velocities Q_2, Q_3, and Q_4 can be resolved into a normal component and a tangential component. Using the same notation as for Q_1, the destructive components become:

$$Q_{2R} = 2\pi NP \cos 180/T. \tag{12}$$

$$Q_{3R} = 2\pi NP. \tag{13}$$

$$Q_{4R} = \pi NP \operatorname{cosec} 180/T\, [\cos (35 + 60/T) + \sin (420/T - 55)]. \tag{14}$$

Equation 14 is based on the assumption of the theoretical pressure angle given by Eq. 7.

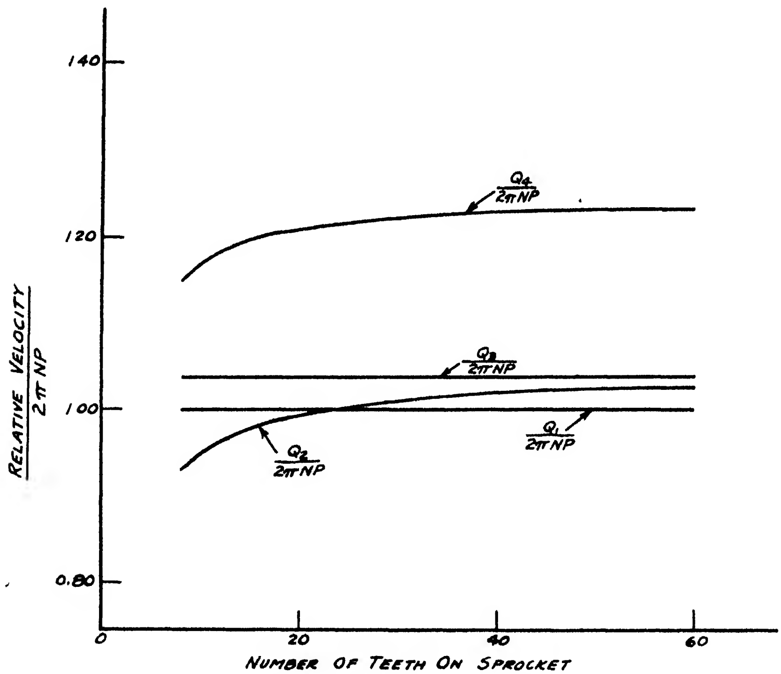

FIG. 8. Comparison of different velocities of impact.

A comparison of the different resultant velocities is shown in Fig. 8 for the American Standards Association tooth form and a pitch of 1 in. The ordinate relative velocity divided by $2\pi NP$ is a dimensionless ratio in any consistent set of units. For some sprockets there is not much difference among the values of Q_1, Q_2, and Q_3. The corresponding destructive components are plotted in Fig. 9.

The impact kinetic energy is $\frac{1}{2}$(mass)(velocity)^2. Let w equal the weight of the impinging body in pounds and g the gravitational accelera-

tion in feet per second per second. As an example, consider the velocity as Q_1 in feet per second. Then

$$\text{energy of one impact} = \tfrac{1}{2}(w/g)Q_1^2 \text{ ft-lb.}$$

Other velocities can be used. It is difficult to say just how much of the mass of the chain is involved in the impact. As a first approximation, it seems reasonable to consider that the average weight of the single link (which is assumed as concentrated at the roller center) is the weight involved. As an illustration, consider a 1-in. pitch chain, American Standards Association No. 80. From about 90 to 93 per cent of the weight of each link is distributed symmetrically about the centers of the rollers.

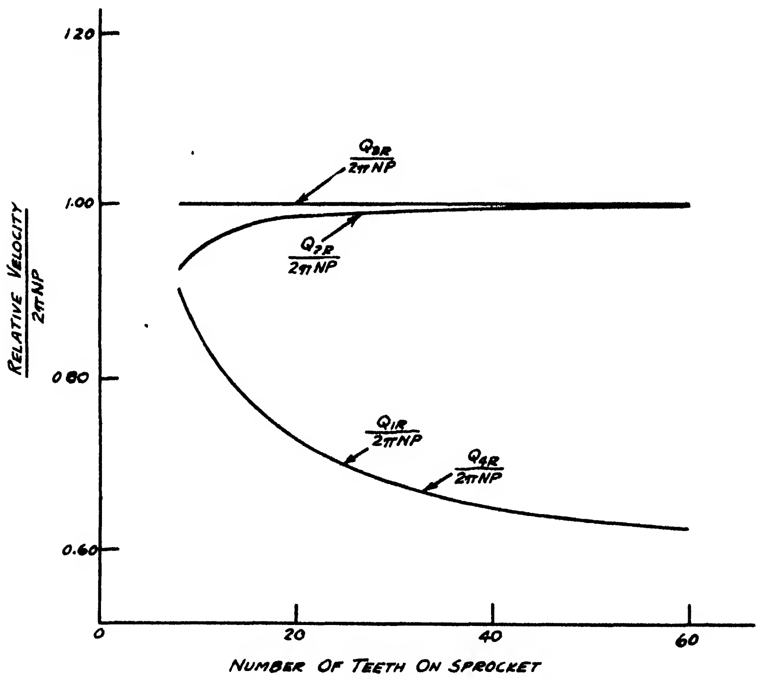

FIG. 9. Comparison of different destructive-component velocities.

The impact energy relation does not consider the effect of chain tension. Since any attempt to include the effect of chain tension would be highly speculative, it was omitted.

For the purpose of capacity rating, the impact energy given on the basis of one of the foregoing velocities can be compared with an allowable impact energy determined by experiment. Such a comparison might be used to determine limiting sprocket speeds on the basis of roller breakage, noise, heating, and sprocket wear. Attempts to reach a conclusion as to what velocity to use in the impact relation for each

particular case are handicapped by a lack of complete information as to just exactly what happens during impact. The foregoing discussion points out some of the difficulties involved. When complete experimental data are available, then some decision may be reached as to just what velocity applies in each case. In the meantime, in regarding roller chain drives, we might well heed the apt expression of O. Heaviside: "Should I refuse my dinner because I do not fully understand the process of digestion?"

LABORATORY TESTS.

Some new data on roller breakage will be presented. These data are helpful in indicating possible trends and methods of organizing and adapting test results.

TABLE I.

Reference No.	Number of Impacts[a] $\times 10^{-6}$	Roller Failure	Coolant Viscosity,[b] Centipoises (100° F.)
1	1.5–3.0	No	1.6
	2.3–3.0	Yes	1.6
2	1.2–1.3	No	1.25
	1.6–2.2	Yes	1.25
	2.1	No	1.6
	7.7	Yes	79
3	1.2	No	1.25
	1.5	Yes	1.25
	1.2–3.2	No	1.6
4	2.3	No	1.25
	2.9	No	1.6
	10.4	No	79
5	3.1–3.3	No	1.25
	2.0–2.2	No	1.6
	28.6	Yes	79
6	1.7	No	1.6
7	1.7–3.1	No	1.6
8	4.1	No	1.6
9	4.0	No	1.6
10	3.3	No	1.6
11	56.0	No	79

[a] Where a number of identical tests were run, the range of impacts is given.
[b] Water = 0.68 at 100° F.; S.A.E. 20 Motor oil = 79 at 100° F.

A series of wear tests were conducted on special $\frac{5}{8}$ pitch chain in the research laboratory of Diamond Chain Co., Inc. In each test the drive consisted of two sprockets of the same size. The drive centerline was horizontal. The top strand of chain transmitted a constant tension in all tests, whereas the bottom strand was slacked as is usual for most drives. All tests were stopped at 1 per cent chain wear elongation (0.006 pin bushing wear per pitch) or 1000 hr., whichever occurred first, as an arbitrary period of duration.

The angular speed of the sprocket, sprocket size, and coolant were varied in these tests; otherwise all drives operated under identical conditions. Some of the pertinent data are listed in Table I. Three

different coolants were used. The number of impacts listed is the number of times a given roller in the chain meshed with the driver sprocket before failure occurred or during the life of the test if failure did not occur. Note that roller failure was found in test references Nos. 1, 2, 3, and 5.

POSSIBLE ADAPTATION AND INTERPRETATION OF TEST RESULTS.

Sufficient and conclusive test data are not available to determine positively the effect of sprocket size, speed, chain pitch, tension, and other factors on roller failure. The limited data, however, show some trends.

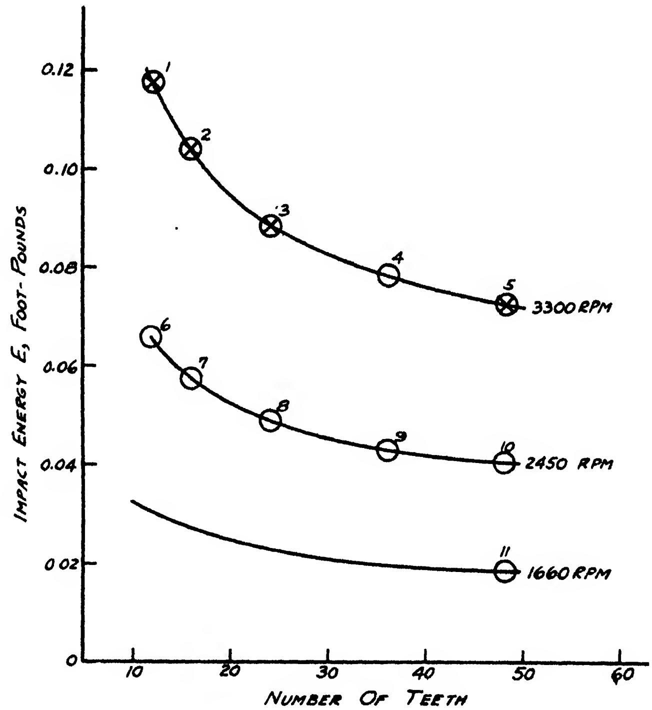

Fig. 10. Impact energy E *versus* number of teeth for $\frac{5}{8}$-in. pitch chain.

Investigation shows that the impact energy E using the velocity Q_{1R}, as given by Eq. 10, gives a factor which might be useful in correlating data. The curves in Fig. 10 show this calculated impact energy E as a function of sprocket teeth for three constant angular speeds of the sprockets. The curves are for the special $\frac{5}{8}$-in. pitch chain. For each constant speed the impact energy decreases as the number of teeth increases. For the same sprocket the impact energy decreases with a decrease in speed. This same general trend for roller breakage has been found in practice, since experience has shown that a sprocket having a

small number of teeth and operating at a high angular speed is destructive to chain rollers. Increasing the number of teeth in the sprocket, while maintaining the same rotative speed, reduces the roller abuse. Thus there is some correlation between the calculated impact energy E and observed roller breakage. This correlation using Q_{1R} in the impact energy is close.

The several points shown by circles in Fig. 10 indicate the sprocket size and speed at which the tests were run. An "X" indicates conditions at which roller fracture resulted; a symbol without an "X" indicates no failures during the test. The numbers at various points on the curves refer to the reference numbers given in Table I.

For this chain one might set a limiting value of about 0.06 to 0.07 ft-lb. for the impact energy E; roller breakage is avoided for impact energies below this limiting value. This limitation, in turn, can be used to establish limiting sprocket speeds.

A number of interesting specific points are brought out by Table I and Fig. 10, as follows:

1. As the number of teeth in the sprocket was increased beyond 24 at 3300 rpm., either the number of failures was reduced or an increased number of impacts to failure was required.

2. Failure of rollers on the 12, 16, and 24 tooth sprockets was eliminated when the speed was reduced to 2450 rpm. This applies only to the results given, which are for a relatively small number of impacts.

3. Coolant viscosity plays an important part in roller impact fatigue. It may be possible that the rate of coolant or oil flow directed at the point of impact is of considerable importance. A comparison of coolant effect is shown in Table I, references Nos. 2, 4, and 5.

4. The effect of chain tension is unknown, since all of the tests were run at the same transmitted chain tension.

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Fundamental Properties of Metals. (*Electrical Engineering*, Vol. 66, No. 11.)—Although great progress has been made in obtaining a sound general knowledge of the forces that hold matter together, scientists up to now have not been able to extend this work to three important classes of materials: the complex substances such as proteins that constitute living organisms; atomic nuclei; and metals and alloys; according to Professor Linus Pauling, director of the Gates and Crellin laboratories of chemistry at the California Institute of Technology.

Speaking before the Southern California section of the American Chemical Society, he pointed out that application of a recently formulated equation has indicated that metallic atoms are bound to one another by electrons which they share. A similar mechanism provides the binding forces that govern the mechanical properties of organic materials such as fats and wood.

Interpretation of metallic structure in accordance with the new equation provides a qualitative explanation of many of the properties of some of the most common metals. It is found that atoms of chromium, tin, and manganese exist in two forms, small atoms with high combining powers, or valences, and large atoms with low valences, and in many metals each atom is attached to some of its neighbors by strong bonds and to others by much weaker links. The association of metallic valences with interatomic distances and properties of crystals has led to the construction of a reasonably satisfactory theory, however the whole treatment is essentially empirical in nature.

R. H. O.

New Process Forecasts Wider Use of Titanium. (*Compressed Air Magazine*, Vol. 52, No. 9.)—Pilot-plant operations at Boulder City, Nev., by the U. S. Bureau of Mines may lead to a wider use of titanium, which is described as a light metal of exceptional chemical and physical properties. Ductile in the pure state so that it can be fabricated and worked, it is said to be superior to aluminum and magnesium on the basis of strength and weight and to possess many of the valuable properties of stainless steel. However, it has been difficult to produce in the pure state, and that has limited its industrial application.

By the new process developed by the Bureau of Mines, pure titanium chloride is dripped onto molten magnesium in the presence of helium under slight pressure. The mixture thus obtained is wet-ground, screened, and leached with dilute hydrochloric acid, yielding a power that is pressed into briquettes and sintered at 1830° F. under a high vacuum to remove the hydrogen picked up during leaching. The sintered material, it is claimed, can be rolled at room temperature with reductions of 20 per cent. before annealing, or of as much as 75 per cent. at 1200° F. Output of the pilot plant is about 100 pounds of pure titanium a week.

Though cost of production by the improved method is still relatively high, the expectations are that large-scale operations will effect some economies. Be that as it may, the simplified reduction furnace promises to make pure titanium available for uses where its favorable properties outweigh price and for the manufacture of airplane parts; diaphragms that are maintained under tension; spindles, spools, and other working components of textile machinery; certain types of springs; and other products where lightness, strength, and resistance to corrosion are vital factors.

R. H. O.

PLANETARY PROPERTIES OF FAST CARS.

BY

A. F. ZAHM,[1] Ph.D.

PREFACE.

In past decades many inventors and writers have advocated designs for super-speed travel round the earth or through cosmic space. One scheme is a vacuum tunnel girdling the globe or a wide continent. Inside it a car speeds many thousand miles an hour suspended by magnetism to avoid mechanical friction. Another scheme is the space car, rocket-driven many miles a second, for journeys round the earth or moon, etc. Both kinds of craft naturally must have air constantly refreshed, as in a submarine.

As such projects are likely to allure men for indefinite future years, it seems worth while to appraise some basic properties of the high-speed car, which always exist and at times may prove dominant. The mass is independent of locality. Also the earth's speed round the sun and his round the Milky Way, though both enormous, can here be ignored as unfelt by mundane vehicles.

WEIGHT AND BUOYANCY.

At any altitude the weight of a body equals its mass times the local intensity of gravity. This latter is given by

$$g/g_0 = R_0^2/R^2 \tag{1}$$

where g is the intensity at the distance R from the earth, and g_0, R_0 like values near the earth.[2] Assuming in turn $R/R_0 = 1, 1.001, 1.002$, etc. gives by Eq. 1 the weight values in Table I.

Thus the loss of weight is shown to be 0.5 per cent at 10 miles height, and 9.3 per cent at 200 miles, say just above the atmosphere. For a

TABLE I.—*Relative Weight and Buoyancy at Various Altitudes.*

Altitude R, miles	Relative orbital radius, R/R_0	Relative gravity, $g/g_0 = R_0/R^2$	Weight of 50-ton airplane, lb.	Loss of weight, lb.	Relative atmospheric density, p/p_0	Buoyancy loss, per cent
0	1.000	1.0000	100,000	0	1.0000	0.00
4	1.001	0.9980	99,800	200	0.5517	0.4494
8	1.002	0.9960	99,600	400	0.2655	0.7356
10	1.0025	0.9950	99,500	500		
40	1.010	0.9804	98,040	1,960		
200	1.050	0.9070	90,700	9,300		

[1] Chief (Retired), Division of Aeronautics, Library of Congress, Washington, D. C.

[2] In elementary mechanics some write $R_0 = 4000$ miles, $g_0 = 32$ ft. per second per second.

transport plane 4 miles high the loss is 0.2 per cent of the gross weight, or say 0.4 per cent of the useful load. This gravitational loss is offset by a loss of about 45 per cent of the sea-level buoyancy. The latter is negligible, especially for a metal plane. For example, the sea-level buoyancy of a ton of aluminum is less than 1 lb., or 0.05 per cent.

RADIAL ACCELERATION.

In normal flight with speed v an avion's acceleration is

$$j = v^2/R \tag{2}$$

away from the earth's center, distance R. Here v is referred to a fixed origin at the earth's center, not on the earth's surface.

In mile-second units Eq. 2 can be written

$$j = v^2/4000, \text{ miles per second per second} \tag{3}$$

and in foot-second units it can be written easily but less simply.

From Eq. 3 one notes that if v is one mile a second, $j = 1/4000$ mile per second per second, or roughly .4 per cent of g at sea level. If v is 1/5 mile a second, j is 1/25 or 4 per cent of g. If v is 5 miles a second, j is 25×4 per cent of g; *viz.* the radial acceleration equals that of gravity. Of these last two examples the first indicates that present-day cars and planes feel no great loss of weight by centrifugal force; the second that a planetary car is apparently weightless at 5 miles a second, and therefore could revolve as an earth satellite permanently without continued propulsion. This alluring fact has been known many decades, even centuries.

Some visionaries foresee craft circling the earth at 100,000 mph. But then the radial acceleration must exceed 30 g. What agent will hold the craft down to a circular orbit? [3] What will save the passengers from pancaking against the ceiling? Or shall car and content hurtle away indefinitely? At one fourth that speed bodies out-climb the earth's gravitation.

For a point fixed on the equator $j = 0.003\ g$, by Eq. 3, since there $v = 1000$ mph. roughly. That is, the inertia lift on a resting plane at the equator is about 0.3 per cent of its weight. Now if the plane skims eastward at an additional speed of 1000 mph., its whole speed is 2000, and inertia lift 1.2 per cent; but if it skims westward at that rate, its whole speed is $v = 0$, and its inertia lift is zero. In said eastward motion a 100-ton plane would have 1800 lb. more centrifugal lift at full speed than when at rest.

By similar reasoning these two cases can easily be generalized to disclose the inertia lift of craft having any speed and direction at any

[3] Shall the centrifugal force, 30 mg., be balanced by the weight and −29 mg. by downward impact of the tenuous air? This negative lift must be 290 tons for a 10-ton vehicle. A nice problem in aircraft design!

place on earth. A table and graph of such values might be serviceable. (Fuller treatment appears in *Aviation* for March, 1945, page 183.)

KINETIC ENERGY AND COST

A mass m with speed v has the kinetic energy

$$E = mv^2/2.$$

Thus at 1 mile a second each pound has the kinetic energy

$$E = (5280)^2/2g = 660^2 \text{ ft-lb.}$$

This equals $660^2/33{,}000$ horsepower minutes, or 0.22 horsepower for 1 hr. At 5 miles a second the last figure becomes $25 \times 0.22 = 5.5$ horsepower hours for each pound mass of the planetary car. That is 11,000 horsepower hours per ton.

At this rate a 10-ton planetary car, rounding the earth just above the atmosphere, would have a kinetic energy of 110,000 horsepower hours, costing at least $10,000. This equals $1,000 each if the mass includes 10 passengers. If they stayed for a hundred trips round the world, the cost of each would be $10 a trip for power, since the energy needs no renewal.

By contrast, one notes that earth riders swing round the sun annually at no mileage cost. A billion stars course the Milky Way free of charge. It is the first cost that counts. At 30 miles a second a pound of meteorite has 36 times the energy above computed, or 198 horsepower hours. That equals nearly a month's toil of a big draft horse. Meteorites come high! The sun sweeps round the Milky Way at 180 miles a second; six times the assumed meteorite speed, involving 36 times the energy per pound, or the output of a draft horse for nearly three years.

CONTROL PROPERTIES.

A space car passively circling the earth outside the stratosphere would retain constant speed of translation and rotation. For it would lack resistance to either motion. The spin velocity could have any magnitude or direction. If desired, some portholes could look always along the earth's radius. Through these an observer would behold the earth's main features sweep past about 300 miles per minute. At 5 miles a second each round trip would take about 1.4 hr. Boston to the antipodes, 0.7 hr.; antipodes back to the world's hub, 0.7 hr.!

The car's normal poise could be adjusted by simple means minus rudders or issuing jets. To turn it positively about an axis some contained smaller body could be turned negatively, thereby generating an opposite reaction able to start the car turning positively. The smaller body, for example, might be the armature of an electric motor. The angular momentum begot in the massive car would be equal and opposite to that of the small armature. Both could be continued for a

while; the car turning quite slow, the armature quite fast. Stopping the motor at the right instant would stop the car's induced displacement at the desired amount.

This principle is employed by a falling cat to land upright, and is proved experimentally by dropping a wooden cat designed to do the same. A good proof can be given with a closed iron box quite free to yaw, enclosing a motor with axis vertical. Running the motor yaws the box; stopping it stops the box but leaves its yaw displacement fixed. Whether still or rotating, their added momentum is zero; for the closed box prevents the armature from gripping external air or earth's magnetism. For linear motion a like principle obtains, as is well known.

LIFE WITH COUNTERACTED GRAVITY.

The assumed planetary car, suitably pressurized and circling with contents uniformly round the earth, just outside the stratosphere, would appear to be weightless, due to radial acceleration. Furniture and passengers would float freely unless somehow anchored or magnetically shod. Fish could pause or swim in the air very much as aquatic ones in sea or lake . . . etc. etc.

A man could pose in any attitude, erect or inverted. With closed eyes he would be unconscious of direction. He could stand on the ceiling, side walls or floor indifferently; or shuffle about with magnetic soles. Chairs, beds, etc. would be superfluous. Invalids could rest day and night without fatigue, insensible of gravity pressures internal or external; a delightful prospect for heavy terrene patients weary from prolonged reclining.

SUNDRY SATELLITES.

Keen astronauts expect to launch space craft set to circle the globe perpetually. If clustered they may appear luminous at dawn or dusk, crossing the sky like a swift star, or a star chain if duly aligned, or a string of moonlets; who shall say? Astronauts are lavish, not bearing the cost of wanted empyrean splendors.

Our moon is roughly 2000 miles across and 240,000 miles away. Hence a moonlet could show as big a face if 240 miles from us and 2 miles across, or a nice little face if 1000 feet across. In this case a few thousand V-bombs could compose the cluster. Or they could emit a trail of snow or white sand to create a moonlet, and return home for more. In due time they should put elegant rings round the earth rivalling Saturn's, yes?

HEAT AND PRESSURE AT STAGNATION POINTS.

An air current sweeping over streamline craft heats their exposed surface by friction and compression. At stagnation points, where the flow comes to a full stop, the friction is null; the compression and temperature rise is calculable by established formulas. At these points the

compression and heating are greatest. Such hot spots may be at the nose or near the front wing edges of aircraft. The calculated pressures and temperatures may be tabulated for convenient reference.[4]

For adiabatic compression of common air on coming to rest from the speed v of a level stream at the initial pressure and temperature p_1, T_1, textbooks give

$$T_2/T_1 = (p_2/p_1)^{2/7} = 1 + nv^2$$

where T is the absolute temperature, p absolute pressure, n a well-known coefficient.[4] For sea-level U.S. standard air $T_1 = 518.4°$ F., and for full impact pressure at speeds up to 1000 miles an hour $n = 0.345259$. Then the temperature rise is

$$T_2 - T_1 = T_1nv^2$$

where $T_1n = 0.000179$, the product of the above numbers. Thus at the hot spots the temperature rise increases as the square of the stream speed, as shown in Table II. The pressure rise there given is taken from the cited paper.[4]

TABLE II.—*Increase of Pressure and Temperature of Sea-Level Air at Stop Points.*

v, mph.	100	200	400	600	800	1000
p_2-p_1, lb. per sq. ft.	25.69	104 09	438.33	1,073.0	2,140.6	3,860.4
T_2-T_1, deg. Fahr.	1.79	7.16	28.64	64.46	114.56	179.0

The table shows that at stagnation points moderate airplane speeds beget perceptible but not baneful increments of pressure and temperature. The highest present-day speeds show increments that invite the attention of designers. Still higher speeds of travel demand special provisions against excessive impact pressure and heating.

For such higher speeds the present formula gives excessive pressure. For $v > 4a$, Rayleigh finds $p_2/p_1 = 1.30v^2/a^2$, a being the speed of sound in the unchecked stream. Thus $T_2/T_1 = (1.3v^2/a^2)^{2/7}$. From these equations we derive Table II extended, where $p_1 = 2{,}116.8$ lb. per sq. ft. – 1,058 tons per sq. ft.

TABLE II. *Extended.*

v/a	4	8	12	16	20	24
p_2-p_1, tons per sq. ft.	20.95	86.97	197.1	351.04	549.1	729.18
T_2-T_1, deg. Fahr.	715.44	1,315.08	1,793.09	2,206.10	2,576.66	2,936.47

The values in the table apply to some rocket missiles and some projected airplanes. At four times the speed of sound the nose tem-

[4] A. F. Zahm and F. A. Louden, "Tables for Pressure of Air on Coming to Rest from Various Speeds," NACA Report No. 316, U. S. Government Printing Office (1929).

perature jumps over 700° F.; the nose pressure over 20 tons per square foot. A few materials can withstand both increments. But no material can endure the two increments listed in the last column. There the speed is roughly 5 miles a second, or that of the touted man moon circling just beyond the stratosphere. Obviously then if such moon bucked common air it would suffer both intense heat and mechanical rupture. Far more severe would be the heat and stress in the visionary jet plane cleaving the troposphere at 100,000 mph.

The values so far obtained are for stagnation points. Further equations and tables are needed for the distribution of velocity, pressure and temperature at other points of practical surfaces. For some geometric figures and flow conditions valid formulas and tables are available. Further researches are under way, especially with missiles and with models in high-powered wind tunnels. Both means can attain speeds many times that of sound, even ten times at present. Thus likely stagnation injuries can be forseen and obviated. Ample references are given in von Kármán's supersonic bibliography.[5]

[5] *Journal of the Aeronautical Sciences*, July, 1947, p. 398.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

DIAMONDS USED TO DETECT ATOMIC RADIATION.

Radioactivity studies conducted by Dr. L. F. Curtiss of the National Bureau of Standards have shown that diamonds are highly sensitive to gamma rays and may be used to detect this radiation in the same way as a Geiger-Müller counter. It has been found that a diamond placed in a strong electric field initiates sharp electrical pulses when gamma radiation is absorbed, and, as with a Geiger counter, a count of pulses gives an indication of the intensity of the radiation. The diamond counter has not yet been tested for beta radiation, but it is expected that a similar effect may be observed in this case.

Gamma rays—electromagnetic rays of very short wave length similar to X-rays—are constantly given off by radioactive materials. Of great value in many lines of research, they are used principally in medicine for treatment of malignancies, in industry for the radiographic examination of metallic castings, and in nuclear physics for the study of the structure and energy levels of the nucleus. The development of the diamond counter is expected to provide an important tool for scientists and technicians in these fields.

To use a diamond as a counter, it is clamped between two small brass electrodes maintained at a difference in potential of about 1,000 volts. When a source of gamma radiation is brought within range of the diamond, there occur across the electrodes pulses of current, which after amplification may be detected and counted on any suitable indicating device, such as an oscilloscope, a current meter, a set of earphones, or a loud speaker. In the apparatus assembled at the Bureau, primary amplification is effected with minimum loss of original intensity through the use of a triode very close to the diamond in the circuit. The output from this tube is then applied to a two-stage amplifier, from which pulses of sufficient magnitude are obtained to operate the detecting instrument.

The pulse-producing property of the diamond is thought to be a result of its highly symmetric crystalline structure, characterized by a very regular arrangement of carbon atoms with relatively large intervening spaces. According to this theory, when a photoelectron is emitted by a diamond atom as the result of the absorption of gamma radiation, the freed electron is accelerated through the interatomic space toward the positive electrode. Within a very short distance it

* Communicated by the Director.

acquires such high velocity that other atoms along its path are ionized by collision with the release of additional electrons, which in turn are accelerated in the same direction. This multiplication of charges repeats itself in rapid succession, producing a sudden avalanche of electrons equivalent to a small pulse of current. The larger the diamond the more electrons would be involved in the sudden pulse that is counted. This means that the gamma-ray sensitivity of a diamond counter should be proportional to the size of the crystal. However, adequate sensitivity is obtained with a comparatively small diamond. Apparently the diamond quickly recovers from its ionized state, as the pulses registered are extremely sharp. The diamond counter is thus a very "fast" counter, capable of indicating a much greater number of pulses per minute than is possible with the ordinary Geiger-Müller counter.

Similar principles have been utilized at the Bell Telephone Laboratories in experiments with alpha particles. Here, because of the poor penetration of this radiation, both electrodes were applied to the same crystal face, and the impinging particles were detected by means of the surface ionization they produced.

While gamma rays may produce photoelectrons in other crystalline substances such as sodium chloride, in most cases the crystal must be cooled to a very low temperature to eliminate background noise, which may be due to continuous ionization of the lattice at ordinary temperatures. The diamond is the only material so far investigated that performs satisfactorily at room temperature.

Industrial diamonds used as counters must be colorless and absolutely free of flaws; about one diamond in forty meets these specifications. Apparently color in a crystal, such as a diamond, indicates a change in the relation of outer electrons to atomic nuclei. Such a condition might tend to inhibit the generation of the required electrical pulse. Obviously, a flaw in the diamond would impede a surge of electrons through the affected portion of the crystal.

Diamonds tested in the Bureau's laboratories have been found to have a sensitivity per unit volume equal to or greater than that of any counter constructed by man. One of these diamonds, measuring about $\frac{1}{8}$ inch on each face, has approximately the same sensitivity for gamma radiation as a laboratory-constructed Geiger-Müller counter of the usual type. Many diamonds are larger and would thus be much more sensitive.

The conventional radiation counter lasts from three months to two years, depending upon how much it is used. A diamond counter, on the other hand, is practically indestructible, although extremely long use might produce discoloration or flaws, with a corresponding loss in sensitivity. There is no appreciable cost difference between the diamond and an ordinary counter. However, one of the important advantages of the diamond counter, in addition to sensitivity and long

life, is its small size, permitting use inside the human body or in small openings in industrial equipment.

Alpha, beta, and gamma radiations are important manifestations of the nuclear changes occurring in the disintegration of radioactive isotopes. Prior to the development of the atomic pile, radio-isotopes were produced in limited amounts in the cyclotron and other "atom smashers." Now, however, they are obtained from a chain-reacting pile cheaply and in quantities sufficient for extensive use as tracers in medical, biological, and industrial research. Basic to the use of a radio-element as a tracer is the sensitive detection of the presence of such an element by means of the radiation it continually emits. The detection of radioactive emissions is also important in experiments involving nuclear radiation, transmission and disintegration, as well as in safe-guarding the lives of those engaged in these projects.

The presence of extremely minute quantities of radioactive material may now be determined by means of the Geiger-Müller counter, one of the most sensitive of scientific instruments, but an increasing demand has resulted in a critical shortage of such equipment. For this reason, many laboratories have found it necessary to construct counters for their own use. In view of the relative abundance and moderate cost of industrial diamonds, as well as the simplicity and apparent indestructibility of the diamond counter, it is probable that Geiger-Müller counters may be replaced by diamond counters for many applications.

COSMIC AND SOLAR RADIO NOISE.

Scientists at the National Bureau of Standards are initiating a project for the observation and analysis of radio noise generated by the sun, a companion project to cosmic radio noise studies already in progress. The new investigation will seek to determine the range of frequencies broadcast from the sun, received intensities, and the correlation of solar noise with other solar, interstellar, and terrestrial phenomena.

Two giant radar mirrors at the Bureau's radio propagation laboratory at Sterling, Virginia, will intercept and record solar noise reaching the earth. These devices are particularly suitable for the investigation because of their size. The reflectors, about 25 feet in diameter, allow the capture of a large amount of energy from solar broadcasts. By automatic control, the mirrors will be directed at the sun constantly throughout the day. The first receiver is now in process of installation and will be used, initially, for studies in the 480- to 500-megacycle band.

With the use of higher and higher frequencies in communication and radar equipment, both solar and cosmic noise have come to be recognized as increasingly important. Recent advances in design for both very-high and ultra-high frequency receivers, which practically eliminate internal set noise, indicate that the limiting factors in the use

of the equipment will be those arising from natural phenomena as solar and cosmic noise.

Three general types of external noise that affect radio reception are of scientific interest. The first is atmospheric radio noise, or "static" originating within the earth's atmosphere which, with its characteristic crackle and crash, is familiar to every radio listener. This type of noise is actually the reception of radio energy produced by a lightning discharge. Extreme and prolonged static in the North American continent is generated for the most part in the Caribbean and South American thunder storm regions. Individual flashes of lightning not only accumulate to produce almost steady noise, but also are transmitted over long distances in exactly the same fashion as manmade signals.

Atmospheric radio noise ceases to be a major problem above about 15 megacycles, but it is at this frequency that cosmic noise becomes noticeable. Unlike the radio noise of terrestrial origin, cosmic, radio noise exists as a low steady hiss. In the case of FM equipment, the FM signal itself tends to suppress this noise within a certain range of the transmitting station. However, as distance from the station increases, the ratio between the strengths of the competing signals changes in the favor of cosmic noise until it completely drowns out the FM signal. The main center for the generation of cosmic noise is the constellation Sagittarius in the Milky Way. Because of this, there is a slow change in noise intensity as the position of the earth changes relative to the constellation.

Because of the similarity of the sound produced in the receiver, it has been suggested that cosmic noise may be due to radiation emitted by the thermal agitation of charged particles. The stars of the Milky Way throw off a large amount of material, which expands and tends to fill the intervening space as a very tenuous gas. Under the action of starlight, these atoms of gas are ionized with the production of positively and negatively charged particles, which radiate visible light and may also serve as sources of radio radiation.

Solar noise, which appears at ultra-high frequencies, has a basic component much like cosmic noise—a steady hiss. However, it also has an undulating component superimposed upon the stable noise. These variations are sometimes of great rapidity and manifest themselves in the form of "puffs" and "swishes" lasting a second or less. The swishes may overlap, giving rise to a grinding noise which may cause streaking on a television screen and picture jumpiness. Intense bursts of solar transmission that last as long as several hours cause a radar to become blind when pointed in the direction of the sun.

In the field of cosmic noise, the two chief problems to be solved are the determination of the intensity-versus-frequency function of the radiation and a more accurate survey of intensity versus position at

a variety of frequencies. Both of these problems are being attached at the Bureau. To investigate the first, a series of measurements are being made over the frequency range from 25 to 110 megacycles by means of a battery of specially designed receivers, each tuned to a particular frequency. The second problem requires the highest possible resolving power, which may be obtained either by going to higher frequencies or by using larger collectors. Both lines of attack are to be employed.

Comprehensive data on radio waves of celestial origin are expected to be useful in several applications. For example, a radio sextant might be built to determine position from the direction of arrival of solar noise. This device would permit navigation by the sun even though the sky is overcast and would have some advantage over loran in that it would be completely independent of ground stations.

RADIO PROXIMITY FUZE IN FIRE FIGHTING.

Airborne fire extinguishers equipped with the radio proximity fuze appear to offer a rapid and practical means for combating forest fires, according to extensive tests in which the Bureau participated, along with other Government agencies, during the summer. Characteristics of the radio proximity fuze [1] make it particularly adaptable for fire fighting. Because it bursts the fire-extinguishing bomb at the desired height above the ground, it sprays the extinguishing material, which may be water or a fire-smothering chemical, over the burning area. If the bomb does not burst until it hits the ground, nearly half of the material in the bomb remains in the crater and the remainder is sprayed over a very narrow area.

The radio proximity fuze, developed at the Bureau during the war, is an extremely small and tough radio sending and receiving station. Immediately upon being released, it begins to transmit radio signals. These signals are reflected back to the fuze from the ground, and when they reach a certain intensity or strength, the receiver triggers an electronic switch that detonates the bomb.

Millions of acres of timber are destroyed each year by forest fires in all parts of the country. In the Northwest forestry area, for instance, an average of 1,200 fires are caused by lightning alone during the months of July and August. Many of these occur in remote areas that cannot be readily reached with ordinary fire-fighting equipment. Aerial bombing, successfully carried out, provides a means for checking such fires as soon as discovered and preventing their spread.

The bombs for these tests were constructed from 165-gallon auxiliary fighter fuel tanks stabilized with the 2,000-pound general-purpose bomb fin. A fuze well was installed in the nose, and a burster well extended

[1] See "Radio Proximity Fuze," NBS Technical Bulletin 31, p. 3 (January 1947).

through the tank. The burster well included a charge to rupture the tank and disperse the extinguishing material after the proximity fuze had functioned.

Fires were purposely started in both brush and dense timber in the mountainous terrain of the Lolo National Forest in northern Montana. A B-29 bomber was used for tests of level bombing and P-47 *Thunderbolts* for dive and glide bombing. Forty-two bombs with proximity fuzes were dropped. Although the containers were not designed for bombing and ballistic data were not available, the accuracy of the bombing, particularly from the B-29, was very good. Improved accuracy, however, should be possible with bombs of known ballistic properties.

Plans for future tests include the use of foam instead of water, which should give a better extinguishing blanket and a better indication of the pattern. Other sizes of bombs will also be used, including a 4,000-pound light-case bomb that holds 260 gallons of foam, and a 500-pound light-case bomb holding 23 gallons; both of these will have proximity fuzes. In addition, tests are to be made with the 100-pound chemical bomb case, with a capacity of 8 gallons of fire extinguishing liquid; these will not be equipped with proximity fuzes.

The tests were conducted cooperatively by the United States Forestry Service and Army Air Forces, with assistance of National Bureau of Standards as representative of Office Chief of Ordnance. National Bureau of Standards prepared and furnished specially prepared VT fuzes and supervised their assembly and use. Flying was done by AAF and ground operations by USFS, centered at the Army Air Base, Great Falls, Montana, and at Forestry Headquarters for the Northwest Area, Missoula, Montana, respectively.

INSTITUTE OF NUMERICAL ANALYSIS ESTABLISHED.

Plans have been completed for the establishment of one of the newest units of the National Bureau of Standards—the Institute of Numerical Analysis—at the University of California at Los Angeles, according to an announcement by Dr. Edward U. Condon, Director of the Bureau.

One of the giant high-speed electronic computing machines, now under development by the Bureau of Standards, will be installed at the Institute when completed. These computers will solve problems in minutes that now take days to work out, and will solve in days problems that are now out of the reach of scientists. Design specifications call for high memory capacity and automatically sequenced mathematical operations from start to finish at speeds attainable only with electronic equipment.

The machines can conceivably revolutionize the field of applied mathematics. Of particular importance both to the physical sciences

and to technical industries will be the fact that the Institute will be able to set up a mathematical counterpart of an actual situation, which permits the situation then to be studied through relatively inexpensive calculating rather than costly experimentation. Great as has been the progress of the past century, the time has come when many problems of great importance, especially in hydrodynamics, aerodynamics, and meteorology, can only be handled by computers working at speeds measured in millionths of a second.

The Institute has two primary functions. The first is research in applied mathematics aimed at developing methods of analysis which will extend the use of the high-speed electronic computers. The second is to act as a service group for Western industries, research institutions, and government agencies. The service function will include not only the use of the machines for problem solving but also assistance in the formulation of problems in applied mathematics of the more complex and novel types. Service operations are to be initiated immediately, using the latest types of commercially available computing equipment.

The decision to locate the Institute at the University of California at Los Angeles was made after a nation-wide survey by the National Bureau of Standards. Centers in the East and Middle West were considered as well as the Far West, but Los Angeles, it was decided, offered the widest range of possibilities for an Institute of Numerical Analysis. Concentration of aircraft industries and the presence of several major scientific institutions were critical in the choice of Los Angeles.

HOME-PLUMBING STUDIES USE PLASTIC PIPE.

Hydraulic engineers at the Bureau are making a study of plumbing systems in which ordinary home plumbing is exactly duplicated except in one important respect—all of the pipes are made of plastic. The transparent pipes are being used so that investigators can see what's actually going on in a typical plumbing installation. This is supplemented with the data gathered from conventional pressure and water-level gages.

The research—directed at some of the more controversial plumbing problems—is being sponsored by the Housing and Home Finance Agency for the Uniform Plumbing Code Committee. Motion pictures are being made of the important tests so that local groups throughout the country can have visual facts to work with when plumbing code revision is under consideration.

Full-scale plumbing systems for two one-story houses and one two-story duplex have been erected. To complete the authenticity of the set-up, building and street sewers of conventional material were installed and means provided for loading the street sewer with a flow of up to 300 gallons per minute.

Three problems so far have occupied the investigators. They are: (1) self-siphonage; (2) stack venting; (3) wet venting. All three are problems connected with the proper venting of fixture traps.

Self-siphonage occurs when the length of the unvented waste pipe from the fixture—for example, the lavatory—is sufficiently long so that the water in the trap is pulled out by suction when the fixture is discharged. In plumbing drainage systems it is necessary to vent, in some manner, each fixture trap. The purpose of the vent is to keep pressures in the fixture drains sufficiently low so that the water seal in the fixture trap is not broken. Stack and wet venting are different methods of accomplishing this result. In stack venting the individual fixture wastes are led directly to the vertical stack, and the stack itself serves, under certain conditions, as a vent for the fixture attached to it. In wet venting, the drain from one fixture serves as a vent for the trap of another fixture.

Each of the above problems is being investigated under many different conditions, involving various lengths, diameters and slopes of fixture drains, various types of traps, and other variables.

That there has been no agreement among the experts in the field is attested to by the variance in plumbing code requirements in municipalities through the nation. It is expected that these investigations will establish facts which will be useful in leading the way to optimum conditions and requirements.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING, WEDNESDAY, MARCH 17, 1948.

The Stated Monthly Meeting of The Franklin Institute was held on Wednesday, March 17, 1948, at 8:15 P.M. in the Lecture Hall. Mr. Richard T. Nalle, President of the Institute, presided. There were approximately 375 persons in attendance.

The President announced that the minutes of the Stated Monthly Meeting of January had been printed in full in the February issue of the JOURNAL and if no additions or corrections were offered from the floor the minutes would stand approved as printed. There was no contrary motion.

The Secretary was then called upon for his report. He announced the following members had been elected to the Institute membership in the month of February.

Active	31
Associate	11
Student	17

It was then reported that the total membership in The Franklin Institute as of February 29 was 5412.

The Secretary then announced the Second Engineering Progress Show, which will be sponsored similarly to the previous one held last April, will be held in Franklin Hall from May 11 to May 16, inclusive, from noon till 9 P.M. each day. In addition to the Show itself there will be lectures in the Hall by distinguished engineers on the evenings of May 11, 12, 13, and 14.

The President presented Dr. E. U. Condon, Director of the National Bureau of Standards, Washington, D. C., who delivered a very comprehensive address on the work carried on by the Bureau of Standards.

Dr. Condon spoke of the inception of the Bureau of Standards and the activities of the Bureau in the fields of mathematics, physics and allied science. He said, however, that no work on medicine, food or biochemistry was done at the Bureau of Standards except on equipment that is used in those fields. Dr. Condon said that a great deal of military work was being done but at the present could not be discussed in detail. He presented a general picture of the testing and calibration services of the Bureau.

HENRY B. ALLEN,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, March 10, 1948.)

HALL OF THE COMMITTEE,
PHILADELPHIA, MARCH 10, 1948.

DR. JULIAN W. HILL in the Chair.

The following reports were presented for final action:

No. 3131: Houdry Process of Catalytic Cracking.

This report recommended the award of a Howard N. Potts Medal to Eugene J. Houdry, of Ardmore, Pennsylvania, "In consideration of his leadership in the development of the im-

portant process of catalytic cracking of petroleum that bears his name, utilizing known basic chemical and physical facts."

No. 3190: Levy Medal.

This report recommended the award of a Louis Edward Levy Medal each to Jan A. Rajchman and William A. Cherry, both of Princeton, New Jersey, "In recognition of their outstanding paper 'The Electron Mechanism of Induction Acceleration', appearing in the April and May 1947 issues of the JOURNAL OF THE FRANKLIN INSTITUTE."

JOHN FRAZER,
Secretary to Committee.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M., Wednesdays and Thursdays from 2 A.M. until 10 P.M.

RECENT ADDITIONS.

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LENT, CONSTANTIN PAUL. Rocketry. 1947.

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FAHSBENDER, MYRTLE. Residential Lighting. 1947.

ASTRONOMY.

ROSEN, EDWARD. The Naming of the Telescope. 1947.

BACTERIOLOGY.

WOLF, FREDERICK A. AND T. FREDERICK. Fungi, Volumes 1 and 2. 1947.

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KAMEN, MARTIN D. Radioactive Tracers in Biology. 1947.

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LEWIS, EUGENE W. Motor Memories. 1947.

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STOUGHTON, B. AND A. BUTTS. Engineering Metallurgy. Third Edition. 1938.

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TAYLOR, FRANK J. AND L. WRIGHT. Democracy's Air Arsenal. 1947.

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Techniques in Experimental Electronics, by C. H. Bachman. 252 pages, 14 × 22 cm., tables, drawings and illustrations. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd., 1948. Price, $3.50.

The Systematic Identification of Organic Compounds, by Ralph L. Shriner and Reynold C. Fuson. Third edition, 370 pages, 14 × 22 cm., tables and drawings. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd., 1948. Price, $4.00.

Stress Analysis and Design of Elementary Structures, by James H. Cissel. Second edition, 419 pages, 15 × 24 cm., tables, drawings and illustrations. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd., 1948. Price, $5.00.

Principles of Jet Propulsion and Gas Turbines, by M. J. Zucrow. 563 pages, 15 × 24 cm., tables, and drawings. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd., 1948. Price, $6.50.

A Practical Evaluation of Railroad Motive Power, by P. W. Kiefer. 65 pages, 14 × 22 cm., tables, drawings and illustrations. New York, Steam Locomotive Research Institute, Inc., 1948. Price, $2.00.

What Electronics Does, by Vin Zeluff and John Markus. First edition, 306 pages, 14 × 21 cm., drawings and illustrations. New York, McGraw-Hill Book Co., Inc., 1948. Price, $3.00.

Problems on Applied Thermodynamics, by Virgil Moring Faires, Alexander V. Brewer and Clifford M. Simmang. 151 pages, 15 × 23 cm., tables and diagrams. New York, Macmillan Co., 1948. Price, $1.70 (Paper).

FM Transmission and Reception, by John F. Rider and Seymour D. Uslan. 409 pages, 14 × 21 cm., tables and drawings. New York, John F. Rider Publisher, Inc., 1948. Price, $1.80 (Paper).

A Text-Book of Practical Organic Chemistry, by Arthur I. Vogel. 1012 pages, 16 × 26 cm., tables, drawings and illustrations. New York, Longmans, Green & Co., Inc., 1948. Price, $10.00.

Microwave Mixers, by Robert V. Pound. 381 pages, 16 × 23 cm., tables and drawings. New York, McGraw-Hill Book Co., Inc., 1948. Price, $5.50.

Broadcast Operators Handbook, by Harold E. Ennes. 265 pages, 14 × 22 cm., tables, drawings and illustrations. New York, John F. Rider Publisher, Inc., 1947. Price, $3.30.

Electrons in Gases, by Sir John Townsend. 166 pages, 15 × 24 cm., tables and drawings. London, Hutchinson & Co., Ltd., 1947. Price, 25s.

The Interpretation of Spectra, by William Mayo Venable. 200 pages, 15 × 23 cm., tables and drawings. New York, Reinhold Publishing Corp., 1948. Price, $6.00.

Electric Powner Transmission, by M. P. Weinbach. 362 pages, 16 × 24 cm., tables and drawings. New York, Macmillan Co., 1948. Price, $5.50.

Proteins and Amino Acids in Nutrition, edited by Melville Sahyun. 566 pages, 16 × 23 cm., tables, drawings and illustrations. New York, Reinhold Publishing Corp., 1948. Price, $7.50.

Hydraulics, by Horace W. King, Chester O. Wisler and James G. Woodburn. Fifth edition, 351 pages, 14 × 22 cm., tables and drawings. New York, John Wiley & Sons, Inc., 1948. Price, $4.00.

Practical Marine Engineering, by Reno C. King, Jr. 470 pages, 15 × 21 cm., drawings and illustrations. New York, Prentice-Hall, Inc., 1948. Price, $6.00.

BOOK REVIEWS.

AMERICAN ANNUAL OF PHOTOGRAPHY, 1948, Vol. 62. 216 pages, illustrations, 18 × 25 cm. Boston, American Photographic Publishing Co., 1947. Price, $2.00.

This standard photographic annual is so well known that there is little need to comment on its interest and value. In scope it follows the practice of past years containing a group of articles, a section of selected photographs, and a directory section.

The lead article by one of the editors, Frank R. Fraprie, illustrates and discusses sixteen of his prints which have met with unusual success in salon showings. J. W. McFarlane contributes a useful article on the making of photographic exploded views describing several methods. Technical problems are considered by H. C. McKay who writes on the art of exposure, by R. J. LeBlanc who discusses sensitometry, and by T. H. Miller who comments on factors to consider in film selection.

Pictorial possibilities of bridges are treated by W. S. Davis with several revealing examples. The section of photographs has many striking pictures. Technical data about these illustrations are also included. The directory section includes a "Who's Who" in pictorial photography, "Who's Who" in color photography, and "Who's Who" in nature photography. There is, in addition, a list of amateur photographic organizations world-wide in scope.

G. E. PETTENGILL.

THE SCIENTIFIC PAPER, HOW TO PREPARE IT, HOW TO WRITE IT, A HANDBOOK FOR STUDENTS AND RESEARCH WORKERS IN ALL BRANCHES OF SCIENCE, by Sam F. Trelease. 152 pages, 13 × 18 cm. Baltimore, Williams & Wilkins Co., 1947. Price, $2.00.

Essentially a revised edition of the author's book "The Preparation of Scientific and Technical Papers," this remains a practical little volume for students and research workers faced with the problem of writing scientific papers. Among the major topics treated are the arrangement of the paper, revision of the manuscript and preparation of final copy, literature citations, abbreviations of periodical publications, and illustrations. This latter is quite detailed considering the correct proportions of the illustrations and different types such as drawings, graphs, photographs and lantern slides. There are many other helpful hints including the use of tenses, acknowledgments, shipping instructions for illustrations, proofreading, and the use of capitals and italics.

A new feature of the work is a section on the use of the library for research purposes prepared by Miss Amy L. Hepburn. Unfortunately there is one quite outdated statement about the planning of a union card catalogue in Philadelphia. Actually the Union Library Catalogue of the Philadelphia Metropolitan Area was incorporated in 1936, now has well over 3,000,000 cards and since 1937 has been successfully fulfilling its primary function of locating books. Mention might well have been made of the fact that the most important union catalogue in the country is that at the Library of Congress and that there are other regional catalogues besides that in Philadelphia. Likewise, in the section on indexing and abstracting tools, *Engineering Abstracts* is not current, having been published only from 1910 to 1913.

These errors do not affect the essential usability of the book, which contains in addition references to other sources for fuller information.

G. E. PETTENGILL.

TIMBER ENGINEERS' HANDBOOK, edited by Howard J. Hansen. 882 pages, illustrations, 19 × 21 cm. New York, John Wiley & Sons, Inc., 1948. Price, $10.00.

During the past decade there has been considerable research on the use of wood as a structural material, bringing with it the need for the correlation of the data developed. The present book attempts to meet that need in providing a ready tool for the designing engineer.

The preliminary chapters are devoted to basic data, including factors affecting strength of wood, grading rules, working stresses, sizes, and properties of sections. One chapter is devoted to terms used in describing standard grades of lumber and to lumber abbreviations. Weights are fully covered, not only weights of lumber but also those of building materials, roofings, nails, screws, and many other varied substances. Load requirements are given for various kinds of construction.

Then follow design information and data on various constructional elements: beams — simple, continuous and trussed; columns—solid and spaced; trusses; fastenings; and floor systems. A long chapter considers buildings and various aspects of both light-frame and heavy timber construction. Glued laminated construction and plywood are likewise treated in some detail.

An appendix actually comprising more than half the nearly nine hundred pages is a table of uniformly distributed loads. This covers spans ranging from 3 to 50 ft. for beams of various sizes and eleven different fiber stresses. Throughout the entire volume an effort has been made to include all woods used in the United States rather than limiting it to those of any one region.

The usefulness of this volume is readily apparent.

G. E. PETTENGILL.

MAGIC SHADOWS, THE STORY OF THE ORIGIN OF MOTION PICTURES, by Martin Quigley, Jr. 191 pages, illustrations, 15 × 23 cm. Washington, D. C., Georgetown University Press, 1948. Price, $3.50.

From the depiction of a boar trotting along in a "motion" still picture on a cave wall 10,000 or more years ago to April 23, 1896 the date of the "most significant motion picture premiere" with Edison's Vitascope, this volume records the gradual realization of the "art of magic shadows." The first identifiable figure in the development of motion pictures is that of Aristotle who made the experiment of "the square hole and round sun" and noticed the existence of after-images. Archimedes used the burning lens, Roger Bacon worked in optics and he or one of his pupils left a description of the room camera. There are other figures mentioned among whom are A. Kircher, who invented the magic lantern; Joseph Plateau and his magic disc; Eadweard Muybridge who made successive still photographs of a horse in motion; and finally Edison.

One chapter brings the story to Philadelphia and concerns the Langenheims who developed a process for making positives on glass slides, and two members of the Franklin Institute, Coleman Sellers and Henry Heyl, who devised machines for showing "posed" motion pictures.

Mr. Quigley has produced a readable, entertaining narrative of the developments leading to the first successful motion picture machine. It is a story of technical achievement and the

culmination of a long desired goal. One chronology, recording in outline form the important dates in the text, forms an appendix and another is devoted to a bibliography of sources consulted.

G. E. PETTENGILL.

THOMAS JEFFERSON AMONG THE ARTS, AN ESSAY IN EARLY AMERICAN ESTHETICS, by Eleanor D. Berman. 305 pages, illustrations, 14 × 21 cm. New York, Philosophical Library, 1947. Price, $3.75.

Among the great figures in American history is that of Thomas Jefferson. He was a man of many interests who achieved his greatest fame as a statesman. Among his Virginia neighbors, his fame as an architect was nearly as great, and he was responsible for the designing of many of the great houses in that state.

In the present volume Dr. Berman has considered Jefferson's relations to all the various arts in which he played a part or in which he was interested. These include painting, sculpture, architecture, gardening, music, oratory, rhetoric and poetry, fiction and letter writing. She traces the derivation of Jefferson's esthetic doctrines and considers each of the arts in relation to one another. Documented, and with illustrations and index, this is an interesting study about our third president.

G. E. PETTENGILL.

ORGANIC SYNTHESES, AN ANNUAL PUBLICATION OF SATISFACTORY METHODS FOR THE PREPARATION OF ORGANIC CHEMICALS, VOL. 27. 121 pages, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1947. Price, $2.25.

To those familiar with earlier volumes of Organic Syntheses the appearance of a new volume with thirty-six additional syntheses will be welcome news. For those who may not know the series, each volume contains some thirty to forty preparations of compounds which are of general interest or which illustrate useful synthetic methods. Compounds are entered under the names commonly used, with the *Chemical Abstracts* indexing name also given if it is different. The formulas indicating the reactions are followed by the names of the submitters and checkers.

The procedure is given with clarity of detail and is accompanied by notes indicating sources of supply, or variations in procedure. Appropriate warning is given of poisonous or otherwise dangerous procedures. The range of yield is reported as are also such data as melting point, boiling range at various pressures, and the refractive index. A section on methods of preparation lists the different methods with citations to the literature. A cumulative index of Volumes 20 to 27 is included.

With each preparation checked to insure accuracy, the series assumes an important place for those who need ready access to tested preparations of organic syntheses.

G. E. PETTENGILL.

THEORY OF SERVOMECHANISMS, edited by Hubert M. James and others. 375 pages, drawings, 16 × 23 cm. New York, McGraw-Hill Book Co., 1947. Price, $5.00.

This twenty-fifth volume of the Radiation Laboratory Series (Massachusetts Institute of Technology) is divided into two sections. The first deals with frequency response techniques of servomechanisms, the second with statistical methods. Each section is provided with an extended mathematical introduction and many illustrations.

HENRY N. MICHAEL.

FUNDAMENTAL ELECTRONICS AND VACUUM TUBES, by Arthur Lemuel Albert, revised edition. 510 pages, drawings and illustrations, 16 × 24 cm. New York, The Macmillan Co., 1947. Price, $6.00.

In his text-book the author has treated electronics as fundamental to both the communication and power fields. Also the theoretical principles are illustrated by applications from both fields. Thus the book is especially suited for college electrical engineering courses.

The first part of each chapter contains the basic information, with the specific information and illustrative applications following.

After discussing basic electronic theory the text takes up specific subdivisions in the following sequence: Emission of Electrons, Thermionic Cathodes, Two-Three, and Multi-electrode Thermionic Vacuum Tubes, Rectifiers, Voltage and Power Amplifiers, Oscillators, Modulators, Demodulators, Photoelectric Devices, and Cathode-ray tubes.

HENRY N. MICHAEL.

TEXTILE BRAND NAMES DICTIONARY, by The Textile Book Publishers, Inc., first edition. 377 pages, 16 × 24 cm. New York, Textile Book Publishers, Inc., 1947. Price, $6.00.

This dictionary is a compilation of more than 4,000 names of fibers, yarns, fabrics and garments, obtained originally from the official *Gazette* of the U. S. Patent office (1934 to 1947). It is divided into three sections. The first section lists alphabetically the brand names with their serial numbers, dates of registration, owners, etc. The second section classifies the most important names under several headings—yarns, rayon piece goods, cotton piece goods, synthetics, silks and others. Part three contains an alphabetical list of the companies owning the brand names, with a cross-reference to the page on which they appear in the first section.

This handy book should prove of definite value to manufacturers, convertors, buyers, patent attorneys, and should be readily applicable whenever questions of ownership, invention and registration arise.

HENRY N. MICHAEL.

WATERBURY'S VEST-POCKET HANDBOOK OF ENGINEERING, fourth edition, revised by H. W. Reddick and others. 386 pages, tables, 7 × 14 cm. New York, John Wiley & Sons, Inc., 1947. Price, $2.50.

This handy volume, originally published in 1908 and now appearing in a fourth edition, has been revised and brought up to date. Several new sections have been added covering the fundamentals of mensuration, the fundamental principles of heat transfer and the basic principles of illuminating engineering. Other subjects included are various phases of mathematics, mechanics, heat engineering, electrical engineering, and radio and electronic formulas. Five-place logarithmic and trigonometric tables are given, too. Lists of the notations used appear at the beginning of most sections and an index completes the volume.

In revised form this volume should continue to render useful service in presenting in concise, convenient form useful formulas and data for the engineer.

G. E. PETTENGILL.

ALEXANDER DALLAS BACHE, SCIENTIST AND EDUCATOR, 1806–1867, by Merle M. Odgers. 223 pages, port. 14 × 20 cms. Philadelphia, University of Pennsylvania Press, 1947. Price $2.75.

"Its most distinguished member" was the way the Franklin Institute referred to Alexander Dallas Bache on the occasion of his death. Not only was he a distinguished member but he had also been an active member, having served as one of the Board of Managers, as Corresponding Secretary, as first Chairman of the Committee on Science and the Arts and as Chairman of the Committee on the Explosions of Steam-Boilers, among other duties. Undoubtedly the Institute was also interested in the fact that Bache was the great grandson of Benjamin Franklin for whom the Institute was named, and that he revealed something of Franklin's versatility and furnished "the professional complement to Franklin's amateur interest in science."

Bache was born in 1806 and died in 1867 and during that period he lived two separate and distinct careers. After graduating from West Point at the head of his class, he taught there for one year and then served two years in the Army. At the age of twenty-two years he was appointed Professor of Natural Philosophy and Chemistry at the University of Pennsylvania, a position he held until he was appointed president of the newly proposed Girard College in 1836. His first duty in this latter post, and as it developed his primary one, was to tour Europe to study the schools there and to make recommendations as to the establishment of the college. This survey resulted in the publication of his famous "Report on Education in Europe." Subsequently Bache became the first head of Central High School in

Philadelphia, general superintendent of the city schools and finally returned to the University of Pennsylvania in 1842, where he closed his educational career a year later.

During this early period he had been active in scientific research, particularly in his work on the report of the Committee on Explosions of Steam-Boilers and in his researches on terrestrial magnetism. He had established a magnetic observatory at Girard College which functioned from 1840 to 1845 and he had also made many other magnetic observations on his trip to Europe and around Pennsylvania.

In December 1843 Bache was appointed Superintendent of the U. S. Coast Survey and it was during his administration that the Coast Survey was established as an important scientific agency. Not only did Bache contribute his scientific knowledge to the Survey, but he also developed a competent staff and displayed real tact and diplomatic skill in his relations both in and outside the Survey. He served his country in yet other ways, among which may be mentioned his duties as Superintendent of Weights and Measures of the United States and as a Regent of the Smithsonian Institution. In this latter position he had much to do with persuading Joseph Henry to head the Institution and was influential in shaping its policy. Bache was also instrumental in founding the National Academy of Sciences and was its first president.

Mr. Odgers has given a revealing and sympathetic treatment of his subject. Considerable attention is devoted to Bache's European trip with numerous quotations from his journal which are illuminating. Several chapters discuss Bache's work as a scientist but not so technically that it would confuse the general reader. The final chapter, "The Chief," dwells particularly on his personal characteristics and points up the humanness and friendliness of the man. A thoroughly enjoyable volume, which this reviewer read with great pleasure, it should appeal especially to these groups: educators, scientists, members of The Franklin Institute, and the many other organizations with which Bache was associated. As a picture of one of the men who did his share toward the development of education, science and life in America it will be rewarding reading for everyone.

G. E. PETTENGILL.

SUNSPOTS IN ACTION, by Harlan True Stetson. 252 pages, illustrations, 14 × 21 cms. New York, Ronald Press Company, 1947. Price $3.50.

Frequently one reads in the newspaper that sunspots have interrupted radio communications, yet many readers may be puzzled as to how this happens. In his latest volume, Dr. Stetson, Director of the Cosmic Terrestrial Research Laboratory at Needham, Mass., explains what takes place. This involves the nature of the radiation from the sun, particularly the ultraviolet rays which produce ceilings in the earth's atmosphere responsible for reflecting the waves used in long-distance radio. Sunspots can completely disrupt these reflecting layers through the increased radiation from the sun.

The author also considers other disturbers of radio communication as magnetic storms, northern lights, solar eclipses and the moon, which investigation has revealed shows a relationship between its phases and the quality of radio reception.

Sunspots are described and the various hypotheses as to their origin are set forth, although this still remains a puzzle for the scientist to solve. The periodicity of sunspot maxima is shown and the attempts at prediction of the next maximum are given, with the observation that there are still not sufficient data so that observers agree on their interpretation.

A relation between sunspot cycles and long period changes in the weather or climate has been revealed by Professor Douglass's notable study of tree rings which register the solar cycles through the centuries. Various other possible relationships between sunspots, the weather and living things are discussed in addition to a chapter outlining the theories relating to sunspots and economic cycles which have been advanced.

A bibliography and a table of relative sunspot numbers from 1749 to 1947 are included. Written in a readable style and avoiding highly technical matter, this volume adequately fulfills its purpose of informing the layman as to the present knowledge of sunspots and their effects.

G. E. PETTENGILL.

LES CATHODES CHAUDES, THÉORIE ET PRATIQUE, by Charles Biguenet. 183 pages, illustrations, 14 × 22 cms. Paris, Editions de la Revue d'Optique Théorique et Instrumentale, 1947. Price 300 francs.

This volume on the hot cathodes has been written with the aim of appealing to the specialist as well as to the general scientific public. The author commences with a review of industrial vacuum technique including a description of various pumps and vacuum measuring instruments. The following chapters deal with some of the phenomena of adsorption whose theory is important in the working of cathodes, and with the theory of electronic emission, particular attention being paid to modern conceptions.

In considering the electronic emission of pure metals, primary emphasis is given to tungsten, for which many valuable tables and graphs are included. Molybdenum and mercury are also noted. From the pure cathodes the author turns to a consideration of the composite cathodes, treating first the monoatomic coated cathode of thoriated tungsten and then the various oxide cathodes which are of commercial importance. One chapter is devoted to the manufacture and treatment of oxide cathodes. A concluding chapter discusses other kinds of hot cathodes, principally various barium types.

This new book fills a gap in the French literature on the hot cahode and will be welcomed as an up to date concise review of the subject.

G. E. PETTENGILL.

CIRCUITS AND MACHINES IN ELECTRICAL ENGINEERING, by John O. Kraehenbuehl and Max A. Faucett. Volumes I and II. Second edition. 367 and 470 pages, drawings and illustrations, 15 × 24 cms. New York, John Wiley & Sons, Inc.; London, Chapman and Hall, Limited, 1947. Price $4.25 each.

Since the text of the first edition has been used for electrical engineering students as well as for other engineering students, the authors have changed the material of the second edition to meet both purposes. The 1947 edition is divided into two volumes—one on circuits, one on machines.

The material in the first volume emphasizes the circuit as the basic element in the machine and in the study of machine operation, rather than as a part of the communication circuit. The emphasis is on power.

Some of the major changes of the second edition occur in the chapter on electronics, which has been considerably expanded. The material on ignitrons is new.

The first chapter contains some material on the use of the mks system, all the general text material follows the cgs system.

The second volume deals with electrical machinery only. The machines are treated as direct and alternating-current ones, rather than as generators and motors as in the first edition.

The transformer chapter has been expanded to include a discussion on harmonics.

New problems have been added throughout the text and the attempt has been made to give problems which lead to analysis rather than to substitution into expressions which appear in the developments.

HENRY N. MICHAEL.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Some Factors in the Nessler Method of Nitrogen Determination.—JAMES L. LEITCH. The following factors involved in the use of the Nessler reagent for the determination of nitrogen are discussed in a paper by Miller and Miller (1): (1) effect of time and completeness of digestion, (2) effect of addition of hydrogen peroxide and the optimum use thereof, (3) rate of addition of reagent to digested sample, (4) rate of mixing of reagent with sample, (5) stability of color, (6) temperature of final solutions, and (7) effects of interfering substances. The purpose of this paper is to present additional data (*a*) dealing with the effect of the amount of sulfuric acid used for the digestion on the final color intensity, and (*b*) extending the range of the method by the use of various light filters.

Effect of Sulfuric Acid Concentration.—Miller and Miller recommend the use of 0.4 ml. of 1:1 sulfuric acid. When large samples are digested with increased volumes of acid, they dilute the final digest with distilled water and take an aliquot for color development so that it contains the equivalent of 0.4 ml. of 1:1 sulfuric acid. Koch and McMeekin (2) vary the amount of Nessler reagent added in relation to the amount of acid used in the digestion so that the final solutions have approximately an alkalinity equivalent to 0.56 per cent sodium hydroxide.

In attempts to use the Miller and Miller method in this laboratory, it was found that slightly low colorimeter readings were obtained at the higher nitrogen levels. These low values were associated with the appearance of a red precipitate of mercuric oxide suggesting that the final alkalinity was too low.

To study further the effects of the sulfuric acid concentration on the final color intensity, standard ammonium sulfate solutions were analyzed with from 0.2 to 0.7 ml. of 1:1 sulfuric acid for the digestion. The method used in these analyses was essentially the short method of Miller and Miller with the following modifications:

1. The volume of sulfuric acid was varied from 0.2 to 0.7 ml.
2. Before the initial period of digestion, two drops of 30 per cent hydrogen peroxide (Superoxol, Merck) were added to eliminate bumping. Miller and Miller have shown that such a procedure may give low results with certain substances. However, for routine analyses, this slight disadvantage is considered to be greatly outweighed by the elimination of the more serious problem of bumping.
3. Repeated inversion for a period of 40 sec. after the addition of the reagent gave satisfactory results.

4. Filter No. 50 was used instead of filter No. 54.

Typical results are summarized in Table I. As a further check on these results, digested samples, after dilution and addition of Nessler reagent,

TABLE I.—*Effect of Volume of 1:1 Sulfuric Acid Used for Digestion on the Intensity of Color Development with 5.0 ml. of Nessler Reagent.*

Volume of 1:1 H_2SO_4 used in digestion, *ml.*	Colorimeter readings with filter No. 50 Micrograms of nitrogen in samples					Remarks
	0	50	100	150	200	
0.20	8	69,68	137,137	206,206	274,274	
	7	68,69	138,137	209,205	273,275	
0.30	6	69,67	137,138	205,205	273,274	
	7	67,68	136,138	206,206	274,275	
0.40	5	68,69	137,137	204,206	268,266	
	4	67,68	137,137	205,201	270,268	
0.45	4	68,66	135,130	202,200	264,264	Slight red precipitate in some samples
	5	65,66	135,134	199,199	262,261	
	6		135,133		260,261	
0.50	0	36,37	108,110	119,181	206,204	Variable amounts of red precipitate in all samples
	5	36,36	116,30	131,162	199,232	
	0		117,95		227,204	
0.55–0.70	Heavy red precipitate in all samples with little or no color in solution.					

were titrated with normal hydrochloric acid using phenolphthalein as indicator to determine the concentration of alkali in the final sample. Representative results of these titrations are given in Table II.

The data in Table I indicate that a straight-line relationship exists between the concentration of nitrogen and the colorimeter reading, provided not more than 0.3 ml. of 1:1 sulfuric acid is used in the digestion. When 0.4 ml. of acid is used, low and somewhat variable results were obtained as the nitrogen content was increased above 150 micrograms. As the concentration of acid is further increased, the color intensity gradually decreases and becomes more variable until at 0.55 ml. and above, a heavy red precipitate of mercuric oxide is formed with little or no color in solution. The data on residual alkalinity as given in Table II indicate that the straight-line relationship is obtained in the presence

TABLE II.—*Residual Alkalinity of Digested Samples after Dilution and Addition of 5.0 ml. of Nessler Reagent.*

Volume of 1:1 H_2SO_4 used, *ml.*	Residual alkalinity	
	As Normality	As per cent NaOH
0.20	0.249	1.00
0.30	0.157	0.71
0.40	0.122	0.49
0.45	0.080	0.32
0.50	0.037	0.15
0.60	Acid to phenolphthalein	

of 0.71 per cent sodium hydroxide but not below 0.49 per cent. These data agree closely with those of Koch and McMeekin who reported satisfactory results with an alkalinity of 0.56 per cent sodium hydroxide. On the basis of these data it would seem advisable to carry out the digestion with not more than 0.3 ml. of 1:1 sulfuric acid. It is realized that greater volumes of acid could be used, providing the volume of the Nessler reagent was also increased so that the final residual alkalinity

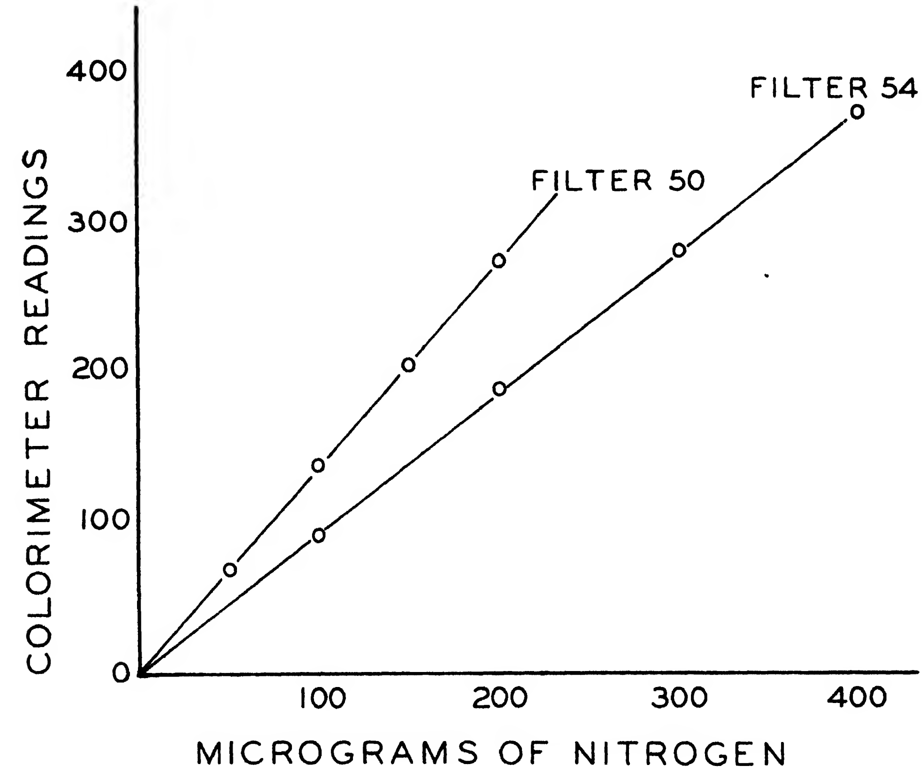

FIG. 1. Calibration curves when filters Nos. 50 and 54 were used.

fell within the optimum range. However, such a change in the procedure would unnecessarily complicate the method.

Relation between Nitrogen Range and Color Filter.—In the method of Miller and Miller, filter No. 54 was used for the determination of nitrogen in the range between 100 and 300 micrograms. In the data reported herein obtained by using only 0.3 ml. of acid for digestion, satisfactory results were obtained with filter No. 54 for samples containing 100 to 400 micrograms, with filter No. 50 for 50 to 200 micrograms and with filter No. 42 for 5 to 50 micrograms of nitrogen, respectively. Representative calibration curves are given in Figs. 1 and 2 for all three filters. One difference was found between the use of filter No. 42 and that of

filters Nos. 54 and 50. With the latter two, a blank reading of from 4 to 8 was always obtained, but the readings of samples containing various amounts of nitrogen did not have to be corrected for this blank value. On the other hand, when filter No. 42 was used for samples containing 5 to 50 micrograms of nitrogen, the data as given in Fig. 2 were obtained only after subtracting the blank value. No explanation can be made from the available data for this peculiarity.

Recommended Procedure.—On the basis of the work reported herein, the following method was finally adopted:

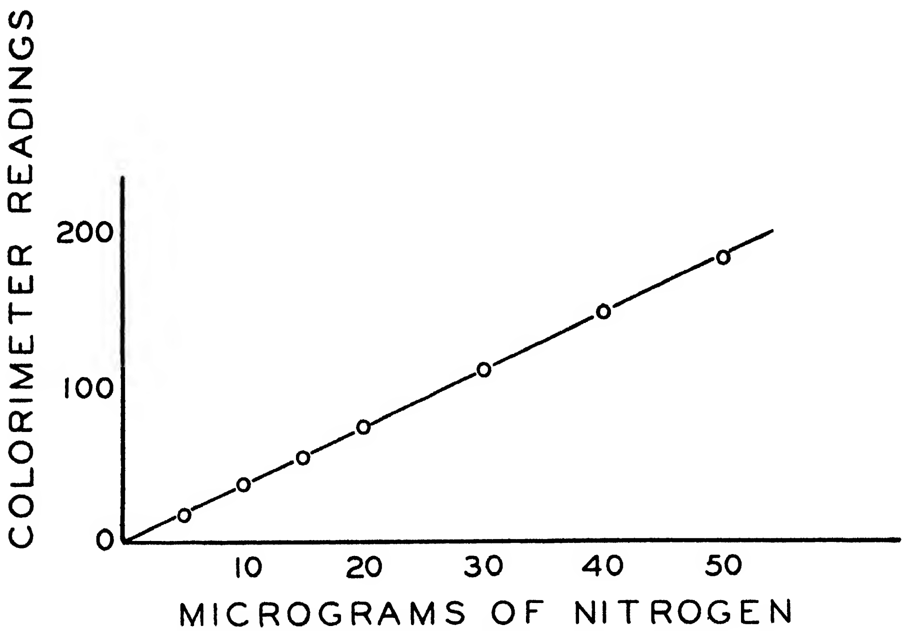

FIG. 2. Calibration curve when filter No. 42 was used.

To the sample (not over 0.5 ml.) is added 0.3 ml. of 1:1 sulfuric acid followed by two drops of 30 per cent hydrogen peroxide (Superoxol, Merck). All digestions are carried out in 18 × 150 mm. Pyrex test tubes. After 5-min. digestion, and cooling for 1 min., Superoxol is added dropwise (usually 2 to 4 drops) to complete decolorization of the sample followed by 2-min. digestion. Cooling, addition of 2 drops of Superoxol and digestion are then repeated twice with 5-min. digestion after the last addition of Superoxol. Additional treatments with Superoxol and additional periods of digestion are used for refractory substances. After completion of the digestion, the samples are allowed to cool for approximately 30 min. before the addition of 20 ml. of distilled water. Then to each sample is added 5.0 ml. of Nessler reagent prepared according to the procedure of Koch and McMeekin. Mixing

of the reagent with the sample is satisfactorily carried out by repeated inversion for a period of 40 sec. after the addition of the reagent. Thirty minutes later, all samples are read with a Klett-Summerson photoelectric colorimeter using the appropriate filter. Analyses were always carried out in duplicate concurrently with suitable standards prepared from ammonium sulfate.

Other volumes of sulfuric acid may be used for the digestion, provided that a volume of Nessler reagent is used sufficient to give a minimum final alkalinity of 0.7 per cent sodium hydroxide.

Summary.

It has been shown that the amount of sulfuric acid used in the digestion of samples for the determination of nitrogen by the Nessler method may have a decided effect on the final color intensity. When 5.0 ml. of Nessler reagent are used for color development, a straight-line relationship between colorimeter readings and nitrogen concentrations is obtained, provided not more than 0.3 ml. of 1:1 sulfuric acid is used for the digestion. Furthermore, the method described herein can be applied to a wide range of nitrogen concentrations, from 5 to 400 micrograms, by appropriate choice of filters.

References.

(1) MILLER, G. L. AND MILLER, E. E., *Ind. Eng. Chem., Anal. Ed., In press.*

(2) KOCH, F. C. AND MCMEEKIN, T. L., *J. Am. Chem. Soc.*, **46**: 2066 (1924).

CURRENT TOPICS.

Iron Ore Supply.—Plans by Lake Superior district iron mining companies for large-scale refining of low-grade iron ore to guarantee American industry a continued supply of high-grade iron are attracting the interest of the engineers of the General Electric Company.

Iron ore companies are turning to the refining or "beneficiation" process, as it is popularly known in the industry, on an industry-wide basis because the supply of high-grade ores will be exhausted in from 20 to 30 years at the present rate of consumption, engineers said.

With beneficiation, the low-grade ores, now plentiful but unmarketable because of their low iron content, will be refined near the mines to increase the grade prior to shipment. Only by this method can the Lake Superior district, which includes the rich Minnesota Mesabi range, continue to supply about 85 per cent. of the nation's annual 60 to 70 million-ton-supply of iron ore, engineers explained.

Electric equipment will be needed by the mining companies in crushing and screening, concentration, and sintering plants, where most of the equipment will be electrically powered, engineers said. This equipment will include generators, motors, transformers, unit substations, switchgear, controls, and other electric apparatus.

General Electric is now supplying some of this equipment for pilot plants in the early stages of the beneficiation program. The company has assigned a special staff of engineers to study approved and new electric systems for the process, and experience is being gained from those industries which have used beneficiation methods for many years.

The new development will also have an important bearing on the power companies in the mining district. Engineers said the new process would require from 60 to 80 kilowatt hours per ton as compared to 1½ to 4 kw.-hr. per ton average for the present methods, which, in most cases, consist only of crushing and washing some ores before shipment.

Two major wars and the great demand for steel products already in the 20th century have resulted in a drain on the high-grade deposits, which were naturally beneficiated by the forces of Mother Nature. Aware they are facing a time limit on high-grade ore production, the mining companies are taking over nature's job on the low-grade ores, and directing their research and economic efforts to beneficiation, it was explained.

The low-grade ores contain about 30 per cent. iron. Known in the iron ore industry as "taconite," it must be refined for smelting, engineers pointed out. Of all Lake Superior ore produced today, only 20 per cent is now beneficiated and shipped to supplement the balance of high-grade ores.

When suitable processes have been worked out, this percentage will become higher, and engineers predict that nearly 100 per cent. of all the ore produced three decades from now will be the low-grade variety, and will require beneficiation.

R. H. O.

Electric Furnace Brazing Facilitates Manufacture of Insecticide Bombs.—Electric furnace brazing greatly facilitates the fabrication of insecticide bombs at the Aer-a-sol Division of the Bridgeport Brass Company, Bridgeport, Conn. Adoption of this method of brazing has inproved the quality of these bombs, simplified their construction, and lowered manufacturing costs. The containers, which are made of SAE-1010 steel .043–.049 inch thick and contain a liquid insecticide under about 90 pounds pressure, are fabricated at an average of approximately 10,000 per eight-hour shift. Two General Electric continuous furnaces are used.

Originally, these containers were fabricated by a bonding method which involved locally heating each joint and using brazing metal and flux. With this method, considerable difficulty was encountered after brazing in removing deposits of charred flux from the interior of the bombs. This was necessary since loose particles tended to clog the .008-inch diameter orifice of the specially designed dispensing valve of the bombs. In addition, there was the ever-present danger of corrosion from moisture trapped in the bomb. Furthermore, a large percentage of the containers leaked.

With electric furnace brazing, inexpensive copper is employed as the brazing metal, and no flux is required. Consequently, the bomb assemblies come from the furnaces clean and bright, no oxides or foreign matter are present, the corrosion and moisture hazard is eliminated, and the bonds are uniformly tight and strong, testing almost 100 per cent. free from leaks. In addition, all four of the assembly joints are bonded simultaneously in a single trip through either of the two continuous furnaces.

With this brazing process, the assemblies are put together with the braz-metal preplaced near the joints to be brazed. As the containers pass through the heating chamber of the furnace, a reducing atmosphere frees the metal from any oxides present, prevents the steel from oxidizing, and thus prepares the parts to be wetted by the molten copper. When the brazing metal melts, it is drawn into the joints of the assemblies by capillary attraction and forms alloys with the steel. Transferred to the adjoining controlled atomosphere cooling chamber, the solidifying alloys develop great strength, and the bombs gradually cool to a temperature at which it is safe for them to come in contact with the outside air without danger of discoloration because of oxidation.

Each of the roller hearth copper brazing furnaces used in this applica-ation consists essentially of a heating chamber nine feet in length and an adjoining water-jacketed cooling chamber 30 feet long. The heating chamber is equipped with electric heating units rated 180 kw., divided into two 90-kw. zones, each with separate power and temperature control. The heating units are made of heavy rolled ribbon, formed in sinuous loops, mounted in the roof of the chamber and on the side walls above and below the roller conveyor running through the chamber. The cooling chamber is made of two concentric rectangular steel shells to provide a water jacket, and is divided lengthwise into three sections, each with a separate water circuit. The first section of the cooling chamber has automatic cooling water temperature control, to prevent condensation during idling periods.

Hollow cast alloy rolls are used throughout the charging vestibule, heating chamber, and the first section of the cooling chamber. Solid steel rolls are employed in the remaining sections. Water-jacketed, self-aligning roller

bearings are mounted on the sides of the casing except for the last two cooling chamber sections, which do not require cooling. The rolls are motor driven through a chain and sprocket mechanism.

Each furnace is capable of brazing 700 containers per hour, which is a production of 700 pounds net, or 1,352 pounds gross per hour. For this rate of production, the power consumption is about 157 kw.-hr. per hour. Consumption of combusted gas for the furnace atmosphere is about 1,500 cubic feet per hour, which is made by burning about 675 cubic feet per hour coke oven gas.

Throughout, the bombs are fabricated almost entirely on an assembly line basis. Belts or roller conveyors carry them from one operation to another until they arrive at the furnaces, where they are manually placed in the brazing trays. The trays are then automatically charged into the furnaces and travel slowly through them on motorized conveyors. After being discharged from the furnaces and inspected, the dispensing valves are added and the bombs are air-tested, marked, and charged with liquid insecticide and then with Freon under pressure. Attached to an overhead chain conveyor, they are cleaned, lacquered, and again inspected. A belt conveyor carries them to the final station, where they are labelled, visually inspected, and finally packed for shipment.

R. H. O.

50 Per Cent of Perishables May Be Frozen by 1960. (*Refrigerating Engineer*, Vol. 53, No. 5.)—Government economists recently declared that within 10 years, 50 per cent of all perishable foods will be preserved by freezing. Today, though the amount of food frozen has increased 300 per cent since pre-war, only 2 per cent of the country's perishable foods is frozen.

Frozen food interests look to trucks for a big share of the unprecedented task of moving literally millions of tons of food at temperatures of zero or colder, the Refrigeration Equipment Manufacturers Association has declared. Frozen food economists estimate that more than 10,000 new refrigerated trucks and trailers are needed at once for coast-to-coast shipments and door-to-door deliveries and that lighter, smaller, and more economical refrigeration equipment for truck installation must be produced in quantity.

In production today are units for trucks of all types which supply the necessary zero temperatures. Temperatures as low as −10 F. with outside temperatures of 110 F. are obtained within properly insulated reefers by means of so-called packaged units. For the large semi-trailers, models supplying two tons of refrigeration are most frequently used, while for the somewhat smaller direct mounted bodies there are models at one ton capacity.

Push-button operation and automatic temperature control require little attention from the truck driver. Cargoes of fresh produce can be transported on one stretch of the trip and by changing the dial setting for lower temperatures, a load of frozen food can be carried on a return trip. Dial settings on this equipment make it possible to provide a wide range of temperatures, depending upon the kind of perishable produce to be transported.

R. H. O.

New Process Control Instruments.—New electric measuring instruments, capable of providing a continuous analysis of flowing streams of fluids and gases, together with automatic control of those processes, give promise of advances for the chemical industry.

This was recently revealed in a paper presented before the annual meeting of the Institute of Chemical Engineers. The speaker was Everett S. Lee, chief engineer of General Electric's General Engineering and Consulting Laboratory at Schenectady, N. Y.

Speaking on "New Process Control Instruments for the Chemical Industry," Mr. Lee described several instruments recently developed "which measure basic quantities to give opportunity for continuous analysis of a flowing process stream." He asserted that chemical industry spokesmen long have pointed to such devices as the industry's greatest need in the field of instrumentation.

"A particularly interesting example of the new process control instruments for the chemical industry is the X-ray photometer, which provides economical and rapid means of making chemical comparisons," Mr. Lee said. "With this instrument it has been found possible to determine satisfactorily the tetraethyl lead content of gasoline, the concentration of an acid in water, the per cent chlorination of a plastic, or the per cent ash in coal. These determinations are made by measuring and comparing X-ray absorption of a sample and a reference."

An outstanding advantage of this method of comparison lies in the fact that X-ray absorption always is the same for a given material, whether it is hot or cold, gaseous, liquid, or solid, or whether the element being measured is alone or in chemical combination. Too, analysis is made without loss or alteration of the sample tested.

R. H. O.

Oregon Installs Radiant-Heated Road. (*Engineering News Record*, Vol. 139, No. 18.)—To end winter skidding danger on an 8 per cent highway grade separation at Klamath Falls, the Oregon Highway Department is now building a section of radiant-heated pavement. This steep grade will be winter-proofed by an adaptation of radiant panel heating making use of nearby natural hot springs as a heat source.

A similar idea was used last winter in Belmont, Mass.

A 10-in. well 250 ft. deep near the highway provides natural hot water, with temperatures up to 220 F. A 2-in. main runs from the well to the pavement and parallels it for about 400 ft. In the pavement slab is placed $\frac{3}{4}$-in. pipe, 18-in. o.d., in 60-ft. panels. Each panel will be connected with the 2-in. hot-water main by valves which will assure an even flow of water through each panel.

The 2-in. main and the series of 60-ft. panels of $\frac{3}{4}$-in. pipe will comprise a closed circuit. Hot water from the wells will not be allowed to enter this circuit. Instead, the circuit will be filled with ordinary water in which antifreeze solution will be introduced. This water will be warmed by means of a heat transfer unit comprised of 2-in. pipe extending close to the bottom of the well in order to obtain maximum immersion. Inserted in the closed circuit will be a pump, thermostatically controlled. When air temperature drops to

the freezing point, the pump will automatically start, circulating the water through the closed circuit. At the same time, another pump will begin exhausting hot water from the well at a rate sufficient to maintain the maximum temperature in the well. Unless hot water is so exhausted, constant circulation of the cold water through the heat transfer unit will have a tendency to cool off the well.

The pumps will continue to operate until air temperatures rise above the freezing point.

R. H. O.

New Airplane Catapult. (*Electrical Engineering*, Vol. 66, No. 10.)—The "electropult," an unusual type of induction motor manufactured by the Westinghouse Electric Corporation, Pittsburgh, Pa., is designed for launching airplanes that require long take-off runs from small landing fields without the high inertia impact imparted by other catapults. The secondary winding of the motor is the track of the catapult, and its primary winding is the car to which the airplane is harnessed.

Two electropults are now in use by the Navy, one at Mustin Field, Philadelphia, Pa., and a larger one at the Naval Air Test Center, Patuxent River, Md. The track of the larger unit is 1,382 feet long, and is made up of 76 sections with 12-inch active core width, that are set flush with the surface of the runway. Resistance of the first 1,000 feet of track is decreased progressively in four steps. This graduation of resistance enables the tractive force to be held substantially constant as the speed is increased. The last 382 feet are used for stopping the car by dynamic braking and application of direct current.

The car projects only $5\frac{1}{2}$ inches above the surface of the runway. On the under side of the car is the 3-phase primary winding in a flat core at a working air gap of $\frac{3}{16}$ inch. The car wheels run on rails recessed in slots which straddle the stationary secondary winding, and a set of rails above the wheels prevents the car from being lifted. A space under the track houses the bus bars and collector rails that carry approximately 7,000 amperes during acceleration and 10,000 amperes during braking. The current collecting problem is extremely difficult because the car attains a speed of 225 miles per hour. The 12 collector shoes per phase are held against the collector rails by spring pressure. Provision is made for the pilot of the airplane to release the harness at the proper time.

One of the features of the launcher is a short-time power demand of approximately 12,000 kw. The power supply consists of a 2,200-horsepower aircraft engine coupled to a 750-kw. d-c generator which supplies a d-c motor connected to a 24-ton flywheel and 3-phase alternator that supplies the catapult car. During the short time required to launch an airplane, the flywheel loses speed to furnish approximately 95 per cent. of power needed. Slowing down of the flywheel and alternator reduces the frequency from 216 to about 192 cycles per second.

The electropult has launched jet-propelled airplane at 116 miles per hour in 4.1 seconds after a run of only 340 feet. Unassisted, the same airplane requires a take-off run of approximately 2,000 feet.

R. H. O.

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THERMODYNAMICS, PART II: WORK, HEAT, AND TEMPERATURE CONCEPTS, AND AN EXAMINATION OF THE TEMPERATURE SCALE.

BY

J. L. FINCK, Ph.D.[1]

ABSTRACT.

In this paper the concepts of work, heat, and temperature are examined from the standpoint of a complete equation (1)[2] for the internal energy of a system. In an incomplete system energy necessarily remains with the system, which results in the inefficiency of a process. This inefficiency is, of course, appreciable for processes taking place at a finite speed, but is not necessarily eliminated by means of a quasistatic process.

The Kelvin temperature scale is shown to be based on an incomplete equation for the internal energy and, therefore, offers certain difficulties. A temperature scale based on a complete equation is shown to have advantages over the Kelvin scale, and it is further shown that the temperature of absolute zero as based on a complete equation is lower than that on the Kelvin scale

A suggestion is offered whereby an actual system may be treated as complete and a temperature scale may be based on this.

1. INTRODUCTION.

The present considerations are based directly on the results developed by the writer in a previous paper (1). For this reason it may be well, at the start, to state briefly the conclusions which were arrived at in this paper, which are as follows:

(*a*) The *number* of independent variables which are selected to define a given thermodynamic system is crucial to the concepts of "equilibrium" and "reversibility," as well as to the second law of thermodynamics. If this number of independent variables is *not less than the minimum* required to define the system completely, we shall have complete conversion of heat into work, and work into heat. In this case, the equation

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[2] The boldface numbers in parentheses refer to the list of references appended to this paper.

of state for the internal energy E, written as

$$E = \phi(\alpha_1, \cdots, \alpha_n), \tag{1}$$

where $\alpha_1, \cdots, \alpha_n$ are the n independent variables, is called a *complete* equation of state.

(*b*) If the number of independent variables is less than this minimum number, the equation of state for E will be *incomplete*, and heat cannot be converted completely into work, although work can be converted completely into heat. That is, the Kelvin-Planck and Clausius principles hold only for incompletely described systems, which is the case in practice.

(*c*) Suppose that of the n variables in Eq. 1 the variables $\alpha_1, \cdots, \alpha_i$ are actually observed and controlled, whereas we are not aware of the existence of, or for practical reasons we decide not to use, the remaining variables $\alpha_j, \cdots, \alpha_n$. Consider a change in E given by

$$\begin{aligned} dE &= \left[\frac{\partial E}{\partial \alpha_1} d\alpha_1 + \cdots + \frac{\partial E}{\partial \alpha_i} d\alpha_i\right] + \left[\frac{\partial E}{\partial \alpha_j} d\alpha_j + \cdots + \frac{\partial E}{\partial \alpha_n} d\alpha_n\right] \\ &= dE_{\text{I}} + dE_{\text{II}}. \end{aligned} \tag{2}$$

If in a cyclic process there results a net conversion of heat into work, the writer has shown that

$$\int_{q \to W} dE > 0, \tag{3}$$

where $q \to W$ indicates a process where the net effect is a conversion of heat into work.

In the present paper we wish to analyze the heat, work, and temperature concepts in detail from the same point of view, and shall also examine the temperature scale. The Kelvin absolute temperature scale is based on Carnot cycle, and the perfect gas is the thermometric system which is used for establishing this temperature scale. Actual gaseous systems deviate somewhat from the perfect gas law, and so correction terms have to be applied to the observations. This temperature scale, it is claimed, has the desirable characteristic that it is independent of the nature of the material system which is used for the Carnot cycle. However, the writer has shown (1) that there are serious difficulties in the use of Carnot cycle when a single phase system is subject to more than two independent variables. For example, the efficiency of a Carnot cycle operating between two fixed temperatures is not necessarily unique, but may have many arbitrary values. Then, again, from a practical standpoint, it is desirable to use for our thermometric system a gas which deviates as little as possible from the perfect gas law. If not, the correction terms for reducing our observations to the Kelvin scale become too large and the accuracy very poor. The fact that Kelvin's scale is theoretically independent of the nature of the material

system is no great comfort to the experimenter, for he cannot use systems where the temperature is measured by electric resistance, or by thermo-electric emf., or magnetic susceptibility, and reduce the results directly and rigorously to the Kelvin absolute scale. Practically, he is still restricted to the so-called permanent gases, and above all to systems which are subject to only two independent variables. At the very low temperatures, where matter manifests many unusual characteristics, it would seem to the writer that a rational temperature scale would be of extreme importance, and one which is lacking at present.

2. WORK AND HEAT CONCEPTS FOR COMPLETE AND INCOMPLETE SYSTEMS.

Let us assume that the internal energy E is defined *completely* by Eq. 1. On the basis of the first law of thermodynamics we must assume E to be a single-valued point function of the generalized space $(\alpha_1, \cdots, \alpha_n)$. The total differential will be

$$dE = \frac{\partial E}{\partial \alpha_1} d\alpha_1 + \cdots + \frac{\partial E}{\partial \alpha_n} d\alpha_n = -(A_1 d\alpha_1 + \cdots + A_n d\alpha_n), \quad (4)$$

where $A_p = -(\partial E/\partial \alpha_p)$ by definition. E is then a potential function and the *complete* system is conservative (see reference (1), Section 6). Each A will, in general, be a function of $\alpha_1, \cdots, \alpha_n$. Some of the α's will be configuration coordinates, so that the corresponding A's will be mechanical forces. Other α's may be electric charge and magnetic flux, so that the corresponding A's may be the electric and magnetic field intensities. In addition to these, some α's may be the degrees of transformations, with their corresponding A's as the latent heats; also, some α's may be the masses of the chemical components, and the corresponding A's the chemical potentials of Gibbs.

To the experimenter ϕ is complete only when he is able in some way to observe and measure each α which enters into Eq. 1, although we may consider in theory the consequences of certain α's being present. The terms of Eq. 4 which have reference to mechanical and electromagnetic quantities we shall, as is customary, refer to as work and the corresponding A's as generalized forces. Those terms of Eq. 4 which refer to inter- and intra-phase transformations we shall refer to as (latent) heat. Those involving masses and chemical potentials may be classified as one or the other, depending on which is more prominent. It is interesting to note the following: since each A is, in general, a function of all α's, it is possible to realize changes in mechanical and electrical properties by chemical changes, and *vice versa*. Also, it is possible to realize changes in electric and magnetic properties by mechanical changes, and *vice versa*. All these various types of phenomena are actually observed, and if the system is complete in the sense defined by Eq. 1 the conversion of one form of energy into another will also be complete.

Suppose, however, that the system is *incomplete*—meaning that only

the variables $\alpha_1, \cdots, \alpha_i$ are observed and measured, whereas the variables $\alpha_j, \cdots, \alpha_n$ have been neglected either unconsciously because we are not aware of their existence, or because we know of no means whereby to measure them; or for practical reasons we decide to neglect them. Whatever the reasons may be, we have no information on the magnitudes and changes in $\alpha_j, \cdots, \alpha_n$ and $A_j, \cdots, A_n$. Should we attempt to form a closed cycle by varying only the quantities $\alpha_1, \cdots, \alpha_i$ and finally returning them to their original values, in general the quantities $\alpha_j, \cdots, \alpha_n$ and their corresponding A's will not return to their original values. Furthermore, since $A_1, \cdots, A_i$ are, in general, each a function of all n independent variables, when we return only the quantities $\alpha_1, \cdots, \alpha_i$ to their original values we do not thereby necessarily return $A_1, \cdots, A_i$ to their original values *entirely*. We see then that for an incomplete system the energy represented by

$$- \sum_{p=j}^{n} \int A_p d\alpha_p \tag{5}$$

can no longer be recovered as work, or latent heat of a specific transformation. Also, of the total energy represented by

$$- \sum_{p=1}^{i} \int A_p d\alpha_p \tag{6}$$

some portion will, in general, not be recoverable. In going through the apparent closed cycle the integrals of Eq. 6 will not necessarily vanish completely. Let us write

$$- \delta \sum_{p=1}^{i} \int A_p d\alpha_p \tag{7}$$

for those portions of the energy (work and latent heats) which remain with the system as a result of the apparent closed cycle. The total energy remaining with the system as the result of the apparent closed cycle may then be written as

$$q' = - \delta \sum_{p=1}^{i} \int A_p d\alpha_p - \sum_{p=j}^{n} \int A_p d\alpha_p \tag{8}$$

and this is the energy which produces the "heating up" of the system and is responsible for the inefficiency of the process.

For the remaining portions of the energy of this cycle we have

$$q - W = - \sum_{p=1}^{i} \int A_p d\alpha_p + \delta \sum_{p=1}^{i} \int A_p d\alpha_p, \tag{9}$$

where q is the heat added to the system and W the work done by the system. For this apparent closed cycle we have, then

$$q - W = q' \qquad \text{or} \qquad q = W + q'. \tag{10}$$

Let us consider here a few examples. First, for a single gaseous phase we should write, according to classic thermodynamics,

$$E = \phi_1(p, v) = \phi_2(p, T). \tag{11}$$

In carrying the system through a closed cycle at a finite rate, we find experimentally that the system "heats up," and this effect is greater for, say, ammonia vapor as compared to that for helium or hydrogen. We have actually neglected a variable, and to make Eq. 11 complete (or more complete) we should add the degree of transformation x. As the writer has shown (2), this variable x, by its absence, accounts for metastable states and such states are more pronounced in the case of ammonia as compared to that for helium or hydrogen.

A second example may be a plastic, such as rubber. In this case we have many coefficients of elasticity which are independent of each other, so that an equation of the form $E = \phi(s, l)$, where s is the linear tension and l the linear strain, is surely incomplete. In addition to the coefficients of elasticity we might, under certain circumstances have to add x's to describe chemical or physical transformations.

A third example may be the hysteresis cycle of, say, a piece of steel. By writing $E = \phi(H, B)$, where H is the magnetic field intensity and B the magnetic flux, we have neglected mechanical, crystalline and possibly other essential properties of the system, and we know from experience that the energy of a hysteresis cycle depends on the past history as well as on H, B.

3 ACTUAL AND QUASISTATIC PROCESSES.

Let us now examine the significance of an actual process which takes place at a finite rate, and that of a quasistatic process.

Equilibrium was considered in a previous paper (1). Assuming that n independent variables define our system completely, we may say that when all α's are held constant with respect to time, no work of any sort can be done by or on the system, for the α's include all the configuration coordinates. Furthermore, there will be no internal changes such as chemical transformations (inter- and intra-phase changes) or changes in mechanical, electric or magnetic properties. We may then define *true equilibrium* as that state of the system where $\alpha_1, \cdots, \alpha_n$ are constant with respect to time.

For a complete system there is necessarily no difference whether changes in the α's are made very slowly or at a finite rate, for in either case the resulting change in E is clearly defined and of the same value. But, now let us consider an incomplete system where only $\alpha_1, \cdots, \alpha_i$

are observed and controlled. The quasistatic process calls for changes in $\alpha_1, \cdots, \alpha_t$ that are infinitesimally slow. We are, however, not certain that the variables $\alpha_j, \cdots, \alpha_n$ have not been altered, for they are not observed. If changes in $\alpha_1, \cdots, \alpha_t$ are made at a finite rate we know from experience that the cycles are not closed and that they depend more or less on the past history of the system. From the above considerations this is understandable because, since $A_1, \cdots, A_t$ are functions of all α's, then if $\alpha_j, \cdots, \alpha_n$ vary in some unknown way, at the beginning of each successive cycle we start with different values of $A_1, \cdots, A_t$. It is conceivable that changes in $\alpha_j, \cdots, \alpha_n$ may be very small, or even nil, when the changes in $\alpha_1, \cdots, \alpha_t$ are infinitesimally slow. However, that must be proven for each particular system, and cannot be taken as a generalization. We have no basis for assuming that a process is reversible only when it is quasistatic. Thus the conception of a quasistatic process, even in theory, does not overcome the difficulties encountered by an incomplete system. In fact, this concept adds serious restrictions to thermodynamics for it limits our study only to static systems.

As an alternative it would appear to the writer that the study of systems solely from the point of view of degree of completeness is clearer and much more informative. We must recognize the fact that certain indeterminacies which exist with systems are unavoidable, and can be diminished only as we derive more knowledge of the systems. From our present knowledge of systems it would appear that the rarer the system the more completely could it be defined, as, for example, rarified charges in an electromagnetic field, or rarified matter in a gravitational field. Such systems are conservative. Cycles involving gases at low pressures are more "efficient" than cycles involving liquids, and the latter are more "efficient" than cycles involving solids. Also, since crystalline media are more clearly defined than plastics it would appear that cycles involving crystalline solids are more "efficient" than those involving plastics.

4. THE "TRUE" TEMPERATURE SCALE.

We shall assume that Eq. 1 defines the internal energy of a thermodynamic system completely. As regards temperature, we shall assume that we know in a general way (empirically) what this means. Temperature may then be one of the independent variables of the system. To be able to do this, however, we should have to establish a temperature scale which is independent of the other α's of the system under consideration; in other words, temperature must be defined on some scale not related to the system.

It should be noted that the essential requirement for an equation of state to be complete is that the number n of independent variables shall be *not less than the minimum required to define the system completely.* The

particular choice of variables to be used may be arbitrary and differ with circumstances. We may, for example, desire to write for our temperature

$$T = \psi(\alpha_1, \cdots, \alpha_n), \tag{12}$$

where T will be a *derived* quantity and will depend on the system. The system may then be used as a thermometer to define its particular temperature scale. On this scale, temperature will be a function of, say α_1, *only if* $\alpha_2, \cdots, \alpha_n$ *are held constant.* In practice, when we state that T is a function of (p, v) for a fluid, or of the electric resistance, or of the thermoelectric emf., or of the magnetic susceptibility, we employ incomplete relations, for on close examination we may find other independent quantities that should have been introduced into the equation for each particular system. In other words, in practice we do not use all required α's to define T, and for that reason each system will, by itself, establish a temperature scale which, in general, will be different from the others.

Suppose we consider two systems, A and B, each to be defined completely by the same set of n independent variables. Should one system require a lesser number of variables, we may still use n for both in our formal considerations, bearing in mind that some of these variables may be of constant value or zero. If A is defined completely by $\alpha_1, \cdots, \alpha_k$, and B by $\alpha_l, \cdots, \alpha_n$, we may still retain our formal Eqs. 1 and 12 above, recognizing that for B we have $\alpha_1, \cdots, \alpha_k$ constant or zero, and for A we have $\alpha_l, \cdots, \alpha_n$ constant or zero.

If the temperature scale for each system is given by a ψ function, for equality of temperature let us write

$$T = \psi_A(\alpha_1, \cdots, \alpha_n) = \psi_B(\alpha_1, \cdots, \alpha_n). \tag{13}$$

Assuming that A and B are any two systems of the entire class whose equations of state are complete, we may utilize each of these systems to establish a temperature scale, and with Eq. 13 all of these scales will be identical. Let us refer to this as the "true" temperature scale.

Is the Kelvin absolute scale a "true" temperature scale? The Kelvin scale is entirely restricted to a two-variable system, both from the standpoint of Carnot cycle and from the perfect gas law. We know from experience that no material system, including a single gaseous phase, is completely defined by only two independent variables (2). Therefore, Kelvin's scale rests on a hypothetical system where, say, in Eq. 12, $\alpha_1 = p$, $\alpha_2 = v$, and $\alpha_3 = \cdots = \alpha_n = 0$. Since actual systems do not meet these requirements, we should be inclined to state that the Kelvin scale is not a "true" temperature scale in the sense defined by Eqs. 12 and 13.

Logically there appears to be no objection to adopting Kelvin's as our standard temperature scale. In doing so, temperature will be an independent quantity, not derived from a complete equation for an

actual system. Kelvin's scale will be independent of material systems in the sense that further knowledge of the behavior of systems, which may lead to a more complete equation of state, will not yield additional information nor accuracy with regard to this scale. Kelvin's scale fixes the absolute zero, and from the classic standpoint this scale should have no negative values.

The objections to Kelvin's scale would come mostly from practical considerations, but those are quite serious. In spite of the fact that Kelvin's scale is designed for a perfect gas, we must actually work with gases which are more or less imperfect. At the very low temperatures, such as the liquefaction point of helium, our working gas is very imperfect; at still lower temperatures we may not have any gas to work with. We certainly cannot interpret directly and rationally (except by extrapolation) measurement of, say, magnetic susceptibility into temperature on the Kelvin scale. At the very low temperatures we are forced, for practical reasons, to consider T as a derived quantity, meaning that we must resort to an incomplete equation (that is, Curie's law).

At the higher temperatures, say above room temperature, objections may be raised as to the way observations on actual gaseous systems are reduced to Kelvin's temperature scale. These objections may be based on the fact that it is assumed that $T = f(p, v)$ for the gas, whereas actually it should be a function of more than two variables (2).

Should we decide to use the "true" temperature scale in our investigations, we should of course also encounter difficulties, for all equations of state which are used in practice are incomplete. That is the basis for the validity of Kelvin-Planck and Clausius principles (1). In practice we do not use or know all n independent variables, and our systems are studied under conditions in which uncertainties do exist. It would appear then that there must be uncertainties in our evaluation of the "true" temperature T. However, the experimenter is always confronted with inaccuracies in his measurements, and intuitively he seeks more and more controls (or more and more α's) in order to make his measurements more reproducible and more accurate. As the science of thermometry progresses we should expect, then, that "true" temperatures as measured by different systems will come closer and closer in agreement, whereas Kelvin's scale is set once for all time and very little more can be done for it. For these reasons the writer would be inclined to favor our considering temperature as a derived quantity based on Eq. 12 and to define our scale on the basis of Eqs. 12 and 13.

5. CARNOT CYCLE AND THE INCOMPLETE EQUATION OF STATE.

We shall consider a system whose state may theoretically be completely described by Eqs. 1 and 12, but in practice we shall operate it only as an incomplete system. Let us assume that $\alpha_1 = p$, $\alpha_2 = v$ (that is, $i = 2$) are the variables which are observed and controlled.

We shall carry this system through an apparent closed cycle in a manner such that the net heat absorbed by, and the net work done by the system are both positive. As far as possible we shall try to make this a Carnot cycle, and shall proceed as follows.

Assume that two large heat reservoirs are available, each of which is kept at a constant temperature as measured on some arbitrary scale which we shall indicate by θ. Let the first reservoir be at the temperature θ_1, and the second at θ_2, where $\theta_1 > \theta_2$. We shall make a distinction between the scales θ and T, where the former is to serve only as a means for detecting fluctuations and to aid in keeping the reservoirs at a constant temperature, whereas T is the true temperature scale defined by our system.

Referring to Fig. 1, let us assume that along the path AB the system is in direct contact with the first reservoir. During the time of this

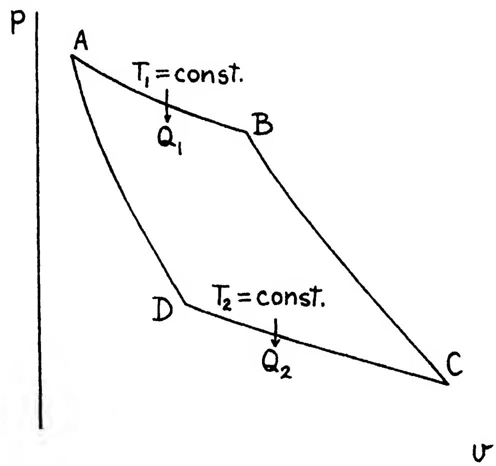

FIG. 1.

contact we change (p, v) of the system, keeping T constant and equal to T_1. Along this path the internal energy will change in value, so that the heat taken by the system from the first reservoir will be, by Eq. 10,

$$\int_A^B dq = Q_1 = \int_A^B dW + \int_{A'}^{B'} dq'. \tag{14}$$

We write the limits for the last integral as A', B' instead of A, B because, in general, there would be some uncertainty in the states defined by the limits of $\alpha_3, \cdots, \alpha_n$, and A' may be somewhat different from A, and B' from B.

From B to C we desire to subject the system to an adiabatic process. For this we write

$$\int_B^C dq = 0 = \int_B^C dW + \int_{B''}^{C''} dq', \tag{15}$$

where the last integral is to take on the uncertainties, and B'' is somewhat different from B', B and C'' somewhat different from C.

Along the path CD we place the system in direct contact with the second reservoir, which if measured by the system is at the temperature T_2. The process along this path is an isothermal, but in the reverse sense to that along AB. The path DA is an adiabatic similar, but in the reverse sense, to BC. Thus,

$$\int_C^D dq = -Q_2 = \int_C^D dW + \int_{C^{\text{III}}}^{D^{\text{III}}} dq' \tag{16}$$

and

$$\int_D^A dq = 0 = \int_D^A dW + \int_{D^{\text{IV}}}^{A^{\text{IV}}} dq'. \tag{17}$$

Adding these four equations we have

$$\oint dq = Q_1 - Q_2 = W + \Delta q'. \tag{18}$$

It will be noted that if the system were completely defined by p, v, then $\Delta q' = 0$ and the process would be a true Carnot cycle regardless of whether the process were quasistatic or not.

6. KELVIN'S TEMPERATURE SCALE AND ABSOLUTE ZERO.

Kelvin's temperature scale is defined on the basis of Carnot cycle by writing

$$\frac{Q_1 - Q_2}{Q_2} = \frac{T_1 - T_2}{T_2}. \tag{19}$$

If our thermometric system consists of a single phase, this temperature scale rests on the assumption that $T = \psi(p, v)$, or any other two independent variables. If the system is *complete* on the basis of these two variables, then

$$Q_1 - Q_2 = W \tag{20}$$

and

$$\frac{W}{Q_2} = \frac{T_1 - T_2}{T_2}. \tag{21}$$

However, if the system is incomplete on the basis of two independent variables, we must use Eq. 18 and write

$$\frac{W + \Delta q'}{Q_2} = \frac{T_1 - T_2}{T_2}. \tag{22}$$

Suppose we let T_1 represent the boiling point of water, and T_2 the ice point, both taken under standard conditions. Write

$$T_1 - T_2 = 100. \tag{23}$$

Substituting Eq. 23 into Eq. 21, we have for a complete system

$$T_{2C} = 100\frac{Q_2}{W}. \tag{24}$$

Substituting Eq. 23 into Eq. 22, we have for an incomplete system

$$T_{2I} = 100\frac{Q_2}{W + \Delta q'}. \tag{25}$$

T_{2C} is the ice point on the temperature scale defined by the complete equation $T = \psi(p, v)$, whereas T_{2I} is the ice point on the temperature scale defined by the incomplete equation $T = \psi(p, v)$, where the complete equation should be $T = \psi(p, v, \alpha_3, \cdots, \alpha_n)$. From Eqs. 24 and 25, we have

$$\frac{T_{2I}}{T_{2C}} = \frac{W}{W + \Delta q'} < 1 \tag{26}$$

since $\Delta q' > 0$. Therefore

$$T_{2I} < T_{2C}, \tag{27}$$

which states that the *ice point as determined by a complete equation of state is higher than that determined by an incomplete equation of state.* Or, with reference to the ice point, *the absolute zero as determined by a complete equation of state is lower than the absolute zero as determined by an incomplete equation of state.* In both cases the equation *as used* involves only two independent variables. Complete equations of state of any number of independent variables, as we have seen above, should yield the same value for absolute zero, so that $-T_{2C}$ is the ultimate lowest temperature below the ice point that can be reached. This is most likely unattainable for the reason that as we examine a given system more and more closely, we may find more and more independent variables and will therefore never attain a complete equation of state.

7. A SUGGESTION FOR A TEMPERATURE SCALE.

We have seen that Kelvin's temperature scale rests on the notion that we have a complete equation of state for a system, and that this equation is made up of just two independent variables for each phase. It is hardly likely that we shall find these conditions satisfied over the entire temperature range, high and very low, even to a moderate degree of accuracy. In view of this, and in the light of the above considerations, the writer wishes to propose a new temperature scale for the reader's consideration. This is, of course, merely a suggestion, subject to further study both from the theoretical and experimental standpoints.

We have seen above that a change in the internal energy dE of a system will, in general, arise from the following three sources: (1) work of a mechanical, electric or magnetic character done by or on the system, which is observable and can be measured; (2) latent heats of trans-

formations and heats of chemical reactions, both of which are observable and can be measured; and, finally, (3) energy which remains with the system due to the incompleteness of the system and which cannot be attributed to any particular form of work or latent heat.

Suppose we subject a system to energy changes in a manner such that no work of any sort is done by or on the system. As a specific example, let us take a cylinder of quartz in which is installed a fine electric heater. We can impart energy (heat) to this system without this energy resulting in any external work. Whatever transformations or chemical changes take place in this system, will be measured by the heat added to the system. In fact, the system may be incomplete as far as our knowledge of it is concerned, and still the heat added to the system will measure *all* energy, provided that the work is zero. In this way we can treat the system as complete and accurately measure its total energy changes. All necessary experimental precautions, such as enclosing the system in a vacuum and surrounding it with radiation shields, may be taken to prevent heat losses, and we can then write for the complete system $dE = dQ$, with $dW = 0$, where dQ is the measureable heat input.

Let us, then, define the new temperature scale θ by writing the following:

$$dE = dQ = Md\theta, \tag{28}$$

where dE is the increase in the internal energy when $dW = 0$, dQ is the heat added, and M the total mass of the system under consideration. From classic thermodynamics we should have the relation

$$d\theta = C_v dT, \tag{29}$$

where C_v is the specific heat at constant volume, and T the temperature on the Kelvin scale. Since C_v is not constant, θ will not necessarily be proportional to T; however, C_v is always positive, so that an increase in T is always accompanied by an increase in θ. In expressing $d\theta$ as proportional to dQ we do nothing which is essentially new, for we see that in Eq. 19 Kelvin makes a similar assumption. The difference, of course, rests in the treatment of dW.

To calibrate such a thermometric system let us proceed as follows: First allow the system to come to equilibrium in contact with melting ice under standard conditions, and assign to it the arbitrary state E_0, θ_0. Then place this system into a carefully shielded vacuum space and apply heat to the system until a differential thermocouple indicates no temperature difference between it and steam under standard conditions. The heat, Q_1, added to the system will satisfy the relation

$$Q_1 = E_{100} - E_0 = M(\theta_{100} - \theta_0), \tag{30}$$

where E_{100}, θ_{100} represent the state of the system at the steam point.

In a second experiment we may start at the ice point and apply heat

Q until the system reaches a temperature θ defined by

$$Q = E - E_0 = M(\theta - \theta_0). \tag{31}$$

Dividing Eq. 31 by Eq. 30, we have

$$\frac{Q}{Q_1} = \frac{E - E_0}{E_{100} - E_0} = \frac{\theta - \theta_0}{\theta_{100} - \theta_0}. \tag{32}$$

Suppose we place

$$\theta_{100} - \theta_0 = 100. \tag{33}$$

Then,

$$\theta = \theta_0 + 100\frac{Q}{Q_1}, \tag{34}$$

which is the fundamental equation for θ. Since all n variables of the system are taken into account, we operate as though the system were

TABLE I
Values of θ.

t°C	Corundum	Quartz	Cristobalite	Vitreous Quartz	Gold	Platinum	Graphite
−200		−116.6	−118.2	−119.0			
−100		−75.5	−75.7	−76.3			
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
100	100.0	100.0	100.0	100.0	100.0	100.0	100.0
200	217.9	217.5	223.4	216.3	201.9	202.1	233.2
300	347.3	348.5	364.5	343.9	305.7	306.3	393.0
400	483.5	489.3	498.6	480.2	411.5	412.6	574.2
500	624.5	638.6	639.0	624.9		521.1	772.4
600	769.2	812.6	782.7	772.3		631.6	984.0
700	917.0	958.8	931.6	923.7		744.2	1208.4
800	1087.2	1106.0	1081.8	1077.4		858.9	1435.8
900	1219.3	1253.5	1235.1	1231.9			
1000		1404.4	1388.7	1390.7			
1100		1557.5	1544.2	1546.9			
1200		1709.9	1698.2	1707.7			
1300		1864.1	1852.8	1876.9			
1400		2021.8	2009.4	2044.8			

complete. For this reason it is to be expected that the θ-scale will be independent of the system that is selected, just so long as the generalized work done by or on the system is zero throughout the range of operation.

With respect to an absolute zero the value of the ice point, θ_0, can be obtained from Eq. 34 by placing $\theta = 0$; thus

$$\theta_0 = 100\frac{Q_0}{Q_1}, \tag{35}$$

where Q_0 is the heat added to the system in going from $\theta = 0$ to $\theta = \theta_0$. Substituting Eq. 35 into Eq. 34, we have

$$\theta = 100\frac{Q + Q_0}{Q_1}, \tag{36}$$

which yields our temperature scale with respect to absolute zero.

To investigate the possibilities of such a temperature scale, the writer has selected some published data on systems which might be considered suitable for our purpose, that is, where $dW = 0$. The data on α-Al_2O_3 (corundum) were obtained by D. C. Ginnings and R. J. Corruccini (4). The data on quartz, cristobalite, vitreous quartz, gold, platinum and graphite are those given in the International Critical Tables (5). Where the average specific heats were given the total heat $Q = \int_0^T C_m dT$ was calculated. For each substance a value of Q_1 was obtained, and then values of θ were calculated on the basis of the equation $\theta = 100 \frac{Q}{Q_1}$, taking $\theta_0 = 0$. In Table I are given the values of θ thus calculated for different Centigrade temperatures.

Even though there are considerable discrepancies in the values of θ as determined from the different substances at the same Centigrade temperature, as a first trial there appear to be possibilities. Undoubtedly the greatest source of error is in the value of Q_1. It would certainly be of great interest to investigate this more fully in the laboratory.

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DIMENSIONAL ANALYSIS: AN APPROACH FROM TRANSFORMATION THEORY AND A CRITERION FOR SCALING MODEL EXPERIMENTS.[1]

BY

J. C. DECIUS.[2]

1. THE INVARIANTS OF CONTINUOUS TRANSFORMATION GROUPS.

In a recent article in this journal, Langhaar (1)[3] has drawn attention to a method of dimensional analysis in which the kinds of physical quantities are explicitly given a vector representation. The crucial theorem of this subject was proved by Langhaar in terms of theorems on homogeneous functions.

The author of the present paper has formalized the subject matter of dimensional analysis from a slightly different mathematical point of view, in which, however, as is natural, the material receives a practically identical representation. Because of the relationship of the following method to a very general method of discussing problems in mathematical physics, it seems worth-while to set it forth here. In addition, since the development is aimed at the treatment of scaling laws for model experiments, a new criterion is given for the possibility of satisfying the scaling laws when certain restrictions are imposed upon the variables.

A fruitful approach to the problems of dimensional analysis may be made by regarding any physical equation as the expression of an invariant under the transformations of some group. Frequently the knowledge of some form of spatial symmetry, the requirement of invariance under permutation of indistinguishable particles, or the necessity of special invariant properties which must obtain under transformations simultaneously involving temporal and spatial coordinates has been used to simplify or actually to advance the mathematical description of the physical world. Although the results of the following application of invariant theory are all rather well known, the method itself is rather elucidating, particularly with regard to the derivation of the laws of similitude which govern the scaling of model experiments.

It will be shown below that all of the results of dimensional analysis follow from the single postulate that all physical relations must be expressible in a form which does not depend upon the "magnitude" of the various physical units of measurement requisite for the description of a given physical situation.

[1] Contribution No 393 from the Woods Hole Oceanographic Institution.

[2] Woods Hole Oceanographic Institution, Woods Hole, Mass ; now at Brown University, Providence, Rhode Island.

[3] The boldface numbers in parentheses refer to the list of references appended to this paper.

Such changes of unit magnitude form the representation of a group of continuous transformations. A group of continuous transformations may be represented by the equations:

$$\begin{aligned} \bar{x}^i &= \xi^i(x^1, x^2, \cdots x^n; \quad \lambda^1, \lambda^2, \cdots \lambda^m), \\ i &= 1, 2, \cdots n \end{aligned} \tag{1.1}$$

in which the barred quantities are the new values of the variables, which were originally $x^1, x^2, \cdots x^n$, after the transformation induced by the set of continuously variable, independent parameters, $\lambda^j (j = 1, 2, \cdots m)$. The usual postulates for a group require that the λ^j can assume such values as will:

1. Induce the identity transformation

$$x^i = \xi^i(x^1 \cdots x^n; \quad \lambda_0^1 \cdots \lambda_0^m).$$

2. Induce the resultant of two or more successive transformations in one step, that is, if

$$\begin{aligned} \bar{x}^i &= \xi^i(x^1 \cdots x^n; \quad \lambda_1^1 \cdots \lambda_1^m), \\ \bar{\bar{x}}^i &= \xi^i(\bar{x}^1 \cdots \bar{x}^n; \quad \lambda_2^1 \cdots \lambda_2^m), \end{aligned}$$

then

$$\bar{\bar{x}}^i = \xi^i(x^i \cdots x^n; \quad \lambda_{12}^1 \cdots \lambda_{12}^m).$$

3. Induce a transformation "inverse" to every given transformation, that is, if

$$\begin{aligned} \bar{x}^i &= \xi^i(x^1 \cdots x^n; \quad \lambda_1^1 \cdots \lambda_1^m), \\ x^i &= \xi^i(\bar{x}^1 \cdots \bar{x}^n; \quad \lambda_{-1}^1 \cdots \lambda_{-1}^m). \end{aligned}$$

It may then be shown that the equations of transformation (1.1) determine a set of linear partial differential operators

$$U_j = \left(\frac{\partial \xi^i}{\partial \lambda^j}\right)_0 \frac{\partial}{\partial x^i} \tag{1.2}$$ [4]

which represent the independent infinitesimal transformations of the group (the subscript zero for $\partial \xi^i/\partial \lambda^j$ implies that the derivative is to be evaluated for those values of the parameters, λ^j, which induce the identity transformation; without loss of generality these values may henceforth be assumed to be zero). The most general infinitesimal transformation is represented by

$$U = \alpha^j U_j \tag{1.3}$$

in which the α^j are arbitrary constants; any finite transformation is

[4] The summation convention of tensor notation is used throughout. Thus $\left(\frac{\partial \xi^i}{\partial \lambda^j}\right)_0 \frac{\partial}{\partial x^i}$ implies $\sum_{i=1}^{n} \left(\frac{\partial \xi^i}{\partial \lambda^j}\right)_0 \frac{\partial}{\partial x^i}$; this is true only if the index appears in one place as a superscript, in another place as a subscript.

generated by an integration of the differential operation signified by (1.3).

The important consequence for the present purposes is that if any function of the variables x^i, say

$$F(x^1, \cdots x^n) = 0, \tag{1.4}$$

is known to be invariant under the group represented by (1.1), then it is always possible to express (1.4) in terms of a complete set of fundamental invariants $y^1, y^2, \cdots y^{n-r}$, which are determined by

$$Uy = 0 \tag{1.5}$$

as functions of the x^i. Thus

$$F(x^1, \cdots x^n) = \Phi(y^1, \cdots y^{n-r}) = 0 \tag{1.6}$$

and the number of independent variables has been reduced by r, which is equal to the rank of the matrix $\left\| \left(\frac{\partial \xi^i}{\partial \lambda^j} \right)_0 \right\|$.

2. DETERMINATION OF THE INVARIANTS OF A GENERAL PHYSICAL EQUATION UNDER TRANSFORMATIONS OF THE UNITS.

The nature of the number, x^j, resulting from a physical measurement of the jth kind of physical quantity is such that it is determined only within a transformation of the type:

$$\bar{x}^j = \exp [\lambda^j] x^j, \qquad -\infty < \lambda^j < +\infty. \tag{2.1}$$ [5]

If it is desired to deduce the possible kinds of functions of the *magnitudes* of a basic set of physical quantities, which give the *magnitude* of a resultant physical quantity derivable from the basic ones, it is possible to apply the principles of Section 1 in the following manner. The resultant magnitude itself must obey a transformation law of the same form as (2.1)

$$\bar{x} = \exp [f(\lambda^j)] x \tag{2.2}$$

in which the value of f may be taken as zero when $\lambda^j = 0$, all j. Then Eq. 1.3 requires that the relation between x and x^j be expressible in terms of a function y determined by

$$\alpha^k \left[\delta_k{}^j x^k \frac{\partial y}{\partial x^j} + \left(\frac{\partial f}{\partial \lambda^k} \right)_0 x \frac{\partial y}{\partial x} \right] = 0. \tag{2.3}$$

The α^k being arbitrary constants, (2.3) is equivalent to the m relations

$$\frac{\partial y}{\partial \ln x^k} = - \left(\frac{\partial f}{\partial \lambda^k} \right)_0 \frac{\partial y}{\partial \ln x}, \qquad k = 1, 2, \cdots m. \tag{2.4}$$

[5] It is clear that the range of the λ^j indicated will contain values satisfying all the group postulates of Section 1

An acceptable solution of this system is

$$y = \prod_{k=1}^{m} \frac{(x^k)^{a_k}}{x}, \tag{2.5}$$

where $a_k = \left(\frac{\partial f}{\partial \lambda^k}\right)_0$ is a constant, so that, y being actually an invariant, which can be assumed to have the numerical value of unity, the acceptable form for the combination of magnitudes is

$$x = \prod_{k=1}^{m} (x^k)^{a_k} \tag{2.6}$$

with the transformation law:

$$\bar{x} = \exp\,[a_k\lambda^k]x. \tag{2.7}$$

It is immediately perceived that the numbers a_k characterize the *kind* of the derived quantity and that they may be regarded as the components of an m-dimensional vector in the basis in which the lth component of the kth fundamental quantity is the Kronecker symbol, $\delta_l{}^k$. In other words, the general symbol, $a_j{}^i$, is the ordinary exponent occurring in the conventional dimensional formula for the ith *kind* of variable relative to the jth fundamental *kind* of quantity; for example, let $i = 1$ designate energy; $j = 1, 2, 3$ correspond to mass, length, and time, respectively, then $a_1{}^1 = 1$, $a_2{}^1 = 2$, $a_3{}^1 = -2$.

Now let x^i be a set of derived magnitudes expressible in terms of the basic set, x^j. Then the transformation laws are:

$$\bar{x}^i = \exp\,[a_j{}^i\lambda^j]x^i, \qquad i = 1, 2, \cdots n; \quad j = 1, 2, \cdots m. \tag{2.8}$$

Now let

$$F(x^i) = 0$$

be the expression of an unknown physical relation in terms of the magnitudes, x^i. Equations 1.2, 1.3, and 1.5 require that the relation be expressible in terms of y^k, the solutions of

$$\alpha^j a_j{}^i x^i \frac{\partial y^k}{\partial x^i} = 0$$

or

$$a_j{}^i \frac{\partial y^k}{\partial \ln x^i} = 0 \tag{2.9}$$

$$\text{for} \quad j = 1, 2, \cdots m.$$

Solution of Eq. 2.9 gives

$$y^k = \prod_{i=1}^{n} (x^i)^{b_i^k}, \qquad k = 1, 2, \cdots n - r, \tag{2.10}$$

where the b_i^k are constants determined by

$$a_j^i b_i^k = 0 \tag{2.11}$$

and where r is the rank of the matrix $\|a_j^i\|$.

This result is, of course, the well-known "π-theorem" as described by Buckingham (2) or Bridgman (3). The virtue of having used the transformation theory lies chiefly in the following two points:

1. The "alias-alibi" duality of any transformation immediately allows the conclusion that although the deductions were based on the assumption that the "absolute" magnitude of the physical quantities were unchanged during the transformation, the invariants, y^k, will certainly satisfy the equation

$$\Phi(y^k) = 0$$

throughout a transformation in which the fundamental magnitudes are fixed and the magnitudes of the x^i are "actually" changed but in such a way as to keep the y^k constant. In other words, empirical physical information satisfying

$$F(x^i) = 0$$

on a given *scale* may be used to predict a physical relation on any other scale, provided only that the x^i are varied in such a way as to maintain the constancy of the y^k.

2. The simplification of a physical problem brought about by the reduction of the degrees of freedom, n, by the number r, is not mathematically different in kind from the complete solution of the problem (the determination of the form of the unknown function, F) which is obtained when $(n - 1)$ independent transformation parameters are obtained.

3. A FORMAL SOLUTION FOR b_i^k.

Equations 2.11 are equivalent to the matrix equation

$$AB = 0, \tag{3.1}$$

where $A = \|a_j^i\|$, $B = \|b_i^k\|$. The rank of A is r; by mere rearrangement of rows and columns, the non-vanishing determinant can be made to appear in the upper right-hand corner of A (under the last r columns and the first r rows; it will be supposed that the subscript is the row index, the superscript the column index). Call this portion of the matrix A_1, call the first $(n - r)$ columns and first r rows A_2; let the last $(m - r)$ rows be called A_0 (if $r = m$, A_0 does not exist).

Make a corresponding partition of B, calling the first $(n - r)$ rows

B_2, the remaining rows B_1. Then Eq. 3.1 is equivalent to

$$\left\| \begin{array}{c} A_2B_2 + A_1B_1 \\ \hline A_0 \left\| \begin{array}{c} B_2 \\ \hline B_1 \end{array} \right\| \end{array} \right\| = 0. \tag{3.2}$$

But since the rows of A_0 are linearly dependent upon the rows of $\|A_2 \vdots A_1\|$, any solution of

$$A_1B_1 = -A_2B_2 \tag{3.3}$$

automatically makes $A_0 \left\| \frac{B_2}{B_1} \right\|$ vanish. Since $|A_1| \neq 0$, A_1^{-1} exists and

$$B_1 = -A_1^{-1}A_2B_2. \tag{3.4}$$

The elements of B_2 are now completely arbitrary, and it is convenient, as will be shown below, to let B_2 equal the negative unit matrix, whence

$$B_1 = A_1^{-1}A_2. \tag{3.5}$$

Combination with Eqs. 2.10 and 2.11 gives the result that the invariants are:

$$y^k = \prod_{p=1}^{r} \frac{(x^{n-r+p})^{b^k_{n-r+p}}}{x^k}, \tag{3.6}$$

that is, since the y^k are to be held constant, the first $(n - r)$ variables must be individually proportional to a set of products involving only the last r variables which may be changed arbitrarily.

4. THE SCALING CRITERION.

In the application of these results to the design of model experiments certain difficulties arise which have not previously been considered in the general case. It frequently occurs that some of the variables essential to a given problem may not be readily changed with scale. As examples, the acceleration of gravity, or, in certain cases, even the properties of liquid and solid media cannot be readily altered in general so as to satisfy Eqs. 3.6 and must therefore be regarded as fixed. In order to set up a general criterion for determining whether scaling is possible, it will be useful to classify the variables in three types: those which are fixed, those which are to be arbitrarily (and independently) scaled, and those which are unrestricted. We shall use the subscripts f and s to designate the first two types respectively; r_s, r_f, r_{sf} will stand for the rank of the sub-matrix of A corresponding to all the variables of types s, f, and both s and f, respectively: t_s, t_f, t_{sf} will stand for the number of y^k which involve variables of type s, f, s and/or f, respectively; and n_s, n_f are the numbers of variables of type s and of type f. A neces-

sary and sufficient condition that scaling be possible is that

$$n_s + r_f = r_{sf}. \tag{4.1}$$

To prove the necessity of the condition, suppose $n_s + r_f \neq r_{sf}$. Since r_{sf} cannot exceed $n_s + r_f$, we must suppose $n_s + r_f > r_{sf}$ or $n_s + n_f - r_{sf} > n_f - r_f$, but since

$$t_{sf} = n_s + n_f - r_{sf}$$

and

$$t_f = n_f - r_f$$

we should have

$$t_{sf} > t_f$$

so that there would be either or both of the following types of y^k:

1. those containing arbitrarily scaled variables only,
2. those containing arbitrarily scaled and fixed, but not unrestricted variables,

neither of which types can be allowed.

To prove that the condition is sufficient, distinguish the two possible cases:

1. $r_{sf} = r$
2. $r_{sf} < r$.

In case (1), that part of A_{sf} with the $r_{sf} \times r_{sf}$ non-vanishing determinant can be taken as A_1; consequently (since such a determinant can be found which contains columns from all the arbitrarily scaled variables) all the arbitrarily scaled variables will appear in the set x^{n-r+p}, the fixed variables (if any) not contained in A_1 will be expressible in terms of fixed variables in A_1 only, and there will be just enough restricted variables to complete the set of $(n - r)y^k$'s with one such variable to each y^k involving arbitrarily scaled variables.

If, on the other hand, case (2) obtains, the same proof will hold (with the modification that some unrestricted variables will appear in the set x^{n-r+p}) provided only that the $r_{sf} \times r_{sf}$ non-vanishing determinant contained in A_{sf} is contained in some $r \times r$ non-vanishing determinant of A.

That this latter requirement is always satisfied follows, for example, from the definition of rank in terms of linear independence as shown by Birkhoff and MacLane (4). This completes the proof of the scaling criterion.

In concluding this section it should be remarked that the criterion furnished by Eq. 4.1 is independent of the basis of fundamental units adopted, since the only quantities appearing are numbers of variables and the rank of various sub-matrices, taken by columns, which are, of course, all invariant under the group of homogeneous linear transformations on the $a_j{}^i$, $A \rightarrow \bar{A} = TA$, $|T| \neq 0$, which corresponds to all conceivable choices of a basic set of fundamental units.

5. APPLICATION TO HYDRODYNAMICS.

Consider the dimensional composition of the variables appearing in the analytical description of fluid flow. The Navier-Stokes equation and the equation of continuity contain variables of the following dimensional types: v = velocity, p = pressure, R = any length, t = time, ρ = density, μ = viscosity. If the external forces are due to gravity, the acceleration of gravity, g, is included. In addition, there exists an equation of state which expresses ρ as a function of p alone in case the flow is assumed to be isothermal or adiabatic. The implication of this last condition is important in the consideration of possible scaling solutions: although the specific form of the equation of state is unknown, it must be expressible in terms of variables, such as the bulk modulus, whose columns in A are linear combinations of the columns in A representing ρ and p. In order to maintain, temporarily, the generality of the discussion, it will be supposed that the density is given by the series:

$$\rho = \sum_{i=\alpha}^{\omega} \rho_i (p)^i \tag{5.1}$$

with $\alpha \leqq \omega$.
The A matrix is then as follows:

A	v	p	t	ρ_i	μ	g	R
M	0	1	0	$1-i$	1	0	0
L	1	-1	0	$-3+i$	-1	1	1
T	-1	-2	1	$2i$	-1	-2	0

A model experiment will now be considered in which R plays the role of the deliberately scaled variables: $n_s = 1$. Then the criterion of Eq. 4.1 requires $r_{sf} = r_f + 1$. If the medium is unchanged with scale, in the general case the rank of (ρ_i, μ) is 3 so that, since r_{sf} also equals 3, scaling is impossible. Even for an incompressible fluid ($\rho_i = 0$ except for $i = 0$) a scaling solution with fixed g is impossible if μ is fixed. The idealizations which lead to the familiar scaling approximations in terms of the Froude, Reynolds, and Mach numbers are described in Table I. In each case the set of variables designated as "ignored" are shown to be

Table I.

Scaling Laws for Hydrodynamics.

Typical Invariant	Ignorable Variables	v	p	t	ρ_i	μ	g	Fixed Variables
Froude Number	$\rho_i(i \neq 0)$, μ	$\frac{1}{2}$	1	$\frac{1}{2}$	$(-i)$	$(\frac{3}{2})$	0	ρ_0, g
Reynolds Number	$\rho_i(i \neq 0)$, g	-1	-2	2	$(2i)$	0	(-3)	ρ_0, μ
Mach Number	μ, g	0	0	1	0	(1)	(-1)	ρ_i
—	$\rho_i(i \neq 0, 1)$, μ	$\frac{1}{2}$	0	$\frac{1}{2}$	$[-1]$	$(\frac{1}{2})$	0	$\frac{\rho_1}{\rho_0}$, g

required to follow rather impractical scaling laws, but by assuming that their influence on the flow is unimportant in a certain range of velocities, a solution is obtained by omitting them from further consideration. The dependence of the variables is expressed in terms of the exponent b_i^R in the expression

$$x^i = x^{io}\left(\frac{R}{R^o}\right)^{b_i^R}, \tag{5.2}$$

where x^i is the appropriate value of the ith variable on any scale with a typical length R related to the model scale with corresponding values x^{io} and R^o. The computation of the b_i^R requires only the determination of that row of A_1^{-1} which corresponds to the column of A_1 in which R appears.

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New 108,000-kw. Generator for Grand Coulee. (*Civil Engineering*, Vol. 17, No. 10.)—A new 108,000-kw. generator as big as a six-room house has been completed by Westinghouse Electric Corp. for installation at Grand Coulee Dam, where it will increase generating capacity by the equivalent of 146,000 hp. This new giant and six similar machines already installed—the world's largest waterwheel generators—could supply power for all the industrial and residential requirements of a city twice the size of Pittsburgh.

Eventually the Grand Coulee program calls for 18 waterwheel generators, each of 108,000-kw. capacity. This tremendous concentration of energy will be used for pumping irrigation water to 1,200,000 acres of arid farm land in the Columbia Basin of central Washington and will set up a huge reservoir of electric power for industrial use in the West.

Completion of the 238-ton stator was the final step in the manufacturing job. Other parts of the generator, including a 72-ton steel shaft 32 ft. long, and a 464-ton rotor, already have been shipped. The generator is being assembled at Grand Coulee and is expected to go into operation early in 1948. Into its construction went approximately 750 tons of steel and more than 100 miles of copper wire.

R. H. O.

Airport Snow Removal. (*Heating and Ventilating*, Vol. 44, No. 10.)--A Pittsburgh engineer, John B. Sweeney, who has an annual battle with snow and ice at the airports in Pittsburgh under his control, has come up with a device designed to wipe out those menaces to year-round operation of airlines. The device operates like a gigantic flatiron, weighing 12 tons and literally ironing out the ice and snow completely on a 12-ft. swath of runway or roadway. It melts the snow and ice by means of two wide combustors which focus a blast of oil-fired, 2000 F. air on the area to be cleared.

It travels at the rate of 10 to 15 miles an hour. It is mounted on an ordinary grader, and water that the great heat does not evaporate is wiped from the pavement by a large squeegee blade, working like an ordinary window cleaner. A relatively small amount of water is formed by the melting snow, since the water content of snow is only 2 per cent of its volume. When not in use the snow remover can be removed from the grader in a couple of hours and stored.

The 2000 F. blast is forced down to the runway through blowers operated by an air-cooled engine, the blast gaining its terrific heat from oil burners consuming 30 gal. an hour. Although they produce 8,000,000 Btu. per hour, the pavement is not damaged because of the speed of the machine and the fact that it is used in cold weather. Mr. Sweeney's machine will remove snow at an estimated cost of one cent a cubic yard. Furthermore, the machine makes a clean sweep of the area covered, leaving no icy spots to cause skids, and leaves no drifts to be cleared from the side of the road or runway, melting an extra 5-ft. strip on each side of its path.

The machine is the result of six years' work and experiment, with field test conducted last winter at the Greater Pittsburgh Airport. His first model was equipped with a two-way radio, which would be equally valuable on any kind of snow removal work, airports or roadways. Mr. Sweeney is Allegheny County Director of Aviation.

R. H. O.

A PHOTOMETRIC INVESTIGATION OF DUST.

BY

DOMINA EBERLE SPENCER [1] AND SELMA MALKIEL.[2]

1. INTRODUCTION.

One of the most troublesome effects of dust is the reduction in the amount of light through windows and skylights and from luminaires. The quantitative knowledge of the effect of dust is of practical importance in the design of luminaires and in the prediction of the performance of lighting systems.

A previous investigation [3] of the available data on the subject indicated a need for further experimental research. As a consequence, the present research was instigated with the purpose of determining how the transmittance of a dust film on a smooth surface varies with time and with angle of the surface. Measurements were made on nearly three hundred samples, which were exposed to the collection of dust in three locations. Measured transmittance of these dust films forms the basis for an empirical equation which gives the transmittance as a function of time and of angle.

2. EXPERIMENTAL PROCEDURE.

The dust was collected on plates of glass 5.1 × 5.1 cm. Cover glasses for Kodachrome slides were employed. They are easily obtained and are reasonably flat and uniform in thickness. The glass plates were held by phosphor bronze clips attached to steel frames. The frames were constructed of 6.4-mm. square steel rod, and the dust samples were held only along one edge so that the frames had little shielding effect.

The samples were carefully cleaned and were then mounted in pairs so that dust collected on one surface of each. The frames were constructed to hold the samples at the following angles: $\theta = 0$ (horizontal, facing up), $\pi/6$, $\pi/3$, $\pi/2$ (vertical), $2\pi/3$, $5\pi/6$, π (horizontal, facing down). The 0 and π plates were clamped together, likewise the $\pi/6$ and $5\pi/6$, the $\pi/3$ and $2\pi/3$, and two $\pi/2$ plates.

When dust had collected for a designated time, the samples were carefully removed and each was covered with a clean glass plate of the same size. The edge was then taped, leaving a dust film sealed between two glass plates. This sample could then be handled as much as desired without altering the film.

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[1] Brown University, Providence, Rhode Island.

[2] Tufts College, Medford, Mass.

[3] Parry Moon and D. E. Spencer, "Maintenance Factors," *Trans. Illum. Soc. Eng.*, Vol. 14, p. 211 (1946); and "Lighting Design," Addison-Wesley Press, Cambridge, Mass., 1948, Chap. VII.

Dust was collected in three locations. Thirty supporting frames were built, and fifteen were placed in a closed room at M. I. T. (Location B). This investigation was started on June 10, 1946. One set of samples was collected each week for four weeks. Thereafter, collections were made every four weeks. The last set of samples was collected on May 12, 1947, thus covering a period of 336 days.

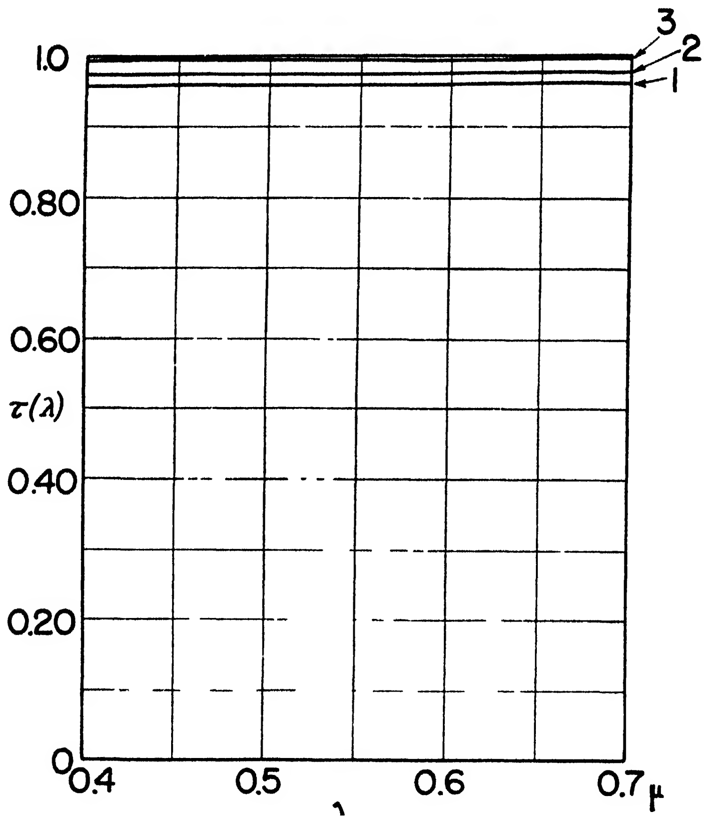


FIG. 1 Plot of transmittance τ of a dust film as a function of wavelength λ. Curves obtained by the Hardy photoelectric spectrophotometer

1 $\theta = 0$
2 $\theta = \pi/3$
3 $\theta = 2\pi/3$

The other fifteen sets of samples were assembled in a similar closed room at Tufts (Location C) on June 11, 1946 and collected on a corresponding schedule until May 13, 1947.

When about half of the collections of dust had been made and there were a number of vacant frames, these were put to use in a third location. Location A was Room 10–218 at M. I. T., a busy office in which the windows face a gravel parking lot and where dust is plentiful. A

set of glass plates was assembled in Location A on November 25, 1946. These were collected every four weeks until May 12, 1947 and provide data covering a period of 168 days.

Figure 1 shows curves of the transmittance of dust samples for Location B at 112 days. These spectral transmittance curves were obtained by means of the Hardy photoelectric spectrophotometer.[1] It is to be noted that the dust film is neutral in color.

A more direct method of measurement, which utilizes a bar photometer and a Lummer-Brodhun contrast photometer head, was finally decided on. Two 200-w. incandescent lamps of the same type were placed at the ends of the bench at a separation of 280 cm. A slide of clean glass was placed before the left lamp. The slide to be measured was placed before the right lamp. The instrument was calibrated by obtaining a visual balance with a clean glass slide also at the right end of the photometer. The same procedure was then repeated with a dust-film slide on the right. The transmittance of the dust film is

$$\tau = \left(\frac{l}{280 - l}\right)^2 \left(\frac{280 - l_s}{l_s}\right)^2, \tag{1}$$

where τ = transmittance of dust film,

l_s = distance from photometer head to right lamp when clean slide is at right, cm., and

l = distance from photometer head to right lamp when slide containing dust film is at right, cm.

Five readings were obtained for each slide by balancing the bar photometer five times. The values of transmittance given in Tables I, II, and III are computed by averaging the five readings of l and substituting the average value into Eq. 1. An idea of the variation in the readings is given by the probable error. The probable error for a single reading was calculated for forty readings with eight slides collected from Location A after 140 days. The probable error was found to be 0.003 or about 0.3 per cent of the readings. Larger fluctuations in the data are consequently attributable to actual variations in the dust films.

If the values of transmittance given in the tables are compared with those obtained with the Hardy spectrophotometer, a discrepancy will be noticed. For instance, the value of τ in Fig. 1 for Location B at t = 112 days, θ = 0, is 0.960; the corresponding value for the same slide measured on the bar photometer is 0.912. The explanation is that the Hardy spectrophotometer utilizes an integrating sphere and consequently records both scattered and directly transmitted light. The bar photometer, on the other hand, measures only the directly transmitted light and thus gives a lower reading.

[1] We wish to thank the Color Measurements Laboratory at M. I. T. for their kindness in obtaining the spectrophotometric curves of Fig. 1.

TABLE I.

Transmittance τ, Location A.

θ / t, days	0	π/6	π/3	π/2	π/2	2π/3	5π/6	π
28	0.941	0.961	0.971	1.000	1.000	1.000	1.000	1.000
56	0.908	0.925	0.935	0.983	0.984	0.990	0.992	0.990
84	0.837	0.879	0.908	0.986	0.988	0.986	0.986	0.986
112	0.816	0.837	0.897	0.984	0.985	0.987	0.988	0.988
140	0.790	0.839	0.885	0.986	0.987	0.994	0.993	0.993
168	0.787	0.801	0.864	0.978	0.983	0.993	0.984	0.992

TABLE II.

Transmittance τ, Location B.

θ / t, days	0	π/6	π/3	π/2	π/2	2π/3	5π/6	π
7	0.993	0.993	0.991	1.000	1.000	1.000	1.000	1.000
14	0.980	0.983	0.984	0.987	0.992	0.993	0.998	0.989
21			0.982	0.992	0.992	0.988		0.992
28	0.965	0.975	0.978	0.988	0.988	0.985	0.983	0.984
56	0.951	0.972	0.967	0.984	0.979	0.976	0.984	0.991
84	0.938	0.943	0.953	0.973	0.970	0.968	0.967	0.972
112	0.912	0.928	0.942	0.970	0.970	0.954	0.959	0.963
140	0.894	0.902	0.924	0.952	0.959	0.956	0.969	0.962
168	0.877	0.885	0.921	0.957	0.962	0.962	0.946	0.968
196	0.859	0.878	0.902	0.969	0.957	0.954	0.958	0.958
224	0.844	0.852	0.895	0.950	0.852	0.957	0.954	0.948
252	0.834	0.858	0.901	0.973	0.980	0.964	0.967	0.965
280	0.820	0.838	0.900	0.974	0.974	0.974	0.975	0.976
308	0.810	0.822	0.888	0.975	0.976	0.975	0.976	0.976
336	0.799	0.825	0.884	0.976	0.976	0 971	0.976	0.976

TABLE III.

Transmittance τ, Location C.

θ / t, days	0	π/6	π/3	π/2	π/2	2π/3	5π/6	π
7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
14	0.985	1.000	1.000	1.000	1.000	1.000	1.000	1.000
21	0.958	0.982	0.992	0.993	1.000	1.000	1.000	1.000
28	0.973	0.968	0.982	0.982	0.982	0.987	0.982	0.983
56	0.980	0.979	0.988	0.997	0.997	1.000	1.000	0.998
84	0.965	0.979	0.986	0.989	0.986	0.992	1.000	1.000
112	0.932	0.910	0.988	0.951	0.951	0.996	0.946	0.948
140	0.922	0.946	0.952	0.992	0.992	0.984	0.992	0.994
168	0.899	0.918	0.940	0.983	0.978	0.984	0.980	0.982
196	0.893	0.906	0.940	0.996	0.993	0.996	0.994	0.990
224	0.850	0.875	0.913	0.982	0.982	0.980	0 985	0.981
252	0.845	0.871	0.919	0.984	0.989	0.993	0.982	0.992
280	0.818	0.828	0.873	0.987	0.991	0.960	0.995	0.973
308	0.813	0.837	0.890	0.973	0.977	0.988	0.990	0.994
336	0.790	0.830	0.866	0.965	0.968	0.949	0.965	0.968

3. ANALYSIS OF RESULTS.

The experimental results are most conveniently used in the form of an equation. The equation

$$\tau = \delta + (1 - \delta)e^{-\mu t}, \tag{2}$$

where τ = transmittance of dust film,
t = time, days, and
δ, μ = constants,

was previously used to fit data on the collection of dust.[3] If dust collected uniformly with time, the transmittance would vary as $e^{-\mu t}$.

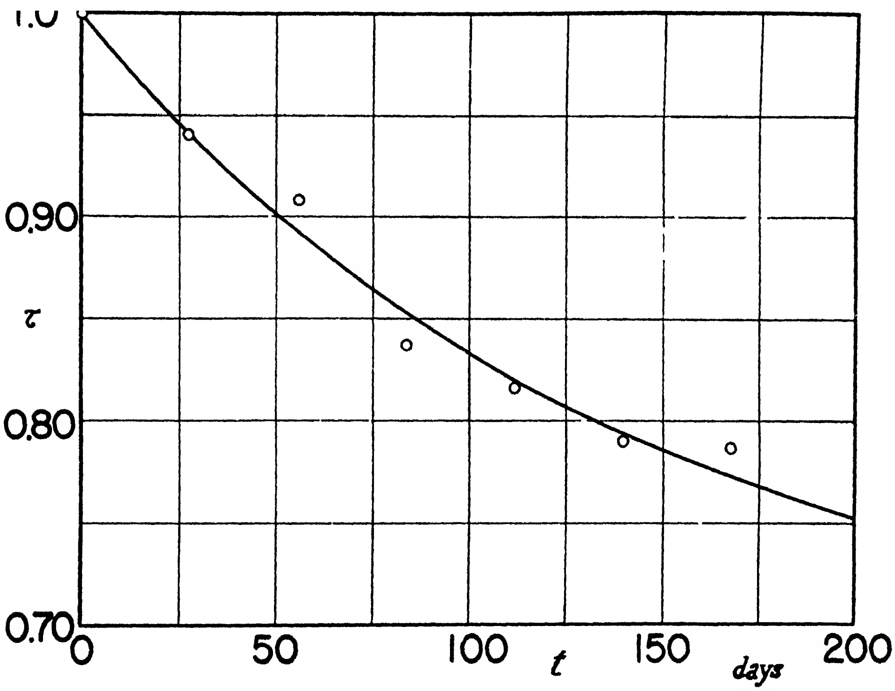


FIG. 2. Transmittance of a dust film as a function of time. Location A, $\theta = 0$.
O, Experimental values,
—, Calculated from Eq. 3.

This is nearly true for a clean surface, but as the dust becomes thicker, there is a tendency for as much dust to leave as arrives. A steady state is finally reached, at which $\tau = \delta$.

Figure 2 shows the experimental data for Location A, plotted as a function of time. The equation for the transmittance for a horizontal surface facing upward ($\theta = 0$) is

$$\tau = 0.681 + 0.319e^{-0.00740t} \tag{3}$$

and the resulting curve is shown in Fig. 2. The curves for other values

of θ were analyzed in a similar fashion, and it was found that all of them could be handled by using the same value of μ and different values of δ. The constant δ is, therefore, a function of θ. The relation may be represented by the equation

$$\begin{aligned} \delta &= 0.634 + 0.047e^{1.255\theta} && \text{if} \quad 0 \leqq \theta \leqq \pi/2, \\ \delta &= \delta(\pi/2) && \text{if} \quad \pi/2 \leqq \theta \leqq \pi, \end{aligned} \tag{4}$$

where θ = angle between the normal to the dust collecting surface and the upward direction (radian).

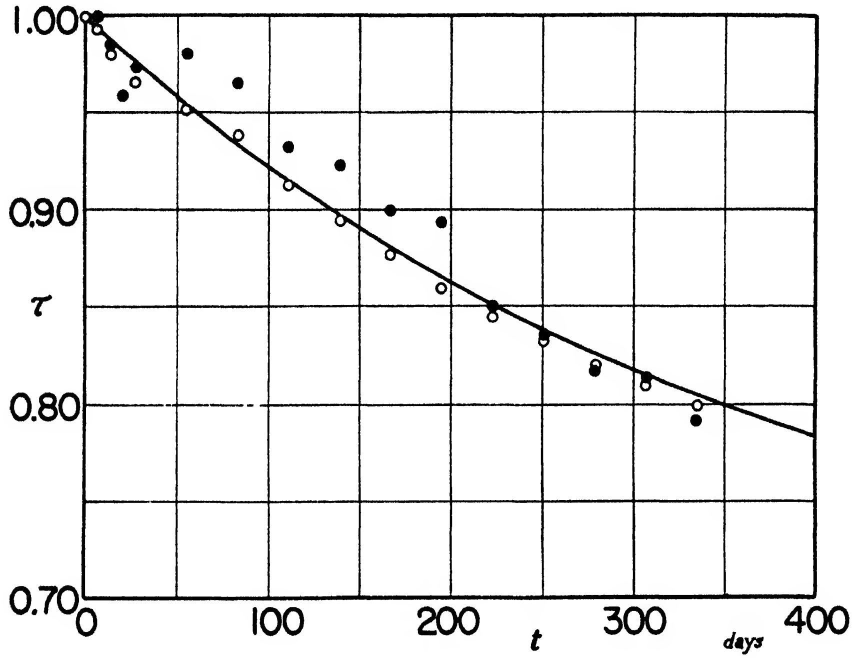


FIG. 3. Transmittance as a function of time. $\theta = 0$.
○, Location B } Experimental Data
●, Location C }
—, Calculated from Eq. 5.

Figure 3 shows data for Locations B and C for $\theta = 0$. The solid curve in Fig. 3 uses the same value of δ as for Fig. 2, but a different value of μ:

$$\tau = 0.681 + 0.319e^{-0.00280t}. \tag{5}$$

The equation for δ found with the data in Location A is found to be applicable to Locations B and C for all angles, but the new value of μ must be used (Eq. 5).

The data of the entire investigation can now be summarized in a single statement:

$$\tau(\theta, t) = \delta(\theta) + (1 - \delta(\theta))e^{-\mu t}, \tag{6}$$

$$\text{where } \delta(\theta) = 0.634 + 0.047e^{1.255\theta} \quad \text{if} \quad 0 \leqq \theta \leqq \pi/2,$$
$$\delta(\theta) = \delta(\pi/2) \quad \text{if} \quad \pi/2 \leqq \theta \leqq \pi,$$
and $\mu = 0.00740$ in Location A,
$\mu = 0.00280$ in Locations B and C.

Figure 4 shows the transmittance as a function of θ for Location A and some of the data which were used to fit this curve. Note that the random variations are appreciable for surfaces facing downward, but

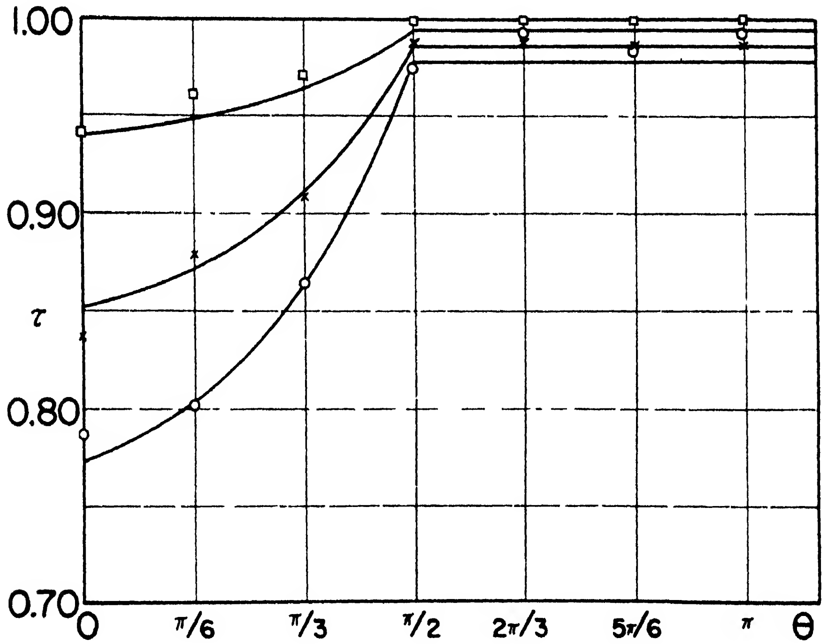


FIG. 4 Transmittance as a function of angle. Location A
○, 168 days; ×, 84 days; □, 28 days — Experimental Data
, Calculated from Eq. 6.

these variations appear to follow no definite trend and are best replaced by a constant. Figure 5 is a plot of δ as a function of θ, calculated from Eq. 4. The physical significance of δ is simple: it is the ultimate transmittance for very large values of time.

An average [8] of previous data for collection of dust on windows was approximated by

$$\tau = 0.720 + 0.280e^{-0.0107t} \tag{7}$$

for a single horizontal film. The corresponding equation for Location A is

$$\tau = 0.681 + 0.319e^{-0.00740t} \tag{8}$$

and for Locations B and C,

$$\tau = 0.681 + 0.319e^{-0.00280t}. \tag{9}$$

Note that the ultimate values of τ are different by less than 6 per cent and the exponents are of the same order of magnitude. Considering the numerous uncontrolled variables of previous data, this agreement can be considered satisfactory.

It is suggested that the data on accumulation of dust in an office are suitable for use in most practical applications. Accordingly, we shall take the data for Location A as standard. Figure 6 shows these

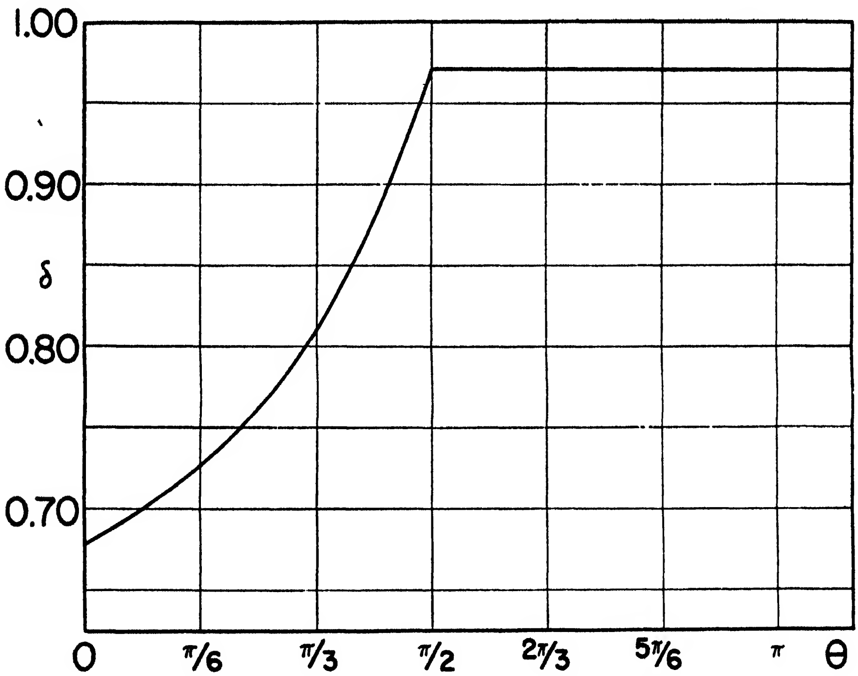


FIG. 5. δ as a function of angle. Calculated from Eq. 4.

data as represented by Eq. 6. The engineer should preferably obtain data for his locality. Lacking this specific information, however, he may find Fig. 6 and Eq. 6 helpful.

4. AVERAGE TRANSMITTANCE.

Equation 6 allows the calculation of transmittance for a dust film on any surface at any time t. In utilizing the results for the prediction of the behavior of luminaires, one needs also the average effect of dust that collects on a half cylinder or on a hemisphere. The former represents the upper half of a fluorescent lamp or a cylindrical enclosure, the latter represents half of an incandescent lamp or a diffusing globe.

The average transmittance τ_c for a half cylinder facing upward and with axis horizontal is obtained by substituting Eq. 6 for Location A in

$$\tau_c(t) = \frac{2}{\pi}\int_0^{\pi/2} \tau(\theta)d\theta$$

$$= 0.781 + 0.219\, e^{-0.00740t}. \tag{10}$$

The average transmittance for a hemisphere facing upward is

$$\tau_s(t) = \int_0^{\pi/2} \sin\theta\tau(\theta)d\theta$$

$$= 0.817 + 0.183\, e^{-0.00740t}. \tag{11}$$

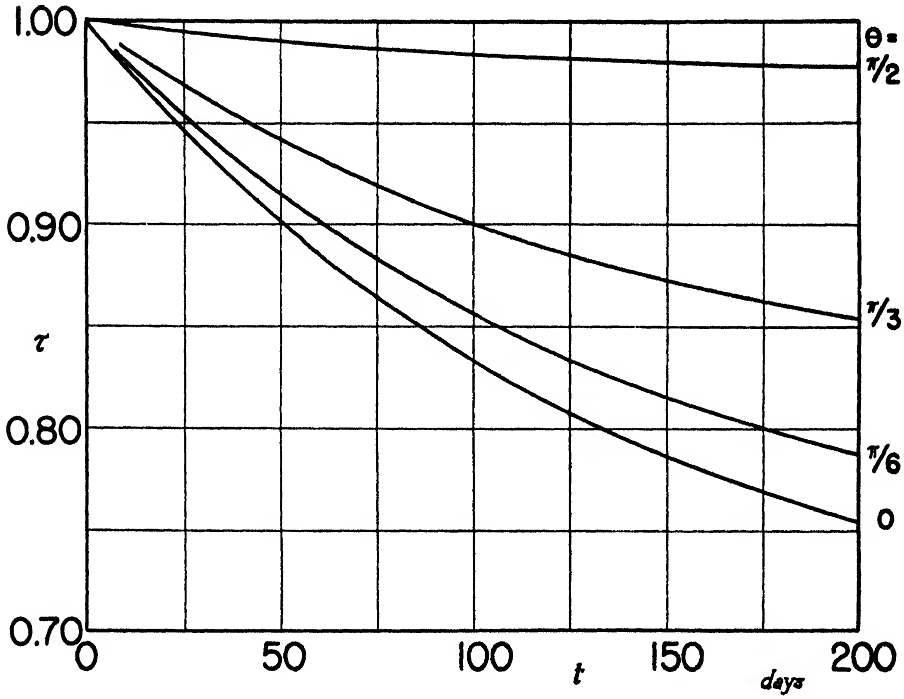


FIG. 6. Transmittance as a function of time (Location A). Calculated from Eq. 6.

The average values of transmittance for these surfaces facing downward are nearly $\tau(\pi/2)$, since the experimental data show that all angles from $\pi/2$ to π give the same transmittance.

All of the previous equations give τ as a function of time. These instantaneous values are not, however, the ones most useful in the treatment of luminaire maintenance. The average value maintained in practice is more useful than the instantaneous value. If the cleaning period for the luminaire is t_c days, then the average transmittance τ over

this period is

$$\tau = \frac{1}{t_c}\int_0^{t_c} \tau(t)dt$$

$$= \delta + \frac{(1-\delta)}{\mu t_c}(1 - e^{-\mu t_c}). \tag{12}$$

Average values of τ computed from Eq. 12 for surfaces at angles of 0, $\pi/4$, $\pi/2$, and for τ_c and τ_s are given in Table IV. The tabulated values

TABLE IV.

Average Transmittance as a Function of Cleaning Time, t_c.

t_c, days	$\tau(0)$	$\tau(\pi/4)$	$\tau(\pi/2)$	τ_c	τ_s
30	0.968	0.976	0.997	0.978	0.982
60	0.938	0.954	0.994	0.957	0.964
90	0.914	0.935	0.992	0.941	0.951
150	0.873	0.904	0.988	0.912	0.927
360	0.792	0.844	0.980	0.857	0.881
∞	0.681	0.760	0.970	0.781	0.817

allow the ready visualization of how much light is lost when lighting equipment is washed infrequently. The economics of luminaire and window maintenance can be handled as in a previous paper.[3]

5. LUMINAIRES.

The new equations are now applied to luminaires. The *luance* or *maintenance factor* of a luminaire is defined as

$$k_m = \frac{\text{Av. pharos (lumen) from luminaire in service}}{\text{Pharos (lumen) from new, clean luminaire}}. \tag{13}$$

It is convenient to employ two luances k_{m1} and k_{m2}:

$$k_m = k_{m1}k_{m2}, \tag{14}$$

where k_{m1} = luance associated with internal blackening of the lamp, and
k_{m2} = luance associated with collection of dust on lamp, reflector diffusor and other parts of the luminaire.

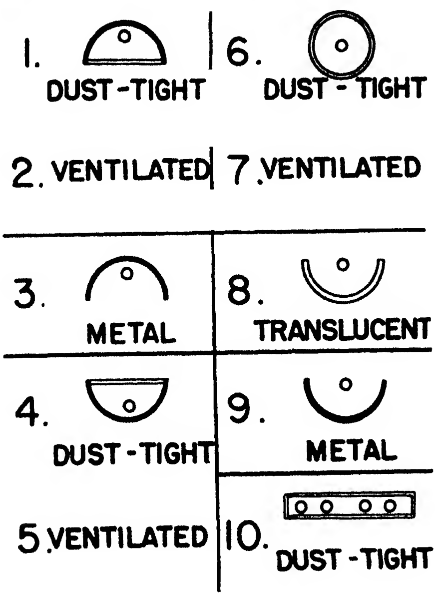
By knowing the distribution of light from the lamp, the luminaire designer can predict the effect of dust for any proposed luminaire. In this way, he can compare various designs, on the basis of maintenance. These calculations can be made to any desired degree of refinement for any specified luminaire, but usually it is sufficient to consider only a few representative types and to omit such refinements as interflections within the luminaire.

It is convenient to obtain a set of equations for luminaires containing fluorescent lamps and another for luminaires with incandescent lamps. Sketches of typical fluorescent types are shown in Table V. The equations follow the table.

TABLE V.

Luance k_{m2} of Fluorescent Luminaires.

Luminaire	t_c = 30 days	60	90	150	360	
1	0.997	0.994	0.992	0.988	0.980	0.970
2	0.948	0.905	0.870	0.810	0.700	0.564
3	0.985	0.970	0.959	0.939	0.902	0.853
4	0.968	0.938	0.914	0.873	0.792	0.681
5	0.932	0.871	0.826	0.748	0.612	0.454
6	0.988	0.976	0.966	0.950	0.918	0.875
7	0.964	0.928	0.902	0.856	0.771	0.664
8	0.974	0.952	0.933	0.902	0.840	0.758
9	0.966	0.934	0.910	0.867	0.788	0.686
10	0.982	0.966	0.958	0.931	0 886	0.826



Fluorescent Luminaires:

1. $k_{m2} = \tau(\pi/2)$

2. $k_{m2} = \tau(0)\tau^2(\pi/2)\left[\left(\frac{F_P}{F_L}\right) + \left(1 - \frac{F_P}{F_L}\right)\tau_c\tau(\pi/2)\right]$

3. $k_{m2} = \tau(\pi/2)\left[\left(\frac{F_P}{F_L}\right) + \left(1 - \frac{F_P}{F_L}\right)\tau_c\tau(\pi/2)\right]$

4. $k_{m2} = \tau(0)$

5. $k_{m2} = \tau_c\tau(0)\tau(\pi/2)\left[\left(\frac{F_P}{F_L}\right) + \left(1 - \frac{F_P}{F_L}\right)\tau_c\tau(\pi/2)\right]$

$$6.\quad k_{m2} = \left(\frac{F_P}{F_L}\right)\tau(\pi/2) + \left(1 - \frac{F_P}{F_L}\right)\tau_c$$

$$7.\quad k_{m2} = \tau_c\tau(\pi/2)\left[\left(\frac{F_P}{F_L}\right)\tau(\pi/2) + \left(1 - \frac{F_P}{F_I}\right)\tau_c\right]$$

$$8.\quad k_{m2} = \tau_c\left[\left(\frac{F_P}{F_I}\right)\tau^2(\pi/2) + \left(1 - \frac{F_P}{F_I}\right)\right]$$

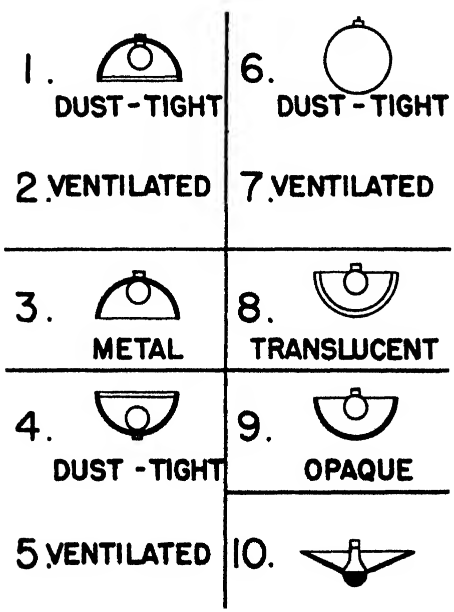
$$9.\quad k_{m2} = \tau_c\left[\left(\frac{F_P}{F_I}\right)\tau_c\tau(\pi/2) + \left(1 - \frac{F_P}{F_I}\right)\right]$$

$$10.\quad k_{m2} = \left(\frac{F_P}{F_I}\right)\tau(\pi/2) + \left(1 - \frac{F_P}{F_L}\right)\tau(0).$$

TABLE VI.

Luance k_{m2} of Incandescent Luminaires.

Luminaire	t_o = 30 days	60	90	150	360	∞
1	0.997	0.994	0.992	0.988	0.980	0.970
2	0.950	0.908	0.874	0.816	0.710	0.575
3	0.985	0.972	0.964	0.946	0.912	0.869
4	0.968	0.938	0.914	0.873	0.792	0.681
5	0.936	0.878	0.838	0.766	0.636	0.483
6	0.989	0.978	0.970	0.957	0.930	0.893
7	0.968	0.937	0.916	0.877	0.804	0.708
8	0.976	0.953	0.936	0.904	0.851	0.766
9	0.970	0.944	0.924	0.888	0.821	0.732
10	0.982	0.964	0.951	0.927	0.881	0.817



In Table V, Luminaires 1 and 2 are boxes with bottoms of diffusing glass or plastic. The reflector is opaque and the diffusing panel is usually flush with the ceiling. Number 3 consists of an opaque reflector with one or more fluorescent lamps. Numbers 4 and 5 are similar to 1 and 2 but are operated with the glass panel facing up. Numbers 6 and 7 are cylinders of glass or plastic surrounding fluorescent lamps. Numbers 8 and 9 are for ceiling lighting, the former being translucent while the latter is opaque. Number 10 is a new design consisting of a dust-tight box containing fluorescent lamps. The calculated values of luance k_{m2} are given in Table V on the assumption that $(F_P/F_L) = 0.50$ in all cases.

A corresponding set of results for luminaires using incandescent lamps is shown in Table VI. The equations for the luance of these luminaires are:

Incandescent Luminaires

1. $k_{m2} = \tau(\pi/2)$

2. $k_{m2} = \tau(0)\tau^2(\pi/2)\left[\left(\frac{F_P}{F_I}\right) + \left(1 - \frac{F_P}{F_L}\right)\tau_s\tau(\pi/2)\right]$

3. $k_{m2} = \tau(\pi/2)\left[\left(\frac{F_P}{F_I}\right) + \left(1 - \frac{F_P}{F_I}\right)\tau_s\tau(\pi/2)\right]$

4. $k_{m2} = \tau(0)$

5. $k_{m2} = \tau_s\tau(0)\tau(\pi/2)\left[\left(\frac{F_P}{F_I}\right) + \left(1 - \frac{F_P}{F_I}\right)\tau_s\tau(\pi/2)\right]$

6. $k_{m2} = \left(\frac{F_P}{F_I}\right)\tau(\pi/2) + \left(1 - \frac{F_P}{F_I}\right)\tau_s$

7 $k_{m2} = \tau_s\tau(\pi/2)\left[\left(\frac{F_P}{F_I}\right)\tau(\pi/2) + \left(1 - \frac{F_P}{F_L}\right)\tau_s\right]$

8. $k_{m2} = \left(\frac{F_P}{F_I}\right)\tau(\pi/4)\tau^2(\pi/2) + \left(1 - \frac{F_P}{F_I}\right)\tau_s$

9. $k_{m2} = \tau_s\left[\left(\frac{F_P}{F_L}\right)\tau_s\tau(\pi/2) + \left(1 - \frac{F_P}{F_I}\right)\right]$

10. $k_{m2} = \tau_s$

The calculated values of Table VI assume $(F_P/F_I) = 0.50$, where F_P = pharos from lower half of lamp, F_L = pharos from entire lamp.

The first and second of the luminaires for incandescent lamps are light boxes with lens plates or diffusing glass for bottoms. The third is an ordinary metal reflector. Calculations are for a hemispherical

reflector but other shapes will give approximately the same results. Numbers 4 and 5 are similar to 1 and 2 but used to illuminate the ceiling. Numbers 6 and 7 are diffusing globes. Numbers 8 and 9 are translucent and opaque reflectors for ceiling lighting. Number 10 is a similar luminaire but with a silvered-bowl lamp.

Tables V and VI are similar to Table III of a previous paper.[3] In the earlier work, however, the effect of angle was not known and rough assumptions were made, based on the data available at that time. The tables of this paper make use of the new data on angle and the average values of Eqs. 10 and 11.

The new values of luance of luminaires are in fairly close agreement with those of the previous paper, differences usually running less than 5 per cent. In using these results it should be remembered that the formulae on which they are based are over-simplifications which neglect interflections. On the other hand, these approximations should be much more reliable than those generally used in industry at the present time.

6 SUMMARY

The paper has presented data on the transmittance of dust films, as a function of time and angle. These results may be summarized by the equation

$$\tau = \delta + (1 - \delta)\, e^{-0.00740t},$$

where $\delta = 0.634 + 0.047\, e^{1.255\theta}$ if $0 \leqq \theta \leqq \pi/2$,

$\delta = \delta(\pi/2)$ if $\pi/2 \leqq \theta \leqq \pi$,

and τ = transmittance of a single dust film (Location A),

t = time, days, and

θ = angle between the normal to the dust-collecting surface and the upward direction (radian).

These results were applied to the prediction of the effect of dust on the performance of various types of luminaires using fluorescent and incandescent lamps.

A MECHANICAL INTEGRAPH FOR THE NUMERICAL SOLUTION OF INTEGRAL EQUATIONS.

BY

PETER L. TEA, E.E., M.A.[1]

INTRODUCTION.

In a previous paper (1),[2] a graphical method was developed for the transformation of the integral of the product of two functions of y, given in Riemann form

$$I = \int^{b} \phi(y)K(y)dy, \qquad a \leqq y \leqq b, \tag{1}$$

into its equivalent Stieltjes form (2)

$$I = \int_{A}^{B} \phi(y)dY(y), \tag{2}$$

where

$$dY(y) = K(y)dy, \qquad Y(y) = \int_{0}^{y} K(y)dy, \tag{3}$$

$$A = Y(a), \qquad B = Y(b), \qquad Y(0) = 0.$$

The Y function was called the *functional line of* $K(y)$. It becomes the *functional surface*, Eq. 4, if it has a parameter x,

$$Y(y, x) = \int_{0}^{y} K(y, x)dy. \tag{4}$$

$$Y(0, x) \equiv 0, \qquad y = 0. \tag{5}$$

The functional surface contains at least one straight line element, the x axis. Planes perpendicular to the x axis cut the functional lines from the functional surface for the values of x cut by the planes.

1. The Conversion of the Riemann Integral, Eq. 1, into Its Equivalent Stieltjes Form, Eq. 2, by Two Successive Projections of Either Function.

Figure 1 shows the functional cylinder, Eq. 3, to the axes $0_1\phi$, $0_1 Y(y)$, and $0_1 y$. The plane of $\phi(y)$ is placed parallel to the plane $\phi 0_1 y$, and in such manner that the origin 0 projects orthogonally from the plane of $\phi(y)$ to 0_1; and $\phi(y)$ projects to the functional cylinder then reprojects to the plane $\phi 0_2 Y(y) \perp 0y$. The second projection will be the Stieltjes curve corresponding to Eq. 2, the area under which between B and A

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

will be equal to the integrals, Eqs. 1 and 2, provided that the scales used in space in Fig. 1 are to the same unit value as that of the planimeter.

2. Shifting Functions for K.

The graphical method. and that of the mechanical integraph are simplified considerably if K is of the form

$$K(y, x) = K(y \pm x); \tag{6}$$

then

$$Y(y \pm x) = \int_0^y K(y \pm x)dy = F(y \pm x) - F(0 \pm x), \tag{7}$$

for $y = 0$

$$Y(0 \pm x) \equiv 0, \tag{8}$$

for $x = 0$

$$Y(y \pm 0) = F(y \pm 0) - F(0 \pm 0). \tag{9}$$

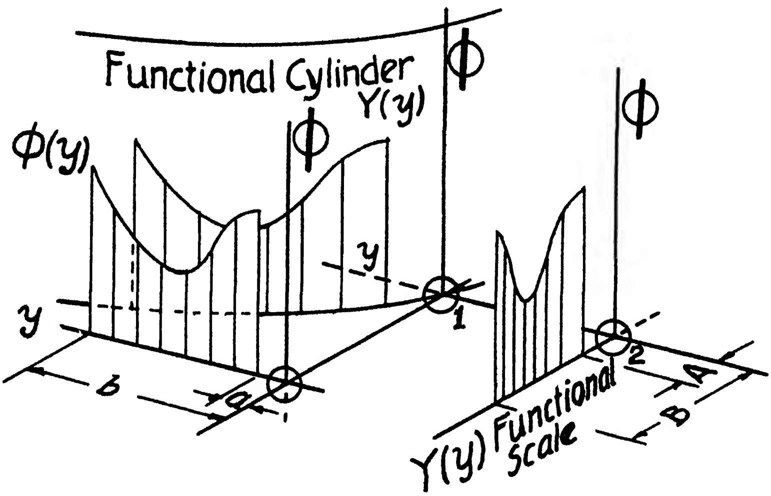

FIG. 1. Two orthogonal projections which convert the integral, Eq. 1, into its Stieltjes form, Eq. 2

The K surface, Eq. 6, is a cylinder generated by the linear motion ot the curve $K(y \pm 0)$ at 45° to the $+y$ axis if the sign of x is $-$, and 135° if the sign of x is $+$, and parallel to the xy plane. An example of such a surface is the symmetrical kernel, Eq. 18, Fig. 5.

The functional surface, Eq. 7, is not a cylinder, but it is composed of two cylinders: cylinder $F(y \pm x)$ at 45° or 135° to the y axis and parallel to the xy plane, like the K surface, and cylinder $F(0 \pm x)$ with elements parallel to the y axis. Cylinder $F(0 \pm x)$ merely displaces the functional lines vertically to pass them through the x axis, without changing their form. The functional surface, Eq. 19 of the kernel Fig. 5, is shown in Fig. 6.

If the functional cylinder $Y(y)$, Fig. 1, has as its base the functional

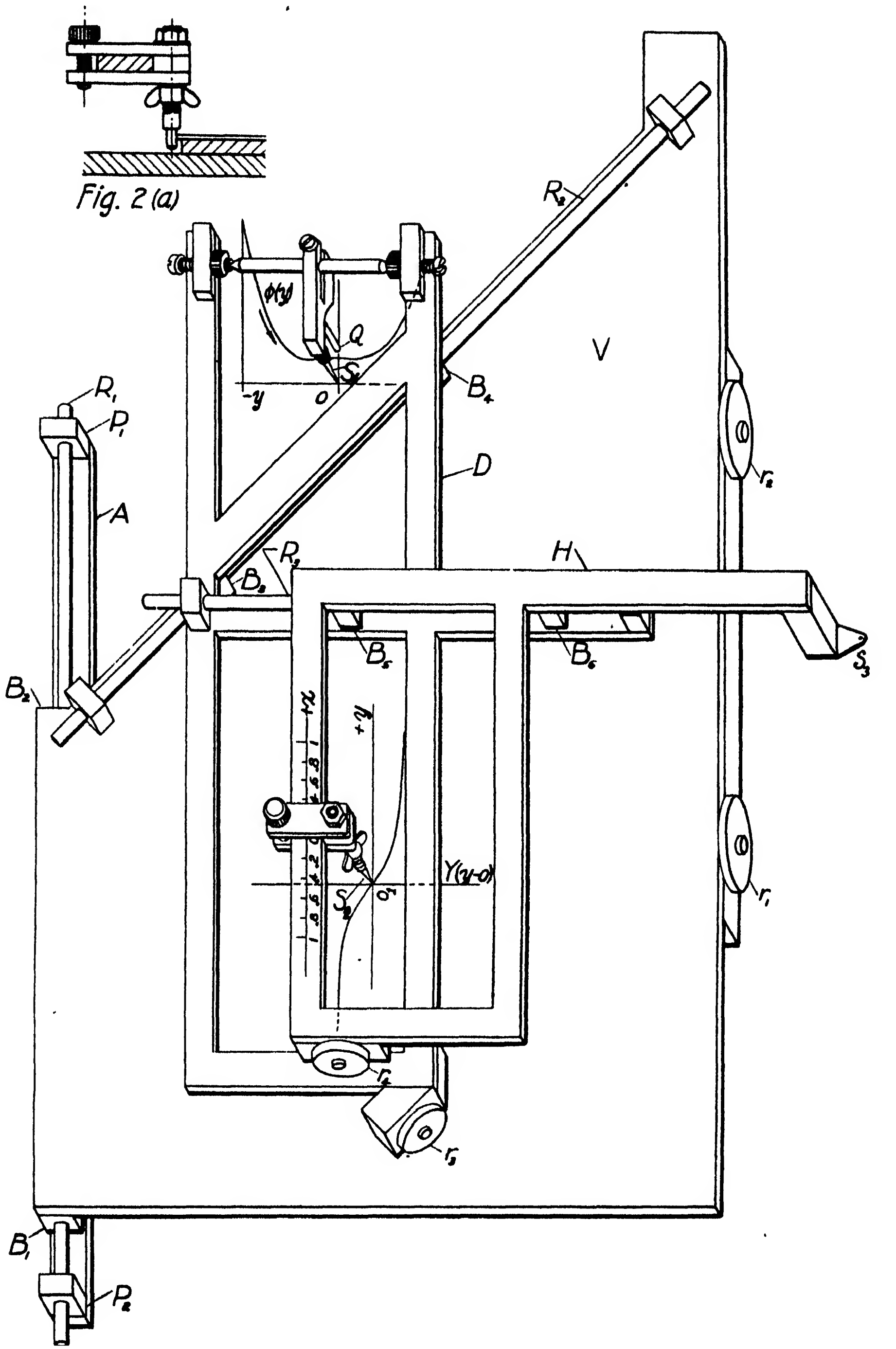

FIG. 2. Schematic drawing of the integraph.

curve Eq. 9 of the shifting function $K(y \pm x)$, then the area under the Stieltjes curve for any value of x is

$$I(x) = \int_A^B \phi(y)dY(y \pm x). \tag{10}$$

The Stieltjes curve is obtained by first shifting the functional cylinder $Y(y \pm 0)$ by the amount $\pm x$, with $\phi(y)$ fixed, and then making the double projections; or $\phi(y)$ can be shifted first, by an amount $\mp x$, with the cylinder fixed, then making the two projections as before.

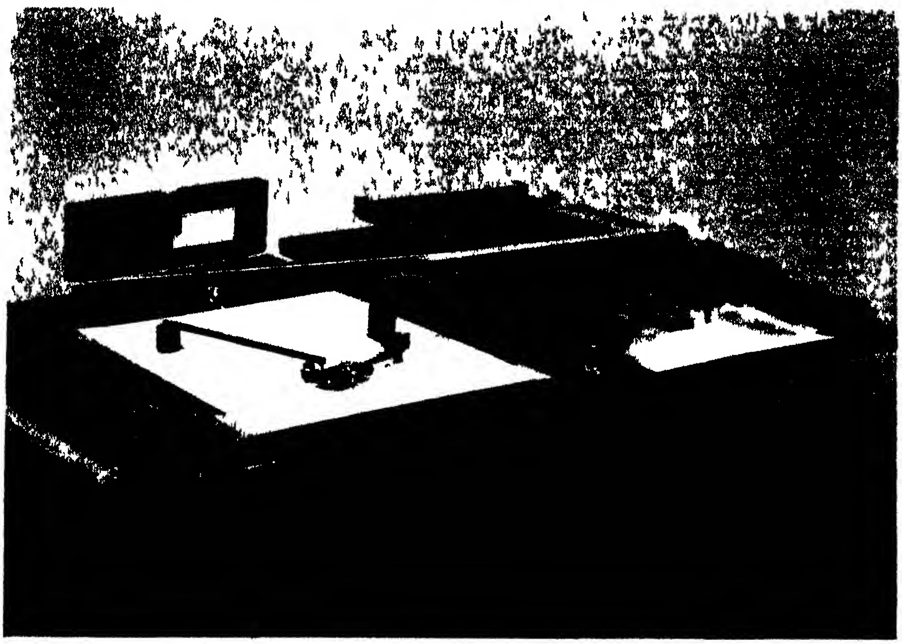

FIG 3 Photograph of the integraph

Analytically expressed

$$I(x) = \int_a^b \phi(y)K(y \pm x)dy = \int_A^B \phi(y)dY(y \pm x) \tag{11}$$

by shifting $Y(y \pm 0)$, and

$$I(x) = \int_{a \pm x}^{b \pm x} \phi(y \mp x)K(y \pm 0)dy = \int_A^B \phi(y \mp x)dY(y \pm 0) \tag{12}$$

by shifting $\phi(y)$.

The active segment of $Y(y \pm 0)$ or $K(y \pm 0)$ is always of length $(b - a)$ along the y axis, and shifts linearly with x. The total length of Y or K involved is $(b - a) + (d - c)$. The $(d - c)$ segment is

added to one side or the other of the $(b - a)$ segment for $x = 0$, depending on the sign of x.

$$a \leqq y \leqq b, \quad B = Y(b \pm x), \quad A = Y(a \pm x), \quad c \leqq x \leqq d.$$

I. THE MECHANICAL INTEGRAPH.

The mechanical integraph consists of three frames: V, D, and H, Figs. 2 and 3. Frame V is supported by two disc wheels r_1 and r_2 and slides on two brass bearings B_1 and B_2 (fixed to V) along the rod R_1 which is fixed to the posts P_1 and P_2, which are fixed to strip A, which is clamped to the drawing board or table. Frame D has one disc wheel r_3 and slides on two bearing B_3 and B_4 along the rod R_2 which is fixed by two posts to frame V. Rod R_2 permits diagonal motion of frame D with respect to V. Rod R_2 is accurately set at 45° to rod R_1. Stylus S_1 is mounted, by means of its holder, on a rod with cone pointed ends along which it can be fixed and with which it can rotate. The weight of the stylus holder and rod combination is supported by the cone bearings and by the rest Q. Rest Q is adjustable in length, and the stylus S_1 is adjustable in the distance of its point from the drawing board on which the $\phi(y)$ curve is fastened.

With a little practice, the two motions: V along R_1, and D along R_2 may be executed using a hand for each, so that stylus S_1 follows curve $\phi(y)$. The rods are lightly lubricated with clear Vaseline. They are drill rods, accurately ground and fairly straight. The slide bearings are of brass, reamed to an easy slide fit to the rods with no shake. Every point of frame D duplicates the motion of S_1 and describes the curve $\phi(y)$ with respect to the table.

Frame H has one disc wheel r_4 and slides on two bearings B_5 and B_6 along the rod R_3 which is fixed by means of two posts to frame D. Rod R_3 is accurately mounted at 90° to rod R_1 and at 45° to rod R_2, Fig. 2.

The functional line $Y(y \pm 0)$ is fastened to the frame V. One operator follows the curve $\phi(y)$ with stylus S_1 from any convenient starting point along the arrow around the perimeter and stops accurately at the starting point, while another operator follows the functional line $Y(y \pm x)$ with stylus S_2. Every point on frame H follows the Stieltjes curve with respect to the table.

A planimeter, placed on the table with its stylus at any convenient point of H, as S_3, will follow the Stieltjes curve and will read the numerical value of the integral.

1. The Setting of the Curve $\phi(y)$ on the Table and of the Functional Line $Y(y \pm 0)$ on V.

The curve $\phi(y)$ is fixed to the table so that its zero ordinate is under S_1 for motion of V only. With the holder of stylus S_2 set for $x = 0$, and with care not to move frame D, the functional line graph is set to frame

V with its zero ordinate line under S_2 for motion of H only. With the setting just described the planimeter reading will be the value of the Stieltjes integral for $x = 0$. For the integral for any value of x the holder of stylus S_2 is moved to that value; or with S_2 fixed at $x = 0$, $\phi(y)$ can be shifted to the x value.

If a shifting function is to have considerable use it is an advantage to cut its profile out of thin sheet iron on a band saw, and to file to the line by means of a jig file or by hand. The profile can be cut so that it can be used for a number of limit values of the integral. The stylus is replaced by a new piece with a No. 00 taper pin instead of the pointed stylus, Fig. 2(a). The sheet metal profile is fixed to a sheet of pressed wood $\frac{1}{8}$ in. thick by means of "Miracle Adhesive," and adjusted on frame V in the manner described. The taper pin is slightly raised from V, and rubs against the profile. A second operator is not necessary if the sheet metal profile is used. To provide a suitable force at the taper pin against the profile, adequate to actuate the planimeter without losing contact, a piece of cotton elastic band $\frac{1}{4}$ in. wide and about 3 ft. long was used with one end tied to frame H and the other end tied to the wall. Better ways can readily be devised, but the elastic band worked very well.

In case K is not a shifting function, then the curves for suitable x values can be drawn, all through the origin 0_1, and each is used in turn without changing the setting of stylus S_2, at $x = 0$.

2. Scales.

With rod R_2 at 45° to R_1, the scales of x and y must be equal. If the unit value of x and y is represented by m inches, and for the ordinate scale one unit of $\phi(y)$ is represented by n inches and one unit of the Y scale is l inches, then the area read on the planimeter is divided by nl. If different scales are desired for x and y the angle between R_2 and R_1 must be changed accordingly.

II. THE APPLICATION OF THE MECHANICAL INTEGRAPH TO THE EXPANSION OF FUNCTIONS IN A SERIES OF ORTHOGONAL FUNCTIONS.

A singly valued function $f(y)$ can be expanded in a series of orthogonal functions

$$f(y) = \sum_{0}^{n} a_q \varphi_q(y) \tag{13}$$

within an interval (a, b) if, *within the interval,*

$$\int_a^b \varphi_m(y)\varphi_n(y)dy = \begin{matrix} 0 & \text{for} & m \neq n, \\ 1 & \text{for} & m = n. \end{matrix} \tag{14}$$

The functions $\varphi_0(y)$, $\varphi_1(y) \cdots$ are said to be orthogonal because all the integrals are zero for $m \neq n$, and are called normalized because all the integrals are equal to unity for $m = n$.

From Eqs. 13 and 14

$$a_q = \int_a^b f(y)\varphi_q(y)dy. \tag{15}$$

The expansion of single valued functions (or arbitrary curves) is greatly simplified by the use of orthogonal functions (3,4,5,6). The mechanical integraph may prove useful in boundary problems where the integration of Eq. 15 may offer difficulties. Functional lines of orthogonal sets of interest can readily be prepared.

III. THE APPLICATION OF THE MECHANICAL INTEGRAPH TO FREDHOLM'S INTEGRAL EQUATION OF THE FIRST KIND.

1. The Magnetic Field Along the Axis of Any Number of Co-axial Circular Loops Carrying a Current i_k, Fig. 4.

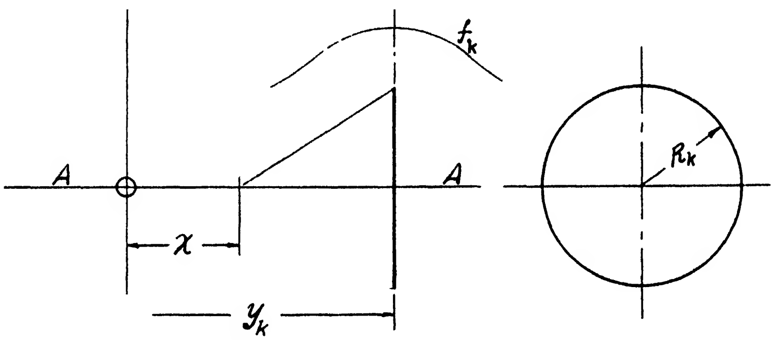

FIG. 4. Magnetic force f_k along the axis AA due to the current through a single turn at y_k.

The magnetic fields add vectorially at any point, and algebraically along the common axis AA,

$$F(x) = 2\pi \sum \frac{R_k^2 i_k}{[(y-x)^2 + R_k^2]^{\frac{3}{2}}}. \tag{16}$$

Helmholtz obtained a quasi constant magnetic field along the common axis between two equal concentrated coils carrying the same current and spaced a distance apart equal to the mean radius of the coils.

2. The Magnetic Field of a Cylindrical Solenoid of Radius 1 cm. Is to be Constant Along the Axis Between the Limits of the Solenoid.

The length of the solenoid is $2L$ and $\phi(y)$ represents the ampere turns per centimeter.

$$f(x) \equiv 2\pi \int_{-L}^{L} \frac{\phi(y)dy}{[(y-x)^2 + 1]^{\frac{3}{2}}} \equiv 2\pi c, \tag{17}$$

where $-L \leqq y \leqq L$ and, for this problem, $-L \leqq x \leqq L$.

This is a Fredholm integral equation of the first kind (3,4,5,7,15). The kernel is symmetrical in x and y, Eq. 18, and is shown pictorially in Fig. 5.

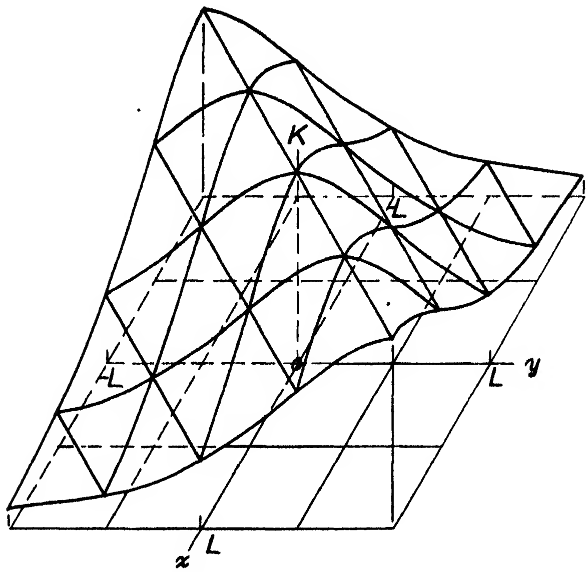

FIG. 5. The surface representing the kernel, Eq 18, of problem III, 2

$$K(y-x) = [(y-x)^2 + 1]^{-\frac{3}{2}}. \tag{18}$$

The functional surface

$$Y(y-x) = \frac{y-x}{\sqrt{(y-x)^2+1}} + \frac{x}{\sqrt{x^2+1}} \tag{19}$$

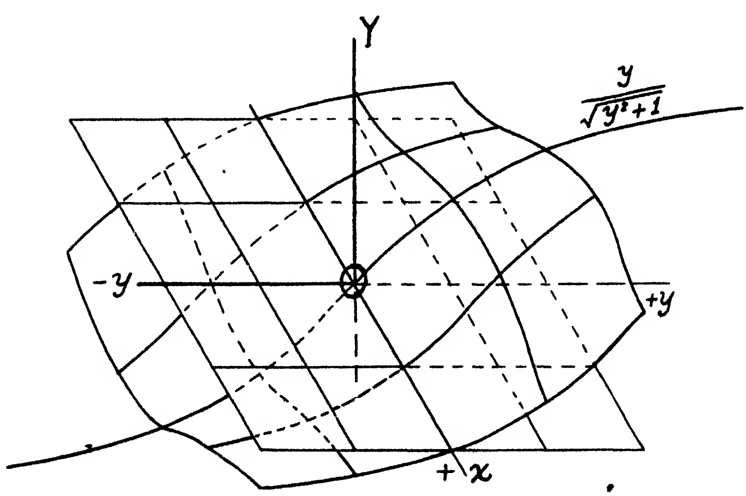

FIG. 6. The functional surface, Eq. 19, of the kernel, Fig. 5.

is shown in Fig. 6. It is generated by the functional line, Eq. 20, with the two motions discussed in the Introduction.

The functional line for $x = 0$, Eq. 20, was cut out of sheet iron of No. 26 gauge and fixed to frame V of the integraph; any value of Eq. 17 could be obtained for any curve $\phi_k(y)$ and any value of x.

$$Y(y - o) = \frac{y}{\sqrt{y^2 + 1}} \tag{20}$$

The integral equation will be considered to be a set of $2n$ linear simultaneous algebraic equations

$$f(x_j) = 2\pi C = 2\pi \sum_{i=1} K(y_i - x_j)\phi(y_i)\frac{L}{n}, \qquad j = 1, 2, \quad 2n. \tag{21}$$

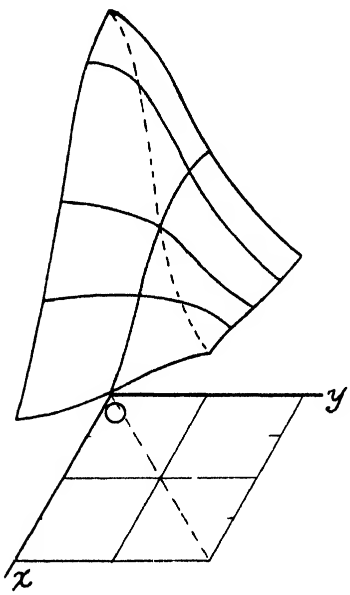

FIG. 7 The matrix of Eqs. 21

The coefficients $K(y_i - x_j)$ for given x, are given in a continuous form by the section of the kernel by the plane $x = x_j$.

The method used for solving Eqs. 21 was suggested by the successive approach to the solution of simultaneous linear equations having a large value of coefficients along the diagonal (8,9,10,11).

Since $\phi(y) = \phi(-y)$ the diagonal coefficient is

$$2\pi M = 2\pi[K(0) + K(2y)] = 2\pi[1 + K(2y)] \tag{22}$$

Half the kernel obtained, by adding the pairs of coefficients for $\phi(y)$, is shown in Fig. 7. The problem was solved for $L = 1$, and Figs. 5, 6, 7 were drawn for the same limits.

3. *Method of Successive Approach.*

The initial value of the ampere turns function was taken to be

$$\phi_0(y) \equiv 1 \tag{23}$$

and, by integration,

$$f_0(x) = 2\pi\left[\frac{1 - x}{\sqrt{(1 - x)^2 + 1}} + \frac{1 + x}{\sqrt{(1 + x)^2 + 1}}\right]; \tag{24}$$

the value for $x = 0$ is

$$f_0(0) = 1.414 = c,$$

which is to be the value for $f(x)$ for all values of x.

The factor 2π was dropped since it cancels out in Eq. 26.

The error is

$$\epsilon_0(x) = c - f_0(x). \tag{25}$$

The correction to be applied to $\phi_0(y)$ is

$$\Delta\phi_0(y) = \epsilon_0(y)1/M, \tag{26}$$

$$\phi_1(y) = \phi_0(y) + \Delta\phi_0(y), \tag{27}$$

$$f_1(x) = \int_{-1}^{1} \phi_1(y)dY(y - x) \tag{28}$$

by means of the integraph.

The process was repeated until the limit of accuracy of the integraph was reached, at ϕ_{10}, after which the method of differences was used, with scale for ϕ multiplied by ten, to the value $k = 18$.

$$\Delta f_k(x) = \int \Delta\phi_k(y)dY(y - x), \tag{29}$$

$$f_{k+1}(x) = f_k(x) + \Delta f_k(x). \tag{30}$$

TABLE I.

Coefficients along the Diagonal.

x	0	0.1	0.2	0.3	0.4	0.5	0.55	0.6	0 7	0.8	0.85	0.9	0.95	1.00
M	2.000	1.943	1.801	1.629	1.477	1.345	1.305	1.263	1.196	1.148	1.130	1.114	1.101	1.089

It is seen from Fig. 7 that the diagonal coefficient is not the largest for x sections except near $x = 0$ and ± 1. However the application of Eqs. 25 to 30 shows a convergence, Fig. 8. The largest coefficient for x values becomes less dominant as x approaches ± 1. The successive approaches for $f_k(x)$ and $\phi_k(y)$ were re-calculated by proportion to anchor $f_k(\pm 1)$ at the value 1.414. Figure 8 indicated a slow convergence. Comparatively large plus and minus changes in ϕ have an almost cancelling effect on $f(x)$. The initial values of $\phi(y)$ show a rough

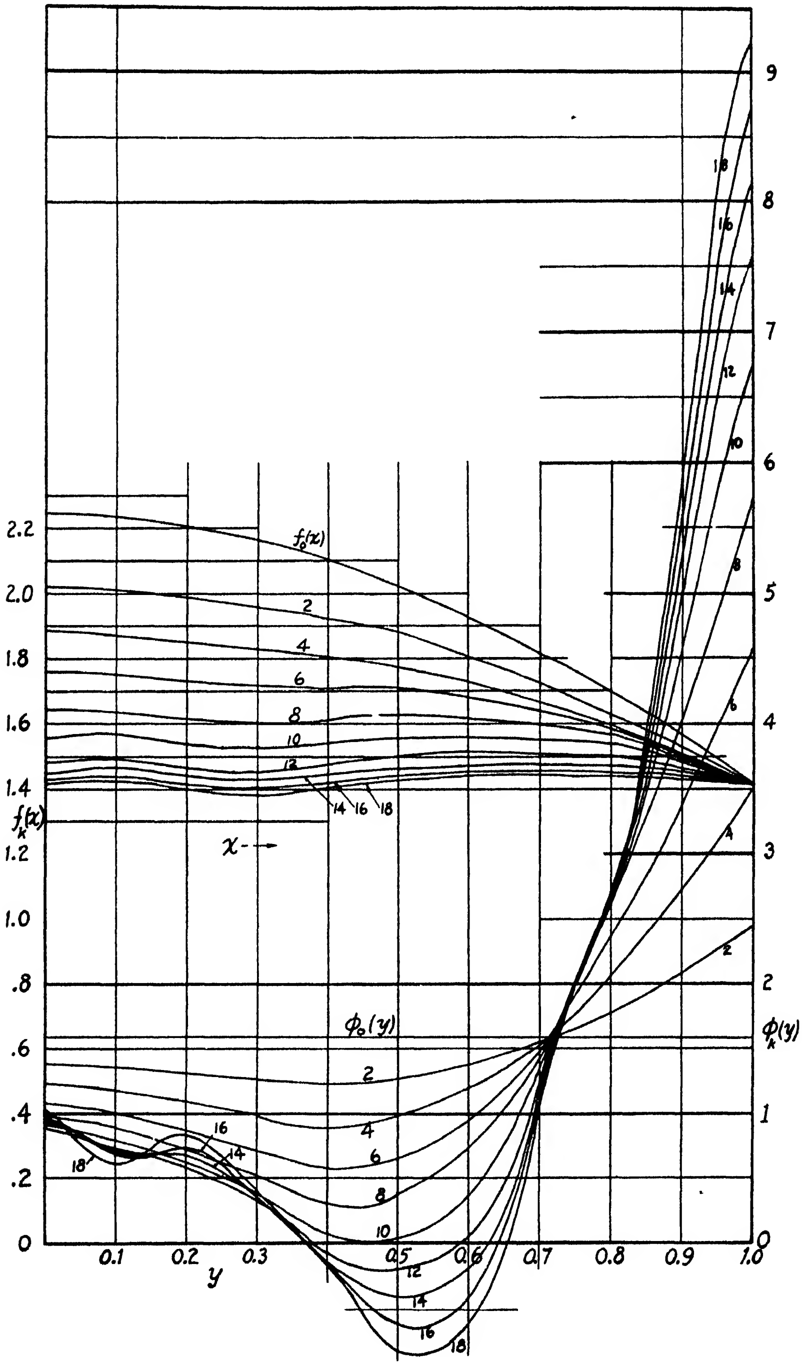

FIG. 8. Curves showing the gradual approach to the ampere turns function $\phi(y)$ as the magnetic force along the axis approaches $f(x) = 1.414$.

overall approach; and later stages indicate local corrections. The process was stopped at $k = 18$.

The Cinema Integraph of the Massachusetts Institute of Technology can record a continuous curve $f(x)$ for any $\phi(y)$ by a continuous variation of the parameter x. The error curve such as Eq. 25, and the $\phi_0(y)$ curve, Eq. 26, and $\phi_1(y)$ would thus be known continuously to within the degree of error of the Cinema Integraph. Errors in drawing the curve through point values of ϕ would be avoided. The advantages of this feature and of the smoothness and celerity are admitted. However it is a simple matter to set the mechanical integraph for intermediate values of x at any time where ϕ indicates the development of sudden changes. The computations were started with point values for $\Delta x = 0.1$; intermediate points were taken at $x = 0.55$, 0.85, and 0.95. On account of the symmetry only positive values of x were used.

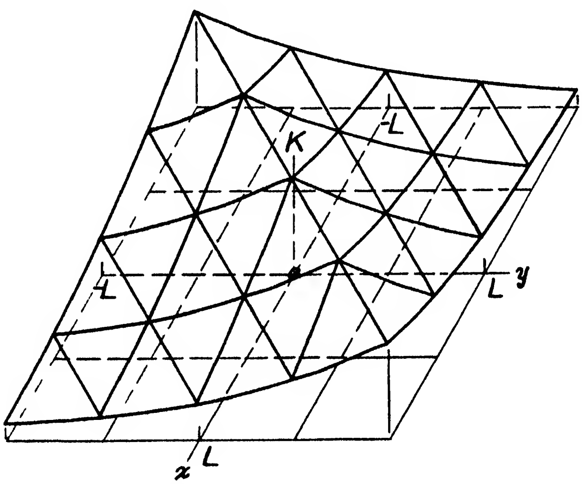

FIG. 9. The kernel in Buckley's interreflection problem.

For longer solenoids the convergence is more rapid, especially near the center. At the ends there are high values of ϕ as in the present problem. Winding of the coil for the ϕ value in Fig. 8 could be only approximate. The solution has no applicable value; it was chosen to show what can be accomplished with the integraph in a problem with slow convergence.

4. Least Squares.

The unknown $\phi(y)$ in Eq. 17 can be assumed represented by a polynomial,

$$\phi(y) = a_0 + a_1|y| + a_2y^2 + a_3|y^3| + \cdots. \tag{31}$$

Curves for 1, $|y|$, y^2, $\cdots$ etc., are used on the integraph, yielding

$$f(x) \equiv a_0\psi_0(x) + a_1\psi_1(x) + a_2\psi_2(x) \cdots, \tag{32}$$

where the ψ values are in graphical form, and the coefficients a_0, a_1, $\cdots$ etc., are to be obtained by the method of least squares.

The normal equations are

$$\begin{aligned}
a_0 \int \psi_0^2 dx \ + a_1 \int \psi_1\psi_0 dx + a_2 \int \psi_2\psi_0 dx + \cdots - \int f\psi_0 dx &= 0, \\
a_0 \int \psi_0\psi_1 dx + a_1 \int \psi_1^2 dx \ + a_2 \int \psi_2\psi_1 dx + \cdots - \int f\psi_1 dx &= 0, \\
\cdots \\
a_0 \int \psi_0\psi_n dx + a_1 \int \psi_1\psi_n dx + \cdots \qquad \qquad - \int f\psi_n dx &= 0.
\end{aligned} \tag{33}$$

The discriminant is symmetrical. If the principal diagonal is dominant the method of successive approximations is practical, even for a large number of terms.

The method is not limited to polynomial expansion. Any series of single-valued functions, orthogonal sets, or any series graphically expressed, can be used. If the assumed expansion is in eigenfunctions of the kernel, then only the diagonal term and the last term remain, and the series of coefficients of the expansion is easily obtained.

5. Accuracy of the Mechanical Integraph.

The accuracy of operation of the mechanical integraph is about the same as that of the planimeter, provided that a profile of the functional line of thin sheet iron is used, and that ϕ does not develop changes of direction that are too many and too steep. A good profile, with offsets every half inch, to better than 0.01-in. accuracy is not difficult to attain, with the aid of a steel depth scale reading to 0.001 in., a mechanic's fine scribe, a watchmaker's glass, and draftsman's triangles and straight edge. But the error involved in drawing the ϕ curves through the point values depends on how sharply the curve changes in direction, where the intermediate points were chosen, and on the skill of the draftsman. In the problem of the solenoid the possible error in some places is estimated to be as high as 4 per cent, due to the sudden changes. Scales used were $m = 2$, $n = 2$, $l = 2$.

6. The Superposition of the Magnetic Fields Due to Co-axial and Uniformly Wound Coils of Finite Lengths.

This subject was discussed in "Vector" (12). A nomogram was designed to facilitate obtaining the force along the axis for any point of uniformly wound solenoids of any length. A line scale of suitable length can readily be made which would be more accurate than the nomogram. The reader is referred to the paper in "Vector" for the

design of long solenoids with concentric coils near the ends, to obtain a fairly uniform magnetic field along the axis for part of the length of the main solenoid.

IV. THE APPLICATION OF THE MECHANICAL INTEGRAPH TO FREDHOLM'S INTEGRAL EQUATION OF THE SECOND KIND.

For the integral equation of the second kind

$$\phi(x) = f(x) + \rho \int_a^b K(y, x)\phi(y)dy, \qquad (34)$$
$$a \leqq y \leqq b, \qquad a \leqq x \leqq b,$$

the successive substitutions

$$\phi_0(x) = f(x), \qquad \phi_{q+1}(x) = f(x) + \rho \int_a^b K(y, x)\phi_q(y)dy \qquad (35)$$

approach the solution as a limit, provided that there is convergence.

The mechanical integraph was used to solve Buckley's problem on the interreflection of light on the mat inner surface of a circular cylinder where $\phi_0(x)$ is the exciting luminosity with no reflection, that is, for $\rho = 0$, and the actual luminosity, with interreflection, is (13,16)

$$\phi(x) = \phi_0(x) + \frac{\rho}{2a} \int_{-l}^{l} e^{-|y-x|}\phi(y)dy. \qquad (36)$$

The exact kernel in Buckley's problem is given in Eq. 37. The kernel in Eq. 36, used by Buckley, is a close approximation to the exact kernel for the limits used. Both forms are shifting functions. The exponential form is convenient in applying Whittaker's method for the numerical solution of Eq. 34, by expanding the kernel in a series of exponentials by the method of Prony, thus effecting a separation of the variables (13,14,15).

$$K(y - x) = \frac{1}{2a}\left[1 - \frac{|y - x|(\overline{y - x}^2 + 6a^2)}{(\overline{x - x}^2 + 4a^2)^{\frac{3}{2}}} \right]. \qquad (37)$$

The values used were $\rho = 1$, cylinder radius $a = 1$, half length of cylinder $l = 1$. In Table II, the first three columns are comparable

TABLE II.

Comparative Values of 0(x) for Buckley's Problem Obtained by:

x	Mechanical Integraph	Graphical Method	Buckley Whittaker	Hedeman, Jr Cinema Integraph
0.00	2.481	2.490	2.500	2.608
0.25	2.450	2.478	2.469	2.548
0.50	2.365	2.384	2.375	2.440
0.75	2.220	2.232	2.219	2.264
1.00	2.009	2.011	2.000	2.046

because the same approximate kernel of Eq. 36 was used; Hedeman, Jr. used the Cinema Integraph of M. I. T. on the exact kernel (16). Comparison of the results of the method of the mechanical integraph and the graphical method with the results of the Buckley-Whittaker method shows that the error is less than 1 per cent.

V. THE APPLICATION OF THE MECHANICAL INTEGRAPH TO THE CALCULATION OF THE RECIPROCAL KERNEL OF VOLTERRA.

The terms of Eq. 35 can be arranged in powers of ρ,

$$\phi(x) = f(x) + \int_a^b \Gamma(\rho^n; y, x) f(y) dy, \tag{38}$$

$$\Gamma = \sum_{n=1}^{\alpha} \rho^n K_n(y, x), \tag{39}$$

where Γ is the reciprocal kernel of Volterra, and

$$K_1(y, x) = K(y, x), \qquad K_q(y, x) = \int_a^b K_1(s, x) K_{q-1}(y, s) ds \tag{40}$$

are the first and the qth iterated function of $K(y, x)$.

Equation 40, in its Stieltjes form, is

$$K_q(y, x) = \int_A^B K_{q-1}(y, s) dS(s, x), \tag{41}$$

where

$$dS(s, x) = K_1(s, x) ds, \qquad S(s, x) = \int_0^s K_1(s, x) ds. \tag{42}$$

The mechanical integraph can be used to calculate the numerical values of the iterated functions, Eq. 41, where x is still a parameter and y is now a parameter, with s as the variable which disappears on integrating.

Equation 41 is written as Eq. 43 in the form for a shifting kernel,

$$K_q(y, x) = \int_A^B K_{q-1}(y, s) dS(s - x). \tag{43}$$

The surface $K_{q-1}(y, s)$ is assumed known, and cut by the planes $y = y_k$, where $k = 1, 2, 3, \cdots$, equally spaced in the interval $(b - a)$, say, and the procedure of Eq. 11 or 12 is used, where s takes the place of y in Fig. 1.

If the kernel is symmetrical, then

$$K_{q-1}(y_k, x) \equiv K_{q-1}(y, x_k). \tag{44}$$

In Eq. 38 the functional line for $f(y)$ can be prepared and each term of the series can be integrated by the integraph, in the form of a series

of curves, functions of x, each term multiplied by the corresponding term of the power of ρ.

Improvements could be incorporated in a new model of the mechanical integraph. Ball bearings could be used, of either type: for rotation about a shaft, or translation along a shaft. The static and dynamic friction values are nearly equal, and less than for sleeve bearings if properly installed without strain. Sealed ball bearings are protected from dust and they contain all the lubricant that they will ever require.

To avoid parallax in using the pointed stylus, Coradi of Zurich, in his precision planimeters, uses "La Loupe Saphyr," consisting of a cylindrical piece of glass, axis vertical, height about $1\frac{1}{2}$ diameters, convex at the top, thus acting as a magnifier of 2:1. In the center of the flat bottom of the cylinder a thin circular disc of hard saphyr is inlaid which rests on and rubs on the paper. In the center of the saphyr disc there is a smaller circular spot which is made to ride the curve. This requires the operator to follow the curve with his eye directly over the cylinder. Van den Akker designed and built a mechanical integraph to integrate the product of two functions of y for a different purpose (17). He uses a small telescope, one for each graph, which projects a small spot of light on the curve. A pair of cross hairs could be used.

ACKNOWLEDGMENTS.

This research was helped materially by a small grant from the Research Corporation. Interesting suggestions were made by members of the Department of Physics at the City College, especially in regard to an optical method based on Fig. 1, which gives promise to develop into a method for obtaining a continuous curve $f(x)$ by moving either $\phi(y)$ or $Y(y \pm 0)$. My colleagues in the Department of Drafting offered valuable suggestions on the mechanical integraph which may be incorporated in a new design.

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Heat-Resistant Dyes. (*Heating and Ventilating*, Vol. 44, No. 10.)—The reaction to infra-red radiation of the dyestuffs used in clothing is an important factor in their warmth or coolness, according to new studies and experiments made at the Philadelphia Textile Institute as part of a research project sponsored by the Philadelphia Section, American Association of Textile Chemists and Colorists.

Experiments.—Wool, cotton, cotton duck, and rayon fabrics identical in every respect, including color, but dyed with different commercial dyestuffs, were used in the experiments conducted at the Philadelphia Textile Institute. Because of difference in the infra-red absorption characteristics of the dyestuffs used, the dyed fabrics, although identical in color, varied as much as 14 F. in temperature under prolonged exposure to sunlight.

Ice-Melting.—The heating effects on fabrics resulting from the varying infra-red characteristics of dyes is illustrated strikingly by the following experiment:

Two pieces of the same wool fabric are dyed black, one with Pontacyl Blue Black RC which is low in infra-red absorption and therefore reflects most of the infra-red in sunlight, and the other with Chromacyl Black W which absorbs most of the infra-red. The two fabrics are identical in appearance. Rectangular pieces of the two fabrics are laid over blocks of ice of the same size. Then they are placed side by side under infra-red lamps, which produce energy similar to that from the sun but at higher intensity and at a uniform rate. The experiment can be conducted equally well in sunlight, but it is slower and more troublesome because of varying sunlight intensity. Under the infra-red lamps, the ice melts very much faster under the fabric dyed with the high infra-red absorbing Chromacyl Black W than under the low infra-red absorbing Pontacyl Blue Black RC.

Burning.—Another interesting experiment also illustrated the heating effects resulting from infra-red absorption:

Two pieces of the same cotton poplin are dyed brown, one with Pontamine Brown BT, which reflects most of the infra-red and therefore is low in infra-red absorption, and the other with Ponsol Brown BB, which absorbs most of the infra-red. The samples are mounted side-by-side under high-intensity infra-red lamps. The fabric dyed with Ponsol Brown BB heats up rapidly, chars and then burns; while the fabric dyed with the low-absorbing Pontamine Brown BT merely becomes somewhat warmer.

In the leaflet entitled "How Cool is Your Color?" the institute points out that new discoveries concerning the wide variations in infra-red absorption characteristics of commercial dyestuffs "may prove of substantial significance to the textile and apparel industries and strongly influence the clothing habits of the American public."

R. H. O.

ADIABATIC FLOW OF HYDROGEN GAS THROUGH A ROCKET NOZZLE WITH AND WITHOUT COMPOSITION CHANGE.*

BY

S. S. PENNER[1] AND D. ALTMAN.[1]

ABSTRACT.

A procedure is described for determining the characteristics of adiabatic flow through a rocket nozzle with and without composition change. The method of calculation is illustrated for the expansion of pure hydrogen gas from a chamber temperature of 3506° K. and a pressure of 20.42 atm. to atmospheric pressure.

The study indicates that the exhaust velocity and temperature are highest for flow where complete equilibrium is reached at each temperature with respect to the reaction

$$H_2 \rightleftarrows 2H.$$

Flow with composition change requires a nozzle exit to nozzle throat area ratio somewhat greater than that determined for adiabatic flow without composition change for the same ratio of chamber pressure to exit pressure.

The residence time in a given temperature range is computed as a function of gas temperature for the two types of flow. The results of this calculation may be used to determine the minimum required reaction rates which allow composition changes during flow through the nozzle.

I. INTRODUCTION.

The performance of a rocket motor depends, among other factors, upon the nature of the flow process through the rocket nozzle. Performance calculations of rockets are generally carried out by assuming adiabatic flow without composition change (**1**).[2] Since reactions involving atoms or free radicals may be very rapid, it is possible that changes in chemical composition associated with atomic and free radical reactions do occur during flow through the rocket nozzle. Kinetics of homogeneous gas reactions in flow systems has been discussed in the literature (**2**). Problems concerned with the existence or nonexistence of equilibrium among internal-energy states are described elsewhere (**3**). Hydrodynamical considerations of flow through the nozzle are examined in a report from the Applied Mathematics Group of New York University (**4**).

The flow process through a rocket nozzle under conditions where complete chemical equilibrium is reached at each temperature is examined in the present paper. The results of this study may be tested for compatibility with the rates of atomic reactions if rate constants for the atomic reactions are available.

* Presented as Paper 27 before the Division of Physical and Inorganic Chemistry, 112th Meeting of the American Chemical Society, New York, N. Y.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

II. ANALYSIS OF EQUILIBRIUM AND CONSTANT-COMPOSITION FLOW PROCESSES.

A. Construction of Pressure-Temperature Plots.

A relation for calculating the pressure as a function of temperature during adiabatic flow through the rocket nozzle may be derived by use of the perfect gas law.

If the initial concentration of hydrogen molecules is 1 mole per cubic centimeter and α moles per cubic centimeter are dissociated at a given temperature, then the corresponding equilibrium constant K_p is given by the relation

$$K_p = \frac{(p_{\mathrm{H}})^2}{p_{\mathrm{H}_2}} = \frac{4\alpha^2 p}{1 - \alpha^2}, \tag{1}$$

where p represents the total pressure and p_{H} and p_{H_2}, the partial pressures of atomic and molecular hydrogen, respectively. Differentiation of Eq. 1 leads to the result

$$d\left(\frac{K_p}{p}\right) = \frac{8\alpha d\alpha}{(1 - \alpha^2)^2}. \tag{2}$$

Since

$$d\left(\frac{K_p}{p}\right) = \frac{K_p}{p}\left(\frac{\Delta \mathrm{H}}{RT^2}dT - d\ln p\right),$$

it follows that

$$\frac{K_p}{p}\left(\frac{\Delta \mathrm{H}}{RT^2}dT - d\ln p\right) = \frac{8\alpha d\alpha}{(1 - \alpha^2)^2}, \tag{3}$$

where ΔH represents the heat required to dissociate 1 mole of hydrogen at the given temperature.

According to Eq. 1

$$\alpha = \frac{1}{2}\sqrt{\frac{K_p}{p}}\sqrt{1 - \alpha^2}.$$

Therefore, Eq. 3 may be rewritten as

$$\sqrt{\frac{K_p}{p}}\left(\frac{\Delta \mathrm{H}}{RT^2}dT - d\ln p\right) = \frac{4d\alpha}{(1 - \alpha^2)^{3/2}}, \tag{4}$$

and if $\alpha \ll 1$

$$\sqrt{\frac{K_p}{p}}\left(\frac{\Delta \mathrm{H}}{RT^2}dT - d\ln p\right) \simeq 4d\alpha. \tag{4a}$$

For adiabatic flow

$$(1 + \alpha)c_v dT + \Delta E d\alpha = -pdV, \tag{5}$$

where c_v is the heat capacity of 1 mole of gas at constant volume and ΔE represents the energy required to dissociate 1 mole of hydrogen.

From the perfect gas law, which is assumed to be valid,

$$pV = (1 + \alpha)RT$$

or

$$pdV + Vdp = (1 + \alpha)RdT + RTd\alpha. \tag{6}$$

Combining Eqs. 5 and 6 leads to

$$(1 + \alpha)c_v dT + \Delta E d\alpha = \frac{RT}{p}(1 + \alpha)dp - (1 + \alpha)RdT - RTd\alpha$$

or

$$c_v dT + RdT + \Delta E \frac{d\alpha}{1 + \alpha} + RT \frac{d\alpha}{1 + \alpha}$$
$$= c_p dT + \Delta H \frac{d\alpha}{1 + \alpha} = \frac{RT}{p} dp. \tag{7}$$

Solving Eq. 7 for $\frac{d\alpha}{1 + \alpha}$,

$$\frac{d\alpha}{1 + \alpha} = \frac{RT}{\Delta H} d \ln p - \frac{c_p}{\Delta H} dT. \tag{8}$$

If $\alpha \ll 1$,

$$d\alpha \simeq \frac{RT}{\Delta H} d \ln p - \frac{c_p}{\Delta H} dT. \tag{8a}$$

Introducing Eq. 8a into Eq. 4a leads to the result

$$\sqrt{\frac{K_p}{p}}\left(\frac{\Delta H}{RT^2} dT - d \ln p\right) \simeq \frac{4RT}{\Delta H} d \ln p - \frac{4c_p}{\Delta H} dT. \tag{9}$$

Equation 9 can be simplified to the relation

$$\left(c_p + \frac{B\Delta H}{RT^2}\right) dT = (RT + B) d \ln p, \tag{10}$$

where

$$B = \frac{\Delta H}{4}\sqrt{\frac{K_p}{p}}. \tag{11}$$

Since Eqs. 10 and 11 were derived on the assumption that $\alpha \ll 1$, they should not be applied at excessively high temperatures where the hydrogen molecules are extensively dissociated.

Equations 10 and 11 permit the construction of a pressure *versus* temperature plot through the rocket nozzle by successive approximation. Results obtained for the flow of 1 mole of hydrogen gas from a chamber temperature of 3506° K. at a pressure of 20.42 atm. to a pressure of 1 atm. are plotted in Fig. 1. Also shown in Fig. 1 is a similar pressure *versus* temperature plot for adiabatic flow without composition change. This curve was constructed (1,5) by use of Eq. 12:

$$\frac{d \ln p}{dT} = \frac{\gamma}{\gamma - 1} \cdot \frac{1}{T}. \tag{12}$$

The results shown in Fig. 1 indicate that the exit temperature for flow without composition change is 1650° K. The corresponding value for equilibrium flow is seen to be 2243° K.

B. Determination of $(1/p)(dp/dT)$ as a Function of Temperature.

The coefficients $(1/p)(dp/dT)$ may be determined as a function of temperature from Eqs. 10 and 12 for equilibrium and constant-composition flow, respectively. The calculated results for the flow of 1 mole of hydrogen from a chamber temperature of 3506° K. at a pressure of 20.42 atm. to atmospheric pressure are plotted in Fig. 2.

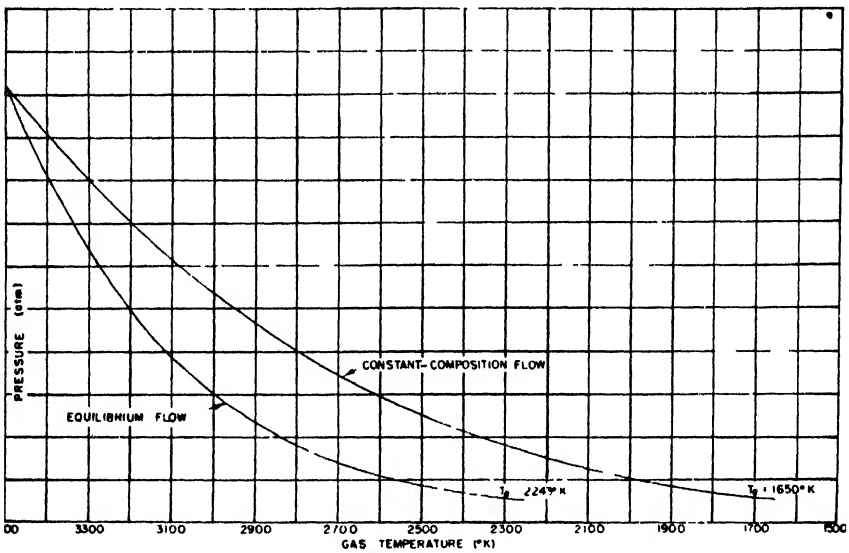

FIG. 1. Pressure-temperature history for the flow of hydrogen gas through a rocket nozzle (T_c = 3506° K., p_c = 20.42 atm.).

Reference to Fig. 2 indicates that $d \ln p/dT$ decreases continuously with increasing temperature for constant-composition flow but appears to increase for equilibrium flow for most of the temperature range from exit to chamber temperature. The coefficient $d \ln p/dT$ is seen to be appreciably greater for equilibrium flow than for constant-composition flow under corresponding conditions.

C. Determination of $-(1/v)(dv/dT)$ as a Function of Temperature.

The coefficient $-d \ln v/dT$, where v represents the linear flow velocity of the gases, can be calculated from the requirement of conservation of energy

$$Mvdv = -c_p dT - \Delta H \frac{d\alpha}{1+\alpha}. \tag{13}$$

Equations 8 and 13 lead to the relation

$$Mvdv = -RT\frac{dp}{p}. \tag{14}$$

Assuming that $v = 0$ in the rocket chamber at $T = T_c$ (this is only a first approximation since the chamber is not infinitely wide), and integrating Eq. 14 from $v = 0$ at $T = T_c$ to $v = v$ at $T = T$, it is found that

$$\tfrac{1}{2}Mv^2 = \int_{T_c}^{T}\left(-RT\frac{d\ln p}{dT}\right)dT = \int_{T}^{T_c} RT\frac{d\ln p}{dT}\,dT. \tag{15}$$

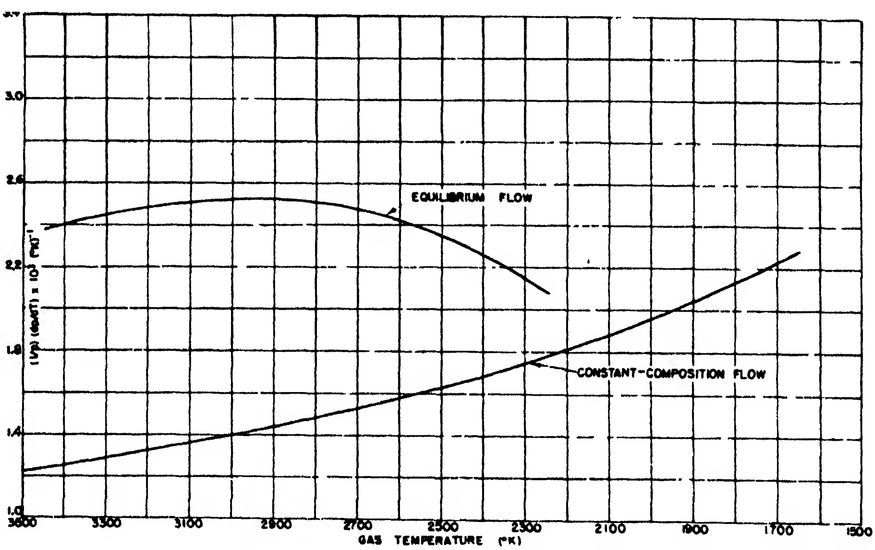

FIG. 2. The coefficient $(1/p)(dp/dT)$ as a function of temperature for the flow of hydrogen gas through a rocket nozzle ($T_c = 3506°$ K., $p_c = 20.42$ atm.).

Dividing Eq. 14 by Eq. 15 leads to the result

$$-\frac{d\ln v}{dT} = \frac{RT\dfrac{d\ln p}{dT}}{2\displaystyle\int_{T}^{T_c}\left(RT\frac{d\ln p}{dT}\right)dT}. \tag{16}$$

For constant-composition flow through the rocket nozzle, Eqs. 15 and 16 should be replaced by Eqs. 17 and 18, respectively (1,5):

$$\tfrac{1}{2}Mv^2 = \int_{T}^{T_c} c_p dT \tag{17}$$

and

$$-\frac{d\ln v}{dT} = \frac{1}{2(T_c - T)}. \tag{18}$$

The gas velocities v and the coefficients $-(1/v)(dv/dT)$ as functions of gas temperature for equilibrium and constant-composition flow are plotted in Figs. 3 and 4, respectively. The exit velocity for equilibrium flow is seen to be 8.58×10^5 cm. per sec. compared with a value of 8.15×10^5 cm. per sec. for constant-composition flow. The higher exit velocity for equilibrium flow leads to the conclusion that better per-

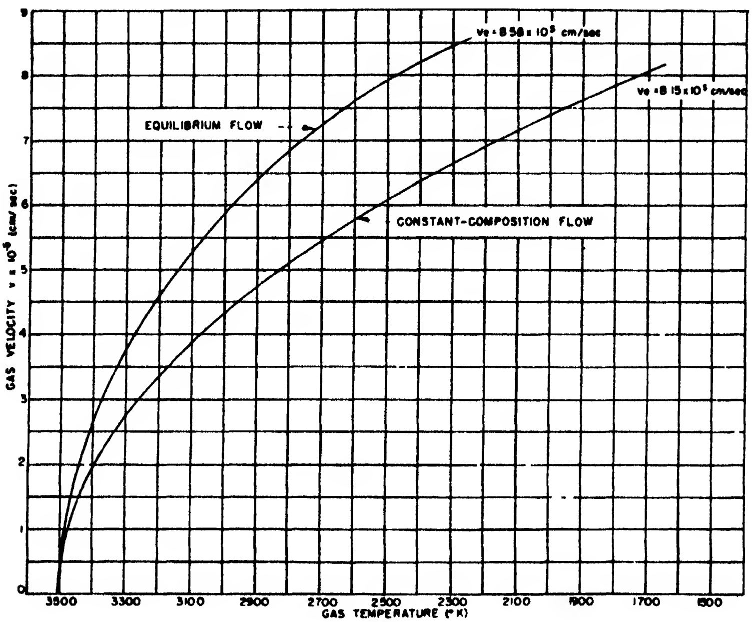

FIG. 3. Gas velocity as a function of temperature for the flow of hydrogen gas through a rocket nozzle ($T_c = 3506°$ K., $p_c = 20.42$ atm.).

formance should be obtained with composition change than without composition change. The coefficients $-d \ln v/dT$ are seen to be of about the same magnitude for both types of flow at the same gas temperature.

D. Determination of the Nozzle Radius as a Function of Temperature.

For the present discussion, a cross section through the nozzle is assumed to be circular. The ratio of the nozzle radius at any point to the nozzle radius at the exit may, by application of the equation of continuity, be determined as a function of temperature.

Continuity of flow imposes the condition

$$\frac{d\rho}{\rho} + \frac{dv}{v} + \frac{dA}{A} = \frac{d\rho}{\rho} + \frac{dv}{v} + 2\frac{dr}{r} = 0, \tag{19}$$

where ρ represents the gas density, A is the cross-sectional area, and r is the radius of a cross section through the nozzle.

Since

$$\frac{d\rho}{\rho} = \frac{dp}{p} - \frac{dT}{T},^{*}$$

Eq. 19 may be written as

$$-2\frac{dr}{r} = \frac{dp}{p} - \frac{dT}{T} + \frac{dv}{v}$$

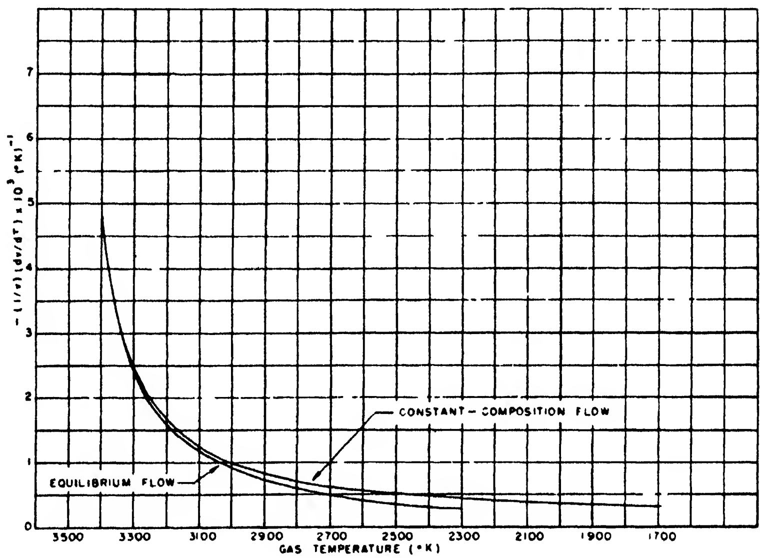

FIG. 4. The coefficient $-(1/v)(dv/dT)$ as a function of temperature for the flow of hydrogen gas through a rocket nozzle ($T_c = 3506°$ K., $p_c = 20.42$ atm.).

or

$$-2\frac{d\ln r}{dT} = \frac{d\ln p}{dT} - \left(-\frac{d\ln v}{dT}\right) - \frac{1}{T}, \tag{20}$$

and, therefore,

$$\ln\frac{r}{r_e} = \frac{1}{2}\int_T^{T_e}\left[\frac{d\ln p}{dT} - \left(-\frac{d\ln v}{dT}\right) - \frac{1}{T}\right]dT \tag{21}$$

where the subscript e indicates conditions at the exit.

Graphical evaluation permits calculation of the ratio r/r_e as a function of temperature for equilibrium flow. The relation corresponding to Eq. 21 for constant-composition flow was derived previously (1,5) and is reproduced in the following equation:

$$\left(\frac{r}{r_e}\right)^2 = \left(\frac{T_e}{T}\right)^{\frac{1}{\gamma-1}}\sqrt{\frac{T_c - T_e}{T_c - T}}, \tag{22}$$

* Actually $d\rho/\rho$ should include a term in dM/M, that is, $\frac{d\rho}{\rho} = \frac{dp}{p} - \frac{dT}{T} + \frac{dM}{M}$. The contribution of dM/M to $d\rho/\rho$ is negligible in the present case. This is consisent with the assumption that $\alpha \ll 1$.

where γ represents the ratio of the specific heat at constant pressure to the specific heat at constant volume.

The ratios r_e/r are plotted in Fig. 5 as a function of temperature for the flow of 1 mole of hydrogen from $T_c = 3506°$ K. and $p_c = 20.42$ atm. to atmospheric pressure.

Reference to Fig. 5 indicates that the required area ratio $(r_e/r_t)^2$ for constant-composition flow is 3.55 while the corresponding value for equilibrium flow is 3.95.

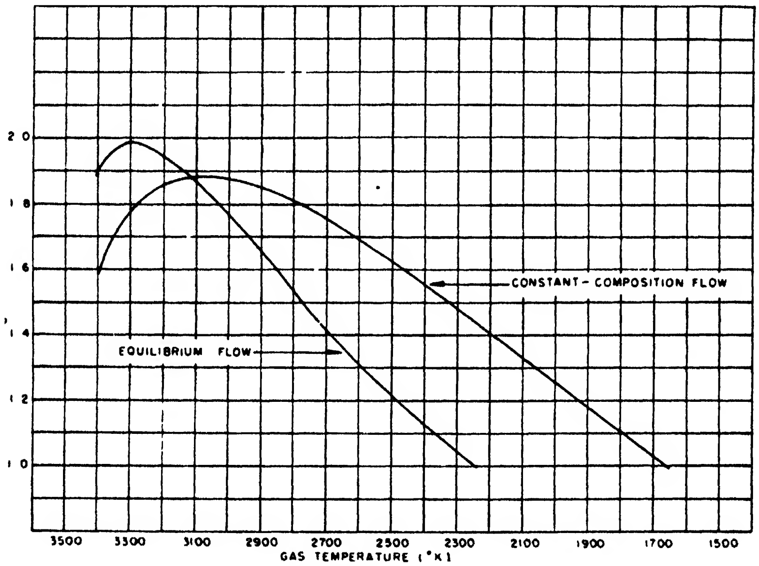

FIG. 5. The radius ratio r_c/r as a function of temperature for the flow of hydrogen gas through a rocket nozzle ($T_c = 3506°$ K., $p_c = 20.42$ atm.).

The coefficient $(1/r)(dr/dT)$ is equal to zero at the nozzle throat since the temperature decreases continuously from the chamber to the nozzle exit while $dr = 0$ at the nozzle throat. Accordingly, the gas temperature at the nozzle throat for equilibrium flow is determined by the condition

$$\frac{d \ln p}{dT} = \left(- \frac{d \ln v}{dT} \right) + \frac{1}{T}. \tag{23}$$

For constant-composition flow, the gas temperature at the nozzle throat is given by the relation (1,5)

$$T_t = \frac{2}{\bar{\gamma} + 1} \cdot T_c, \tag{24}$$

where $\bar{\gamma}$ represents an average heat capacity ratio during flow through the nozzle.

The gas temperature at the nozzle throat for constant-composition

flow is seen to be 3060° K. while the corresponding temperature for equilibrium flow is 3300° K.

E. Determination of the Time Spent in a Given Temperature Interval.

The coefficient $|-dt/dT|$ may be determined for equilibrium flow by following the procedure described for constant-composition flow (5).

If $|dx|$ is the linear distance measured parallel to the axis of the rocket nozzle, then

$$\left|-\frac{dt}{dT}\right| = \frac{1}{v}\frac{|dx|}{dT}. \tag{25}$$

A cross section through a rocket nozzle is shown in Fig. 6.

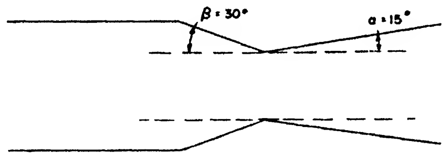

FIG. 6. Cross section through a rocket nozzle.

It is convenient to define $\tan\delta = \tan\beta$ between the nozzle throat and the chamber, and $\tan\delta = \tan\alpha$ between the nozzle throat and exit. It follows from these definitions that

$$dr = |dx|\tan\delta. \tag{26}$$

Replacing dr in Eq. 20 by its value from Eq. 26 leads to the relation

$$-\frac{2}{r}\left|\frac{dx}{dT}\right|\tan\delta = \frac{d\ln p}{dT} - \left(-\frac{d\ln v}{dT}\right) - \frac{1}{T}. \tag{27}$$

Combining Eqs. 25 and 27 leads to the result

$$\left|-\frac{dt}{dT}\right| = \frac{1}{v}\cdot\frac{r}{2\tan\delta}\left[\frac{d\ln p}{dT} - \left(-\frac{d\ln v}{dT}\right) - \frac{1}{T}\right]. \tag{28}$$

From Eq. 15, v is found to be

$$v = \sqrt{2\int_T^{T_c}\frac{RT\dfrac{d\ln p}{dt}}{M}\,dT}.$$

Therefore Eq. 28 becomes

$$\left|-\frac{dt}{dT}\right| = \left(\frac{r}{r_e}\right)\frac{r_e}{2\tan\delta}\left[\frac{d\ln p}{dT} - \left(-\frac{d\ln v}{dT}\right) - \frac{1}{T}\right] \cdot \sqrt{\frac{M}{2\int_T^{T_c}\left(RT\dfrac{d\ln p}{dT}\right)dT}}. \tag{29}$$

The expression for constant-composition flow, corresponding to Eq. 29, was previously shown to be (5)

$$\left|-\frac{dt}{dT}\right| = \left(\frac{r}{r_t}\right)\frac{r_t}{2\tan\delta}\sqrt{\frac{M}{2c_pT_c^3}} \times \left[\frac{1}{2\left(1-\frac{T}{T_c}\right)^{3/2}} - \frac{T_c}{(\gamma-1)T\sqrt{1-\frac{T}{T_c}}}\right]. \qquad (30)$$

Calculated coefficients $|-dt/dT|$, obtained by use of Eqs. 29 and 30, are plotted in Fig. 7. Reference to Fig. 7 also indicates that $|-dt/dT|$

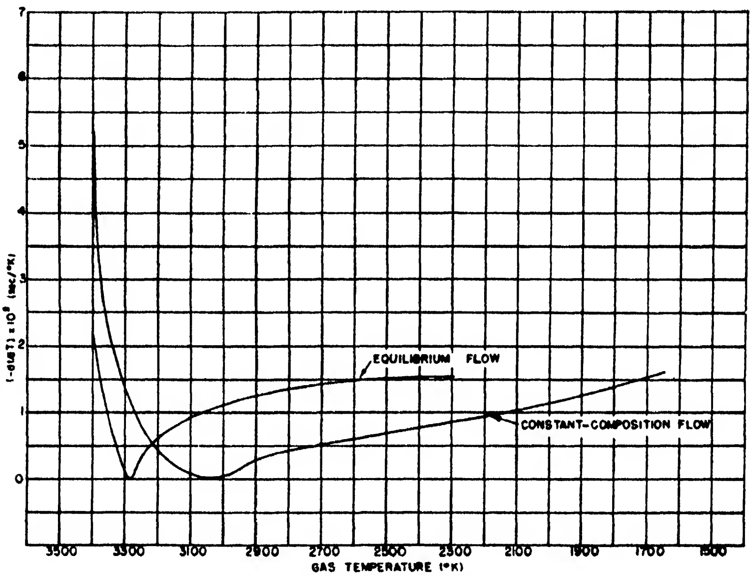

Fig. 7. The term $-dt/dT$ as a function of temperature for the flow of hydrogen gas through a rocket nozzle (T_c = 3506° K., p_c = 20.42 atm.).

between nozzle throat and nozzle exit is greater for equilibrium flow than for constant-composition flow. If this result is generally applicable, then the value of $|-dt/dT|$ for constant-composition flow may be advantageously employed for an estimate of minimum residence times of gases in given temperature intervals.

F. Determination of Residence Time of Gas Molecules in a Fixed Temperature Range.

The total time required for flow through the rocket nozzle cannot be determined accurately because of the approximation that the gas velocity in the chamber is zero.

The time spent in the temperature intervals between 3400° K. (which has been chosen arbitrarily) and any other temperature $T°$ K. may be

evaluated by graphical integration of Eqs. 29 and 30 between $T°$ K. and 3400° K. The results of this calculation for constant-composition and equilibrium flow, respectively, are plotted in Fig. 8.

Reference to the data plotted in Fig. 8 indicates that 1.574×10^{-5} sec. are required during constant-composition flow before the gas is cooled from 3400° K. to the exit temperature of 1650° K. Similarly, 1.411×10^{-5} sec. are required during equilibrium flow before the gas is cooled from 3400° K. to the exit temperature of 2243° K. The time spent by the gases between the nozzle throat and the nozzle exit may

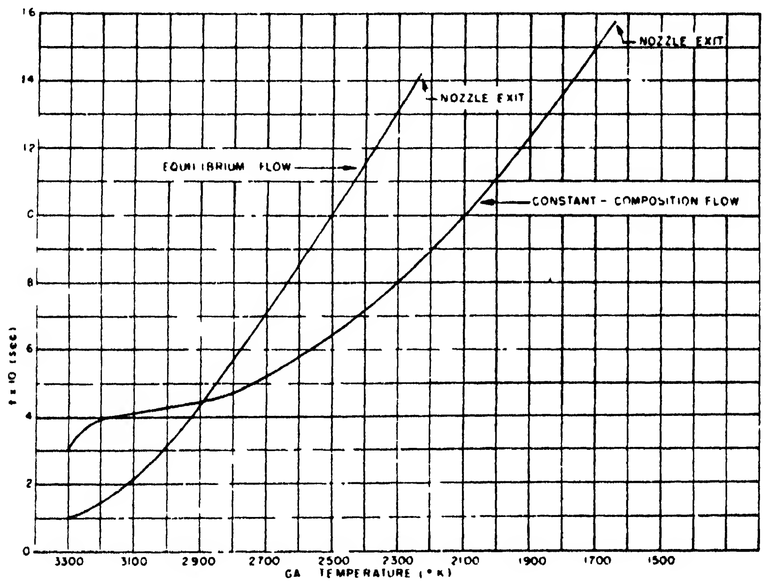

FIG. 8. Time spent in the temperature range between 3400° K. and $T°$ K. for the flow of hydrogen gas through a rocket nozzle (T_c = 3506° K., p_c = 20.42 atm., α = 15°, β = 30°, r_t = 5 cm.).

be read from Fig. 8 by subtracting the time spent between 3400° K. and the temperature at the nozzle throat from the time spent between 3400° K. and the exit temperature. This calculation leads to residence times between nozzle throat and nozzle exit of 1.31×10^{-5} sec. for equilibrium flow and 1.15×10^{-5} sec. for constant-composition flow.

The required residence times in fixed temperature intervals permit the determination of the minimum required reaction rates which allow composition changes during flow through the nozzle. This problem will be discussed in more detail in a subsequent report.

G. Distance Between Nozzle Throat and Nozzle Exit.

The linear distance between nozzle throat and nozzle exit may be determined for constant-composition and equilibrium flow, respectively,

for the chosen values of $r_e = 5$ cm. and $\alpha = 15°$. The ratios of r_e/r_t (Fig. 5) are required for this calculation. The distance L_t between nozzle throat and nozzle exit may be determined by use of the relation

$$L_t = \frac{r_e - r_t}{\tan \alpha}. \tag{31}$$

Calculations indicate that the conditions specified for the present analysis lead to the results

L_t (equilibrium flow) = 9.27 cm.
L_t (constant-composition flow) = 8.74 cm.

The distance between chamber and nozzle throat cannot be accurately determined because of the assumption of zero velocity of flow in the chamber.

The distance between the nozzle throat and the nozzle exit L_t may also be calculated from the relation

$$L_t = \int_{\text{throat}}^{\text{exit}} v\, dt. \tag{32}$$

Equation 32 and all of the other relations derived on the basis of Eq. 25 involve the inherent assumption that flow through the nozzle occurs parallel to the nozzle axis. The error introduced by this approximation would not be expected to be large for the chosen value of α.

III. KINETIC CONSIDERATIONS.

The kinetics of atomic reactions has recently been considered by Schaefer (6) in an analysis of near-equilibrium flow. Application of Schaefer's method to the hydrogen system discussed in this report leads to the conclusion that nearly complete chemical equilibrium is maintained during the adiabatic expansion of hydrogen through the rocket nozzle.

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NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

ELECTRONIC POWER SOURCE FOR ELECTROLYSIS.

A new constant voltage power supply for use in certain electrolytic determination and separation processes has been developed by the Electronic Instrumentation Laboratory of the National Bureau of Standards. This device, operating from a 110-volt A.C. line, supplies a stabilized and continuously adjustable D.C. voltage at current demands up to two amperes or more. Regulation is maintained with respect to a reference potential electrode, a principle recognized in recent years as a useful means of selectively depositing only those metals that it is desired to remove in electrolytic separation.

Electrolysis has been used extensively for the separation and determination of certain elements, particularly copper and lead. Another application consists in removing a large variety of metals from a solution so that the remaining constituents may be subsequently determined by some other method. Thus in the determination of aluminum in steel, the mercury cathode serves to remove iron and certain alloying constituents which might interfere in the analysis.

In most of the above applications no effort is made to control the voltage, and the selectivity of the process is determined by the type of solution in which the electrolysis takes place. Procedures using the more selective method of controlling deposition potential have been limited by the lack of suitable equipment to furnish and maintain the constant voltage required. Although several such devices have been built, they are not entirely suitable because of mechanical features which render their performance uncertain due to such factors as fouling of contacts, or because the electronic circuits used do not permit control under all conditions of operation.

The deficiencies of previous instruments are eliminated by the new power source developed by the Bureau. The voltage is regulated by a carefully engineered and compact electronic instrument that utilizes standard radio-type components. It is designed to produce a controlled output of several volts in loads as low as one ohm, though the same techniques could be used in apparatus built for higher outputs. In many applications, the conventional series-regulated power supply is at a disadvantage because of the high load currents which must be handled directly by vacuum tubes.

In the electrolytic separation application for which the instrument was designed, it is necessary to control the power supplied to a load, the

* Communicated by the Director.

plating electrode, with respect to the potential of a reference electrode. Too high a potential is indicative of excessive load power. Therefore, the function of the circuit is to reduce its output automatically until the control potential is at the desired value.

Internally, the instrument contains an amplitude-controlled oscillator, a power amplifier and rectifier, and two direct coupled stages of amplification. The oscillator, of the multivibrator type, operates at approximately 2,000 cycles per second, producing a variable amplitude square wave which is amplified by a pair of audio beam power tubes in push pull. These in turn are coupled through a step-down transformer to the low-voltage, high-current rectifier. An inductance and capacitance filter reduces ripple and noise to a very low value. This filtered voltage is the output of the instrument.

For regulation, a portion of the output voltage is compared to a voltage stabilized by a gaseous regulator tube. The difference between these voltages is amplified by the two direct coupled stages and used to control the amplitude of the oscillator signal. Connections are such that the oscillator amplitude is increased when the output voltage falls below the reference value. With the two stages of amplification, regulation is very precise, so that the output deviates only slightly from the reference voltage.

The reference voltage is adjustable by a control in the panel of the instrument. This control, which therefore acts as the output voltage regulator, has a range of adjustment from zero to two volts, but higher voltages may be obtained by using only a part of the output for comparison. With a suitable voltage divider, a power supply capable of delivering up to 8 volts may be obtained.

As a result of the high degree of regulation, the instrument has a low effective internal impedance. This is about 0.005 ohm when the entire output voltage is used for comparison with the reference voltage. Hence the voltage drop from heavy load currents is small. The low internal impedance of the instrument makes it suitable as a battery substitute in those applications where constancy of output for load fluctuations is desired. The regulation ratio is approximately 0.5 per cent. In order to duplicate the performance of this instrument with a storage battery at an output of one volt, it would be necessary to employ a voltage divider with 200 amperes fixed drain.

The regulated voltage source may also be connected as a constant current source if desired. The voltage drop due to the passage of the load current through a selected fixed resistance then becomes the index which is compared to the reference potential. When applied in this manner, the instrument offers approximately the same advantages as when used for a voltage source.

One application in which the apparatus has been especially successful is the separation of copper from solutions preliminary to the polaro-

graphic determination of very small amounts of cadmium, lead, tin and other metals. The high degree of control attainable permits separation of elements whose half-wave potentials differ by only a few tenths of a volt. Additional experimentation is in progress in the Bureau's physical chemistry laboratory to determine the full scope of the instrument in electrochemical processes.

ELECTRODEPOSITION OF TUNGSTEN ALLOYS.

A method of producing satisfactory electrodeposits of tungsten alloys on metal surfaces has resulted from investigations conducted by Abner Brenner, Polly Burkhead, and Emma Seegmiller of the National Bureau of Standards. Smooth, thick deposits of tungsten alloyed with cobalt, nickel, or iron have been obtained from hot ammoniacal solutions containing organic acids. These alloys are quite hard and, what is more, retain their hardness at elevated temperatures, just as do the similarly constituted cast metallurgical alloys.[1]

The commercial importance of tungsten alloys is directly related to their use in applications—such as bearings, pistons, cylinders, dies, molds and machine tools—where hardness, corrosion resistance, or high strength at elevated temperatures are essential. Although efforts to electrodeposit pure tungsten have for many years proved unsuccessful, deposits of this metal alloyed with iron, nickel, and cobalt, have been produced by various processes. These methods, however, yielded specimens having unsatisfactory physical properties for many uses. Only a thin layer could be obtained, and this was weak because of cracks and oxide inclusions. The development of an improved process for depositing these alloys was therefore undertaken by the National Bureau of Standards.

Of the alloys investigated by the Bureau, cobalt-tungsten was found to deposit most easily. In some of its properties it resembles the alloy Stellite. A typical solution for depositing cobalt-tungsten contains:

Tungsten	25 g./l. (as sodium tungstate)
Cobalt	25 g./l. (as cobalt chloride or sulfate)
Rochelle salt	400 g./l.
Ammonium chloride	50 g./l.
Ammonia to a pH of 8.5 to 9	

Plating takes place at a temperature above 90° C. and at a current density of from 2 to 5 amperes per square decimeter. The Rochelle salt may be replaced by salts of other organic hydroxy acids. As anodes, either tungsten, the appropriate iron-group metal, or a tungsten

[1] For further technical details see a paper by Abner Brenner, Polly Burkhead, and Emma Seegmiller, "Electrodeposition of Tungsten Alloys Containing Iron, Nickel, and Cobalt," J. Research NBS, **39**, 351 (1947), RP 1834.

alloy may be employed. Tungsten anodes leave the least amount of residue when they go into solution, and may be used without bags. The solutions for depositing the iron- and nickel-tungsten alloys are very similar to that for cobalt-tungsten.

The maximum amounts of tungsten that may be obtained in this way are 35 per cent. in the nickel alloy, about 50 per cent. in the cobalt alloy, and about 60 per cent. in the iron alloy. However, for sound alloys of desirable properties, the tungsten content must be lower than the maximum. For example, the cobalt alloys should have not more than 30 per cent. of tungsten.

The nickel-tungsten and cobalt-tungsten deposits have good adhesion to steel and can be plated up to a thickness of 0.02 inch without becoming appreciably nodular. As formed, they are brittle and show a laminar structure under the microscope, but heating to suitable temperatures results in the disappearance of the laminations as the alloys become ductile.

The most interesting feature of the alloys obtained by this process is their hardness, which in the untreated nickel- and cobalt-tungsten alloys may be between 400 and 700 on the Vickers scale. The iron alloy is still harder, ranging from 700 to 900 Vickers; it is thus comparable in hardness to electrodeposited chromium.

Unlike most other electrodeposits, which soften with heat treatment, these alloys may increase 100 points or more in hardness upon heating to 600° C. The cobalt-tungsten alloy is of further interest in that it maintains a fair degree of hardness even at quite elevated temperatures. For example, at 700° C. the hardness of a carbon steel drops to about 50 Vickers, whereas the cobalt-tungsten alloy at this temperature has a hardness of over 290 Vickers.

The resistance of the nickel- and the cobalt-tungsten alloy to chemical attack is not much greater than that of either nickel or cobalt in the pure state, with the exception that they are much more resistant to nitric acid. In the salt-spray test, coatings of cobalt-tungsten alloy show less porosity and afford better protection to steel than do ordinary nickel coatings. However, the nickel-tungsten alloys are inferior in protective value to nickel coatings.

While tungsten alloys deposited from the solutions developed at the Bureau have not yet been utilized commercially, it is expected that they will find application on surfaces that require hardness and durability at elevated temperatures. Because of the excellent throwing power of the solutions, these coatings are more readily applied to irregular-shaped surfaces than is chromium plate. The electrodeposition section of the National Bureau of Standards will cooperate with those who are interested in plating small metal parts that can be subjected to a service test.

BORON-TREATED STEELS.

It is now recognized that the hardenability of some types of steel is materially improved by the addition of small amounts of boron. Apparently the effectiveness of the treatment with boron depends upon the steelmaking practice, and the amount and form of boron retained in the steels. The optimum effect on hardenability is obtained when boron is added in the form of simple or complex ferroalloys, commonly called "intensifiers," "special addition agents," or "needling agents," to thoroughly deoxidized heats in which the amount of boron recovered is within the range of about 0.001 to 0.005 per cent. Because one of the main roles of alloying elements in steels is to increase the depth of hardening or hardenability, this significant effect of boron suggested the possibility that it might be substituted for a part or all of the strategic elements commonly used as alloying agents in steels. During the war, to offset possible shortages of these critical elements, an extensive investigation was undertaken at the National Bureau of Standards to determine quantitatively the effects of boron on some properties of steels used for armor plate and other military applications.

The program was divided into two main parts; namely, (1) a study of the interrelationship between boron, carbon, and alloying constituents on some of the properties of steels made in the laboratory, and (2) a study of the properties of boron-treated steels of selected chemical composition made commercially under predetermined deoxidizing practices. Special attention was also directed toward the development of spectrographic and chemical methods for the accurate determination of small amounts of boron in steel. The investigation [1,2] was carried out at the National Bureau of Standards under the auspices of the War Metallurgy Committee, National Research Council, to which funds were transferred by the Office of Scientific Research and Development.

The work included a study of the cleanliness and structures of the steels as hot-rolled, normalized, and heat-treated. Determinations were made of the austenite and McQuaid-Ehn grain sizes, hardenability (Jominy), notch toughness (Charpy impact) at room and low temperatures, and tensile properties at room temperature. The investigation was extended with experimental steels to include a study of the effect of deoxidation practice and composition of ferroalloys containing boron, on the recovery of boron and its influence on the above properties; a determination of the recovery of boron on remelting in an induction

[1] For further technical details see "Influence of Boron on Some Properties of Experimental and Commercial Steel," by Thomas G. Digges and Fred M. Reinhart, *J. Research*, Nat. Bur. Standards, Vol. 39, p. 67 (1947) RP 1815. Reprints may be obtained from the Superintendent of Documents, Government Printing Office, Washington 25, D. C., at 30 cents each.

[2] The plans for the investigation were prepared jointly with a representative of the War Metallurgy Committee. The testing program for the commercial steels was prepared with, and approved by, the Subcommittee on Special Addition Agent Steels of the War Engineering Board, Iron and Steel Committee, and Army Ordnance

furnace under both slightly and strongly deoxidizing conditions; and the determination of the effect of boron on the transformation temperatures and weldability of some of the experimental steels.

Approximately 250 experimental steels were made as "split" heats in an induction furnace and 20 commercial steels as a split heat in a basic open hearth furnace. Thus, a steel from each heat was prepared without boron (except in a few special cases) and used as a basis for determining the magnitude of the effect of boron on the mechanical properties.

Boron was determined chemically by a distillation-colorimetric method, using phosphoric acid and turmeric, with an accuracy of ± 0.0002 per cent of boron in the lower part of the range from 0.0005 to 0.006 per cent, and an accuracy of ± 0.0005 per cent in the upper part of this range. The average difference between determinations by chemical and spectrographic methods was about 0.0003 per cent of boron.

Initially normalized specimens, quenched from the usually recommended temperature range, were subjected to end-quench tests for hardenability (Jominy method). Although the relative hardenability of the various steels is obtained by a comparison of the Jominy curves, it is considerably more convenient to show the magnitude of the hardenability effect due to boron (or other variable) from heat to heat by comparing the distances from the quenched end of the Jominy bars corresponding to the same selected hardness values or structures. Comparisons, therefore, are made of the hardenability of the various steels on the basis of the distances from the quenched end of the bar for hardness values corresponding to both 50 and 95 per cent of martensite.

Charpy impact tests for notch toughness were made with quenched and tempered specimens. Tests were carried out in duplicate at room and low temperatures in a Charpy machine of 224.1-ft-lb. capacity, with a striking velocity of the hammer of 16.85 ft-sec. The procedure for making the tests at low temperature consisted of cooling the specimens in an insulated bath containing equal parts of carbon tetrachloride and chloroform for a minimum of 30 min. at the desired temperature, and then quickly transferring to the impact machine and breaking. The desired temperatures were obtained by regulated additions of solid carbon dioxide, and as this material passes directly from the solid to a gas, no dilution occurred in the liquid bath. The temperature of the cooling bath was measured by means of a copper-constantan thermocouple and an indicating potentiometer. Rockwell "C" hardness was measured on the fractured specimens at room temperature.

Variations from nil to 0.006 per cent of boron additions made with either simple or complex intensifiers had no significant influence on the following properties of the steels: (1) cleanliness, except titanium or zirconium inclusions in some steels treated with complex intensifiers (nonmetallic inclusions), (2) hot working (experimental steels), (3)

transformation temperatures, (4) resistance to softening by tempering, (5) weldability (experimental steels), and (6) tensile properties of fully hardened and tempered specimens, except possibly an improvement in ductility when tempered at low temperatures.

Boron lowers the coarsening temperature of austenite. However, steels with relatively high additions of boron can be rendered fine-grained at heat-treating temperatures by the judicious use of grain-growth inhibitors such as aluminum, titanium, and zirconium.

The influence of boron on hardenability and on notch toughness (Charpy impact, V-notched specimens) of fully hardened and tempered steels varied with the base composition of the steels, the composition of the intensifiers, and the amount of boron present. The increase in hardenability due to boron was greater for basic open-hearth than for experimental steels prepared in an induction furnace. The notch toughness at room and low temperatures of the commercial steels fully hardened and tempered at high temperatures was also superior to that of the experimental steels of similar composition heat-treated alike.

The hardenability, as determined by the end-quench test, of many of the experimental and all the steels comprising a basic open-hearth heat were markedly improved by additions of boron. However, no definite correlation was found between the hardenability effect and the amounts of boron added or retained in the steels. In many of the experimental steels the optimum hardenability was obtained with small additions of boron (0.001 per cent or less retained), while in other steels the hardenability increased continuously with increase in boron. In other steels, the addition of boron as a simple or complex intensifier was either without effect or impaired the hardenability. In general, relatively small additions were more effective than large, and the complex intensifiers were more effective than the simple ones. The effectiveness of boron in enhancing the hardenability increased with the amounts (within limits) of manganese, chromium, and molybdenum. The hardenability of the boron-treated steels also varied with the state of deoxidation of the heat, and the final nitrogen content. High soluble nitrogen (and possibly oxygen) was detrimental to the boron effect on hardenability, but it was possible to retain the effect in high-nitrogen steels (low-soluble nitrogen) by fixing the nitrogen with strong nitride-forming elements such as titanium or zirconium.

For the commercial steels, the magnitude of the hardenability effect was independent of the amount of boron added or retained and the composition of the intensifiers.

The hardenability of some of the experimental and commercial steels treated with boron was affected by variation in quenching temperatures. In certain steels, the degree of hardenability was increased by increasing the quenching temperature above the usual recommended range, whereas in other steels the hardenability was not affected or was de-

creased by this change. The magnitude of the hardenability effect due to boron appears to depend upon the form in which it exists in austenite; not necessarily upon the total amount present.

The addition of small amounts of boron was often beneficial to the notch toughness at room temperature of the steels when fully hardened and tempered at low temperatures. When the steels were fully hardened and tempered at high temperatures the presence of boron, especially as relatively high additions with intensifiers containing titanium, was usually either without effect or was detrimental to notch toughness at room and subzero temperatures. Appreciable amounts of boron were retained in steel after remelting in an induction furnace under both normal and highly oxidizing conditions.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING, WEDNESDAY, APRIL 21, 1948.

The Stated Monthly Meeting of The Franklin Institute was held on April 21, 1948 in the Lecture Hall of the Main Building. The President, Mr. Richard T. Nalle, called the meeting to order at 8:15 P.M., with a capacity house in attendance.

The President stated that minutes of the monthly meeting for February had been printed in full in the March issue of the JOURNAL OF THE FRANKLIN INSTITUTE and if there were no corrections or additions the minutes would stand approved as printed. There was no comment.

The Secretary, Dr. Henry B. Allen, was then called upon for his report. The membership increase for the month of March was given as follows:

Active	36
Associate	23
Student	8

The total Institute membership as of March 31, 1948 was given as 5374.

The Secretary then announced the dinner to be given in honor of Mrs. Nellie Tayloe Ross, Director of the U. S. Mint. The dinner is scheduled to be held on April 29, and will be the opening of the exhibit of the new Franklin fifty-cent piece.

The President presented Dr. Hubert N. Alyea, Associate Professor of Chemistry at Princeton University, who spoke on "Chemistry of the Atom Bomb." He described, in a popular manner, the intense efforts of scientists to transmute oné element into another, and how man's triumphant tapping of the energy locked up within the atom was accomplished. He also told how the Atom Bomb works and the peace-time uses to which Atomic Power may be put. Dr. Alyea illustrated with many chemical experiments throughout his lecture.

After a very interesting and edifying lecture the meeting was adjourned by a rising vote of thanks to the speaker of the evening.

HENRY B. ALLEN,
Secretary.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M., Wednesdays and Thursdays from 2 A.M. until 10 P.M.

RECENT ADDITIONS.

AERONAUTICS.

CONWAY, H. M., JR. Principles of High Speed Flight. 1947.
WILKINSON, PAUL H. Aircraft Engines of the World. 1947.

tables and drawings. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $6.00.

Vacuum-tube Circuits, by Lawrence Baker Arguimbau. 668 pages, 14 × 22 cm., drawings. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $6.00.

Soil Mechanics in Engineering Practice, by Karl Terzaghi and Ralph B. Peck. 566 pages, 16 × 23 cm., tables, drawings and illustrations. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $5.50.

Mathematical Table Makers, by Raymond Clare Archibald. 82 pages, 17 × 25 cm., portraits. New York, Scripta Mathematica, 1948. Price, $2.00.

John Couch Adams and the Discovery of Neptune, by W. M. Smart. 56 pages, 16 × 25 cm., portraits, and tables. London, The Royal Astronomical Society, 1947. Price, 5s (Paper).

Vacuum Tubes, by Karl R. Spangenberg. First edition, 860 pages, 16 × 23 cm., drawings and illustrations. New York, McGraw-Hill Book Co., Inc., 1948. Price, $7.50.

Elementary Thermodynamics, by Virgil Moring Faires. 269 pages, 16 × 24 cm., portraits, tables, drawings, illustrations and plates. New York, Macmillan Co., 1948. Price, $4.40.

Centrifugal and Axial Flow Pumps, by A. J. Stepanoff. 428 pages, 15 × 24 cm., plates and drawings. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $7.50.

Loran; Long Range Navigation, edited by J. A. Pierce and others. 476 pages, 16 × 24 cm., tables, drawings and illustrations. New York, McGraw-Hill Book Co., 1948. Price, $6.00.

Microwave Receivers, edited by S. N. Van Voorhis. 618 pages, 16 × 23 cm., tables, drawings and illustrations. New York, McGraw-Hill Book Co., 1948. Price, $8.00.

La Foudre, by Charles Maurain. 215 pages, 11 × 16 cm., tables. Paris, Librairie Armand Colin, 1948. Price, 120 fr. (Paper).

Microwave Duplexers, edited by Louis D. Smullin and Carol G. Montgomery. First edition, 437 pages, 16 × 23 cm., tables, drawings and illustrations. New York, McGraw-Hill Book Co., Inc., 1948. Price, $6.50.

Klystrons and Microwave Triodes, by Donald R. Hamilton and others. First edition, 533 pages, 16 × 23 cm., tables, drawings and illustrations. New York, McGraw-Hill Book Co., Inc., 1948. Price, $7.50.

BOOK REVIEWS.

HEAT, by A. G. Worthing and David Halliday. 522 pages, illustrations, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1948. Price, $6.00.

This textbook covers a wide range of material, making it appropriate for an undergraduate course where introductory physics has been mastered, or for an advanced course of early graduate work, as well as for industrial reference.

The emphasis is on experimental methods. The inductive chapters evaluate the relative importance of the subject in question, then present the pertinent theories, experimental methods, results and problems. The thirteen chapters of this book cover laboratory procedure, temperature measurement, the expansions of liquids and solids, the dynamical theory of heat, calorimetry, specific tests of solids and liquids, thermal conduction, thermal properties of gases, thermodynamics, change of phase, heat engines, refrigerators, convection, and radiant energy.

H. N. MICHAEL.

FUNDAMENTALS IN CHEMICAL PROCESS CALCULATIONS, by Otto L. Kowalke. 158 pages, illustrations, diagrams, 15 × 22 cm. New York, The Macmillan Co., 1947. Price, $2.80.

This textbook is intended for use in the second year of a chemical engineering course. The process calculations discussed are to a large extent related to material balances.

The first portion of the book is made up of a discussion of the measurement of temperature, density and pressure. This is followed by an explanation of methods of expressing compositions of mixtures and solutions. After a study of the gas laws, the student is introduced

to material balances, and examples are given from manufacturing chemistry. Combustion calculations and heat losses make up the final chapters. Some commonly used tables are appended. The book is a careful exposition of the principles of industrial stoichiometry for the beginner. For this purpose it should be excellent.

G. S. GARDNER.

FIELD PRACTICE, by Elwyn E. Seelye. A Data Book For Civil Engineers. 306 pages, tables, drawings and illustrations, 13 × 21 cms. New York, John Wiley & Sons, Inc., 1947. Price $4.50.

Field engineering comprises extensive technological advances, many of which were developed quite recently. This book enables the inspector or field engineer to brief himself as to the essentials in the inspection and supervision of the proposed work. It is also useful as a field manual.

The contents of this volume, conveniently divided into sections on inspection and surveying, are very comprehensive.

The first part of the book deals with the inspection of concrete, masonry, structural steel, welding, bridges, painting, foundations on soil, pile driving, timber ropes and cables, soils, grinding, bituminous paving, pipe laying, and other miscellaneous items, even containing sample blanks of the pay-roll and expense records.

Inclusion of "Rules for Sampling," instructions for field tests, and other material, as well as check lists for work in concrete, bituminous paving, steel, welding and timber, adds to the field value of this book.

The second part reviews surveying and includes such items as construction stakeouts, circular curves, transition curves, vertical curves, earthwork computations, leveling, transit problems, and chapters on instrument adjustment and mapping.

General tables of trigonometric formulas, natural and logarithmic trigonometric functions, and other pertinent tabulated information round out this well-organized volume.

H. N. MICHAEL.

VERY HIGH-FREQUENCY TECHNIQUES, compiled by the staff of the Radio Research Laboratory, Harvard University. Volumes I and II. First Edition. 554 pages and 1057 pages, drawings and illustrations, 16 × 24 cms. New York, McGraw-Hill Book Co., 1947. Price $14.00 two volumes.

This work, in two volumes, represents a summary of the methods, theories, and circuits used by the Radio Research Laboratory, which will be of general interest to radio engineers and physicists. A compilation, drawn from many other sources besides the RRL, it does not pretend to be either a textbook or a comprehensive treatise covering all aspects of the v-h-f field. However, as a reference work, it should save considerable time and effort in the solution of problems in this field.

In its thirty-five chapters and over 1600 pages, the work covers a wide range of subjects.

A representative cross-section from the first volume would include ultra-high-frequency measurements, the various types of antennas, principles of direction finding, homing systems, power generation, and resnatron and magnetron operations.

The second volume includes such topics as filters, receivers, detectors, mixers, oscillators, and amplifiers.

In addition to the foot-noted bibliography the compilation contains a supplementary bibliography of recent articles.

H. N. MICHAEL.

NOMOGRAPHY, by Alexander S. Levens. 176 pages, illustrations, diagrams, plate, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1948. Price, $3.00.

The material presented in this book provides a good working knowledge of the mathematical theory and design of nomograms. When the theory is mastered by the student, he is introduced to practical short-cuts which reduce the time required to design a chart. The lan-

guage of the text is clear and concise. The book itself is based on the material for nomography courses offered by the author at the University of Minnesota and the University of California. However, the practicing engineer who has retained a working knowledge of his algebra, plane geometry and logarithms will find no difficulty in using this book.

H. N. MICHAEL.

CHEMICAL PROCESS PRINCIPLES, by Olaf A. Hougen and Kenneth M. Watson. Volumes 1 to 3, 14 × 22 cm., illustrations. New York, John Wiley & Sons, Inc., 1947.

In the first edition of their work "Industrial Chemical Calculations" which appeared in 1931, the authors selected certain essential portions of physical chemistry and applied them to practical engineering calculations by direct and logical methods. The volume was so successful that it was followed by a second edition in 1936 in which the thermodynamic approach was much more strongly emphasized. The work now reappears in three volumes under a new name "Chemical Process Principles." The first volume covers applications of physical chemistry and the first law of thermodynamics, in much the same manner as in the second edition of the previous work, with much added material, and many new methods. In this volume material balances are discussed. The treatment has been elaborated to cover recycling, by-passing, changes of inventory and accumulation of inerts.

The second volume is devoted exclusively to thermodynamics. This represents a considerable elaboration as compared to the older work. The material is authoritative and carefully selected. The third volume covers a subject not previously considered in the second edition. The volume deals with homogeneous reactions, catalytic reactions, mass and heat transfer in catalytic beds, catalytic reactor design, and uncatalyzed heterogeneous reactions. The treatment is exceptionally fine. In the opinion of the reviewer "Chemical Process Principles" covers the field with a thoroughness and skill not duplicated by any other similar work.

G. S. GARDNER.

ACTIONS OF RADIATIONS ON LIVING CELLS, by D. E. Lea. 402 pages, illustrations, plates, 15 × 22 cm. New York, The Macmillan Co., 1947. Price, $4.50.

This timely subject has been brought very much to the fore by revelations and developments in physics in recent years and the recent spur of research in certain directions in the medical field. It is a span between sciences which so often bring practical results of great value to mankind. This book reviews and summarizes present knowledge and gives pointed discussions by the author who has been engaged in studies of various biological effects of radiation over the past ten years. The radiations with which this book is concerned are the α-, β-, and γ-radiations of radioactive substances, X-rays, protons and neutrons which are grouped as ionizing radiations. Occasionally ultraviolet light is dealt with. Introductory chapters describe the physical properties and dosimetry of different radiations, and chemical effects of ionizing radiations to which is appended possible modes of biological action of radiation. Much of the rest of the book concerns the target theory. The biological effects of radiation to which this theory is applicable are those in which the effect studied is due to the production of ionization by the radiation in, or in the immediate vicinity of, some particular molecule or structure. Application of the target theory is referred to as the class in which the biological effect is believed due to a single ionization and there is interpreted in this manner the inactivation of viruses, the production of gene mutations and the killing of bacteria. A second class of action to which the target theory is applied is the production of certain chromosome aberrations in higher cells by radiations. The book reveals that considerable progress both on the experimental and theoretical sides of the applications of the theory has been made. Special attention is given to the subjects of the production of chromosome structural changes and the mechanism of induction of chromosome structural changes. An account is given of delay in division as an effect of radiation on a cell involving experimental materials such as bacteria, invertebrate eggs, and various rapidly dividing plant and animal tissues. An appendix contains supplementary calculations on the relation between dosage in air and energy dissipation in tissue, spatial distribution of ionization in irradiated tissue, and target theory calculations. There is a bibliography, author and subject index. The book is of value for all those whose work lies in this direction as a text and

reference. A great deal of effort has been put forth to make it usable by a wide circle of interested people, by arrangements for briefing prerequisite knowledge sufficient for a better understanding.

R. H. OPPERMANN.

CHEMICAL ENGINEERING FUNDAMENTALS, by Chalmer G. Kirkbride. 419 pages, illustrations, 15 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1947. Price, $5.00.

The present volume is published as another work in the well known "Chemical Engineering Series." This series comprises many good books in the field of chemical technology. This textbook was written as an introductory work for the second year student in applied chemistry, applied physics, and applied mathematics and was introduced at the Agricultural and Mechanical College of Texas by the author, a former professor of the college. The text is presented in nine chapters. Chapter I is the introduction. Chapter II is on human relations (a matter not usually mentioned in chemical engineering texts). This chapter is well written and worthy of careful reading. Chapter III covers dimensional analysis and graphical presentations. Chapter IV acts as a refresher on some basic physical chemistry and thermodynamics, with a few pages on organic chemistry. Chapters V and VI take up, in a very thorough manner, the important question of material and energy balances. Chapters VII and VIII, two excellent chapters, are on static and dynamic equilibria, evidently a favorite subject of the author. Chapter IX, economic balance and finally Chapter X, presentation of technical results, show very clearly how large is the field covered by the book. The treatment is up to date, carefully written and reasonably accurate. In the opinion of the reviewer, this book is valuable for the student, engineer or chemist.

G. S. GARDNER.

A PRACTICAL EVALUATION OF RAILROAD MOTIVE POWER, by P. W. Kiefer. 65 pages, illustrations, 14 × 22 cm. New York, Steam Locomotive Research Institute, Inc., 1947. Price, $2.00.

Since the railroads today are faced with ever-increasing competition, they must try various ways to meet this rivalry. One of the more important methods is through the development of the ultimate in motive power. Mr. Kiefer, who is Chief Engineer of Motive Power and Rolling Stock of the New York Central System, offers an evaluation of present and proposed railroad motive power. Originally presented as a lecture at the Centenary Celebrations of the Institution of Mechanical Engineers in England, it has been made available in this form because of the present importance of the problem.

Mr. Kiefer begins his discussion with definitions of four fundamentals to be considered in evaluating motive power. These are (1) availability and its dependent counterpart, utilization, (2) over-all costs of ownership and usage, (3) capacity for work, (4) performance efficiency. He then proceeds to describe briefly the characteristics of the present types of motive-power units.

First is the reciprocating-type steam locomotive, still basic on American railroads. The need for fewer failures en route and a lessening of time spent in maintenance, servicing and inspection is commented upon. There have been notable improvements in this type over the past two decades and a table gives pertinent data for seven New York Central and one Pennsylvania Railroad engines during this period. Many of the features of the N.Y.C. "Niagara" 4-8-4 type are considered, this type being the culmination of the work to date on the New York Central. The kind of experimentation being carried on is evidenced by the fitting of one of the "Niagara" type with the Franklin system of poppet valve steam distribution. Exhaustive tests are being run to determine the relative merits of this new system in comparison with the more conventional system.

Of the new experimental types of locomotives mentioned there are the Pennsylvania Railroad coal-burning stoker-fired non-condensing steam-turbine locomotive introduced in 1944, a possible pulverized coal fired steam-turbine locomotive, and both coal and oil fired gas-turbine locomotives scheduled for delivery in 1949.

For electric locomotives the author notes their many operating advantages and the fewer

but more telling disadvantages, a lack of flexibility and a higher first cost and fixed charges for the necessary plant and equipment.

The Diesel-electric, which is the newest type in major use, has proved its value both in switching and road operation. Several different types are commented upon briefly.

The seventh chapter presents the heart of Mr. Kiefer's paper, in that he discusses the different types of motive power in relation to the fundamentals which he has previously defined. Availability of motive power has been the subject of several tests made on the New York Central of steam-engines and Diesels under comparable conditions of service, both passenger and freight. For costs evaluation there are offered in considerable detail actual and estimated annual operating costs for four different locomotives. In respect to capacity for work the author presents several charts showing power characteristic curves for various types of equipment. Performance efficiency is treated in two parts —over-all thermal efficiency at drawbar referred to fuel and over-all performance efficiency.

The concluding chapter presents a table summarizing the author's findings as to the relative evaluations of various motive-power types. It has been necessary to estimate the position of the gas-turbine and steam-turbine locomotives since few data are yet available. The tables indicate that on the majority of factors the straight electric is best, the Diesel electric is median and the reciprocating steam locomotive is last. The whole paper is a well-ordered presentation of the fundamentals to be considered in evaluating motive power and it forms a significant contribution to an understanding of the relative position of the types now in use.

GEORGE E. PETTENGILL

THE CHEMISTRY OF PORTLAND CEMENT, by Robert Herman Bogue. 572 pages, illustrations, 15 × 24, cm. New York, Reinhold Publishing Corp., 1947. Price, $10.00.

The manufacture of portland cement is taken in this book as a distinct chemical industry. The treatment is principally chemical with side sketches of the mechanical phases where necessary to make a well rounded picture. The book is divided into three parts, the first being devoted to the chemistry of clinker formation, a section of ten chapters. This begins with a brief reference to the history and development of the cement industry, an outline of the manufacture of portland cement and proceeds through treatments on early experimentation on the constitution of portland cement, a discussion of the more important methods of investigations adaptable for research and for special examinations of portland cement and heat treatment in manufacture. The application of the petrographic microscope for examination of the principal compounds or phases of clinker, and the crystal analysis approach as accomplished by X-ray structure and character is given particular attention. The combination of cement components, the design and control of cement composition, and the calculation of the potential phase composition of the clinkers produced from raw mixtures of any known oxide composition.

Part two is devoted to the phase equilibria of clinker components. Following the development of the phase rule under the heading of the "Principles of High Temperature Phase Research," descriptions are given of the types of systems commonly encountered, the methods used for expressing the results of phase studies and the interpretation of phase diagrams. The technique of high temperature phase research is next considered which is followed by a detailed consideration of the characteristics of various systems including $CaO—Al_2O_3—SiO_2$, systems containing CaO and H_2O with Al_2O_3 and Fe_2O_3 are studied and this is followed by treatments on the aluminate and ferrite complex salts, the heat of hydration, the structure of hydrated cements, the various aspects of the settling process, and the progress of the reactions in the set cement by virtue of which the paste acquires the characteristics necessary for the development of strength and durability in concrete structures. A final chapter is devoted to tests for cement quality. There is included a subject and author index.

The work is a comphehensive and thorough treatment developing modern concepts based on an historical background.

R. H. OPPERMANN.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

ON ENZYMES IN TUMORS.

BY ERNST WALDSCHMIDT-LEITZ, ELLICE McDONALD AND CO-WORKERS.

(This paper "On Enzymes in Cancer" was published fifteen years ago, in Hoppe-Seyler's *Zeitschrift für physiologische Chemie*, **219**: 115–127 (1933). It attracted little attention for two reasons: it was unfortunately in German and few people were then interested in enzymes, much less enzymes in cancer. Now enzymes in cancer are much studied and this translation is published in order to make this early information available to others.

ELLICE McDONALD.)

The results of O. Warburg's researches, by means of which the peculiarity of malignant tumors was recognized to be a metabolic problem, give the task to cancer research of examining the enzyme composition of tumors. The enzyme systems determine the direction and extent of metabolic changes, and must be described in the course of tumor development in order to follow the exchanges taking place in the tumor animal; to trace, for example, the relation between the enzyme picture in the normal and in the cancerous organs. This problem has as yet been studied but little. No researches of this kind have included a systematic series of functionally important intracellular enzymes.

The present work, which examines the proteolytic systems of organs and of experimental tumors both sarcoma and carcinoma, includes cathepsin, peptidases, arginase, desamidases, phosphatase and catalase. Pathological analyses showing the proportion of different structural elements during the development and aging of the tumors serve as a foundation for the enzymatic research. We distinguish between parenchymatous, fibrous and necrotic portions. The enzyme analysis is, therefore, referred to the structural composition of the tumor, although the figures used for the latter were necessarily based rather on estimations than exact numerical measurements. The results so obtained, however, are uniform enough to show the main trend of alterations in the concentration of single enzymes establishing a definite relation with the structural elements of the tumors; thus at the same time they confirm the authenticity of the pathological findings.

Most of the enzymes studied, namely cathepsin, phosphatase and desamidases, show a steady and pronounced decrease in the enzyme concentration (see Figs. 1 to 4) with the aging of the tumor and the consequent increase in necrotic tissue. It is to be concluded that these

enzymes are confined almost exclusively to the parenchyma of the tumors. Arginase, on the other hand, shows the opposite picture; the amount increases greatly with increasing necrosis (Fig. 5). It appears that arginase is to be found predominantly in necrotic tissue substance

FIG. 1. Cathepsin content and tumor aging.

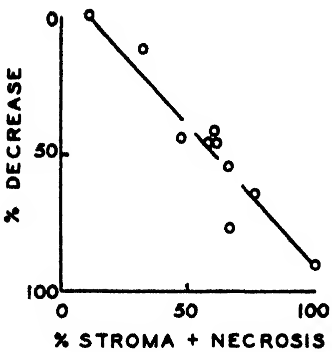
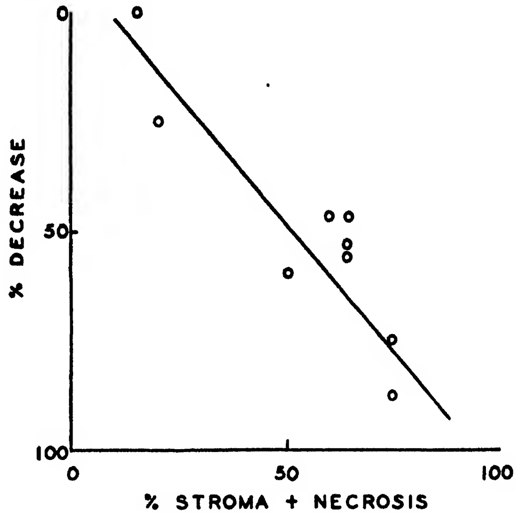


FIG. 2. Phosphatase content and tumor aging.



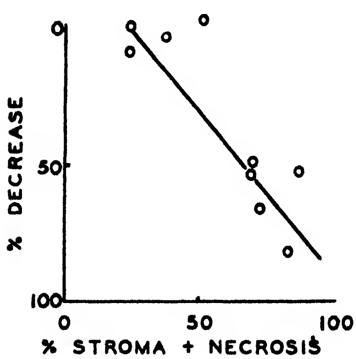


FIG. 3. Adenylic acid desamidase content and tumor aging.

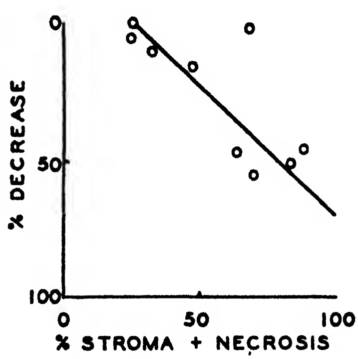


FIG. 4. Guanine desamidase content and tumor aging.

and only in small amounts in growing tissue. An explanation for the rise in arginase, particularly in decomposition tissue, may be sought by the assumption that the action of this enzyme is especially necessary in the later stages of cell decomposition, since, according to Krebs [1] it

[1] H. A. KREBS, Hoppe-Seyler's *Zeitschrift fur physiologische Chemie*, 210: 33 (1932).

plays an important part in the formation of urea. The amount of the peptide-splitting enzymes, on the other hand, amino-polypeptidase and

FIG. 5. Arginase content and tumor aging.

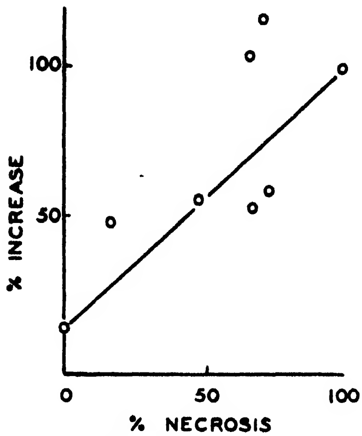
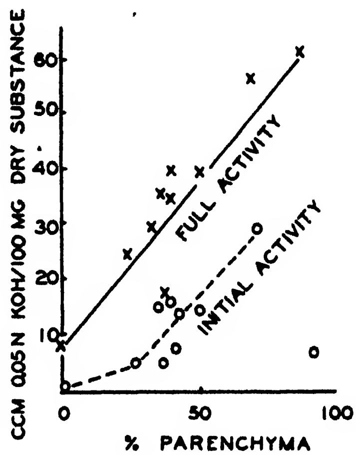


FIG. 6. Degree of activation of cathepsin and tumor aging.



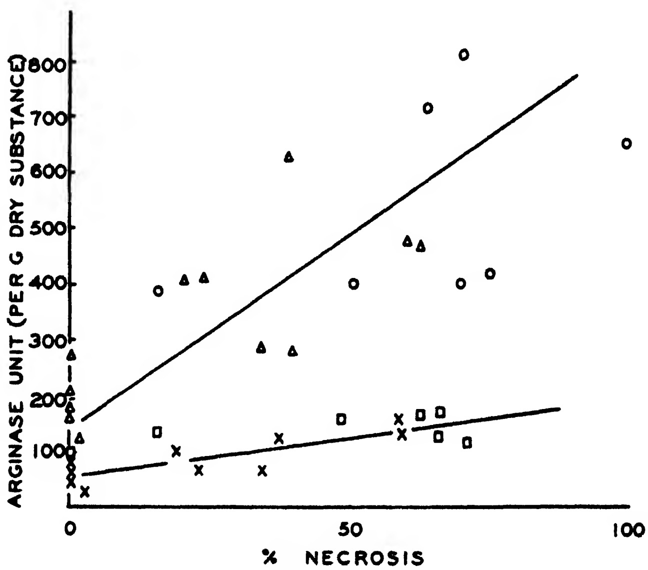


FIG. 7. Degree of activation of arginase and tumor aging.

× = Initial activity No. 1.
□ = Initial activity No. 256.
△ = Full activity No. 1.
○ = Full activity No. 256.

dipeptidase (see Table I of the Experimental Part *), like that of catalase, shows no significant dependence on the degree of aging of the tumor.

* To be published in the June issue of the JOURNAL.

The observed fluctuations may have to be explained by a varying infiltration of the tumors by leucocytes, as could be established by histological analyses in some of the cases observed.

In order to understand the changes of the enzymatic equipment in tumors and their significance, it is especially informative to follow the enzyme content of other organs in the animal. For some of the enzymes studied, as may be seen in the experimental part, significant changes are found in the amount of enzyme in the liver and kidneys of cancer animals. The content of cathepsin [2] and arginase in the liver is reduced, the phosphatase content of the kidneys increased. A profitable task which we have begun would be to follow stepwise the alteration of the organs not affected during the aging of the cancer, to arrive at a balance in the enzyme economics of cancerous animals. The changes of the enzyme composition of musculature, such as the appearance of arginase in cancerous animals as described by Klein and Ziese,[3] are pertinent to this question.

The comparison we have made of the enzyme content in tumors with that in the organs and the musculature of normal animals leads to the conclusion that in almost every case the enzyme content of tumor is similar to that of normal organs, but differs from that of muscle tissue in order of magnitude. Arginase, on the one hand, which is not found at all in normal muscle but occurs in tumor in amounts similar to that in liver, and adenylic acid desamidase, on the other hand, which is found in much greater amounts in muscle than in liver or tumor, may be taken as examples. The fact that in tumor the content of the two desamidases specific for adenylic acid and for guanine corresponds to that of the liver, but not to that of musculature, is also significant in this connection. Among the examples hitherto studied only the unusually high catalase content of the liver is exceptional. It seems that tumor in its enzyme equipment may be regarded as an unspecific organ, in which not only the ubiquitous intracellular enzymes, but also those normally localized in certain organs, are found in amounts similar to those in their specific organs. In every case the enzyme activity of the growing tumor differs entirely from that of the muscle in which it is embedded.

An important observation of a special type is yielded by the comparative study of the state of activation in tumor on the one hand and in the liver of normal and cancerous animals on the other, of the enzymes cathepsin and arginase, which require specific activators. The proportion of activated cathepsin in tumors independent of their age, is not appreciably different from that observed in liver (Fig. 6); this would indicate that a distinctive change in the quantity of cathepsin-activating

[2] Compare this with the observations of E. MASCHMANN AND E. HELMERT, Hoppe-Seyler's *Zeitschrift fur physiologische Chemie*, **216**: 163 (1933).

[3] B. G. KLEIN AND W. ZIESE, *Zeitschrift fur Krebsforschung*, **37**: 323 (1932).

sulfhydryl compounds does not occur in tumor tissue.[4] For arginase, however, the activation of which requires both a heavy metal and sulfhydryl, it has been shown that the amount of enzyme occurring in the activated state in tumors is not greatly increased during aging, while the total amount of arginase present, as measured by maximal cystein-iron activation, is at the same time increased many times[5] (see Fig. 7). Also in livers of cancerous animals the enzyme occurs in only partially activated form. The incompleteness of arginase activation is due, as the interpretation of the research shows, not so much to a lack of sulfhydryl as to a lack of heavy metal. Especially in cases with highly increased arginase content, in advanced necrosis, a striking, often a maximal, activation of the enzyme is attained by the addition of iron alone, while the activating effect of cystein is much less pronounced. This finding seems of great significance. The activation of the arginase is, then, as the researches shortly to be published show, bound up with an oxidation-reduction process in which sulfhydryl-bound heavy metal takes part. Activation of the enzyme by cystein-iron is observed only in the presence of oxygen; it does not occur in hydrogen atmosphere. These observations suggest the idea that the incompleteness of arginase activation in tumors is to be attributed to the same cause responsible for the decrease of normal respiration, possibly to a lack of an activating heavy metal with particular binding properties. To follow this concept farther is one of the objectives of our research.

(To be continued)

[4] Compare this with H. KLEINMANN AND F. WERR, *Biochemische Zeitschrift*, **241**: 108, 140, 181 (1931), P. RONDONI, *Biochemical Journal*, 26. 1477 (1932), E. MASCHMANN AND E. HELMERT, Hoppe-Seyler's *Zeitschrift fur physiologische Chemie*, **216**: 141, 171 (1933). Concerning the findings of the latter authors one of us will shortly enter into a discussion to be published elsewhere

[5] Compare the observations of S. EDLBACHER, J. KRAUS AND F. LEUTHARDT, Hoppe-Seyler's *Zeitschrift fur physiologische Chemie*, **217**: 89 (1933) which were published after the completion of our work

CURRENT TOPICS.

Problems to Face in Testing Jet Engines. (*Heating and Ventilating*, Vol. 44, No. 10.)—Reconversion of an 18-year-old brick cell from its original use in testing conventional piston aircraft engines to testing jet-powered engines was recently completed by the Pratt and Whitney Division of United Aircraft. Principal problems encountered were:

(1) the heat of the exhaust, which reaches 1200 F.
(2) protection from flying particles from burst turbines or compressors, which travel with the velocity of machine gun projectiles.

These problems were studied jointly by engineers of Pratt and Whitney Aircraft Division and United Aircraft Corporation, and by Albert Kahn Associated Architects and Engineers, Inc., who are architects on this and other Pratt and Whitney expansion projects at East Hartford, Conn.

To meet the problem of heat, unsheathed brick walls were adjudged adequate in the exhaust flow at right angles and directed upward through an acoustically lined brick stack. At the point of turning, the 1200 F. at the orifice is reduced to approximately 500 F.

Plastic armor is used to meet the problem of protection from flying particles.

The cell is of brick construction, 12 ft. wide, 40 ft. long, and 20 ft. high. Two brick stacks project above the roof, one at each end, one for air intake and the other exhaust. None of these specifications in the original structure needed to be changed in the reconversion, except that the stacks were fitted with sliding covers to permit all-weather work inside.

Because the jet engine consists only of compressor, turbine and burner, no special new handling problems were entailed. Engines for test are brought in on an assembly stand equipped with wheels, and a one-ton monorail crane is used for service handling. Engines are moved to the test position through a removable section in the air entrance tube.

Plastic armor is used in the engine area to protect operators and control room. Steel plates are additionally used on the control room side. Two observation ports look out of the control room into the rear of the test area where engineers may examine the exhaust flame.

Some idea of the protective qualities required of the armor is the method of testing turbine rotor before assembly. These tests take place in pits where six inches of steel armor are backed by two feet of solid concrete.

R. H. O.

Electric Locomotives for Paulista Railway of Brazil.—Mass production methods are speeding the manufacture of 182-ton, 3,000-volt d-c electric passenger locomotives for the Paulista Railway of Brazil in the locomotive shops of the General Electric Company at Erie, Pa. Twelve of these locomotives have already been shipped.

Built for operation over mountainous terrain involving gradients as great as 1.85 per cent, these 3,000-volt d-c 2–C+C–2 locomotives are rated contin-

uously at 4,050-hp. with an hourly rating of 4,470-hp. With an over-all length of 76 feet, the weight on each of six driving axles is 45,000 pounds for a total of 270,000 pounds on driving axles.

The two-axle guiding trucks and two articulated three-axle driving trucks of these 2–C+C–2 locomotives are completely fabricated by welding. Considerable saving of time and labor is realized by this method in elimination of extensive machining and handling large heavy parts for machining.

Of all-steel, all-welded construction, the cab underframe is composed of two main 21-inch, I-beam longitudinal sills welded to cross sills and further braced and supported in position by heavy floor plates. The main air duct is a chamber formed in the space between the main longitudinal sills.

The cab sides and ends are built of separate assemblies, set in position on the underframe and then welded in place. The roof, consisting mainly of covered hatches, is constructed and assembled on the locomotive in a similar manner.

The high-voltage compartment, entirely enclosed and occupying the center section of the main apparatus cab, is designed to permit the complete installation of electrical equipment before the compartment itself is installed in the locomotive. After assembly inside the cab the complete unit is welded into place and final electrical connections are made. This compartment is equipped with removable side covers to give access to the equipment while end doors furnish entry to the compartment interior.

Designed for 1,500-volt service and insulated to operate two in series on 3,000-volt supply, the six traction motors are of the commutating-pole, force-ventilated type. Each motor is suspended from the axle by two constant-level oil filled waste-packed bearings and by a spring nose support carried on the truck transom. A single wide-faced pinion on the motor shaft engages a a gear of special heat-treated steel to complete the drive.

A three-speed system of control gives traction motor combinations of six in series; three in series, two groups in parallel; and two in series, three groups in parallel. Provision is also made for multiple-unit operation when such service is required.

R. H. O.

Pressure Gasometer Aids in Industrial Health Studies. (*Compressed Air*, Vol. 52, No. 10.)—An unusual application of compressed air has been found in the development of a pressure gasometer by the Bureau of Industrial Hygiene of the Detroit Department of Health. Before gas and vapor measuring devices can be used satisfactorily for the examination of workroom air, each instrument must be checked with a known concentration of the vapor to be determined. This is called "calibrating" the instrument and enables the bureau's engineers and chemists to make accurate determinations when estimating the amount of toxic vapors present in industrial atmospheres.

This type of calibration has always entailed considerable difficulties chiefly because it is not easy to keep constant amounts of solvent evaporating without gross fluctuations. Moreover an air chamber large enough to serve as an experimental room is not within the reach of most laboratories. The pressure gasometer, however, has solved the problem of constant amounts of vapor-air mixtures. The instrument itself consists of a sturdily constructed steel cham-

ber (air-compressor tank) with a capacity of about 4.7 cubic feet. It is provided with an inlet valve for solvent and air, a pressure gauge, a needle valve, and a safety valve.

Setting up and operating the apparatus is comparatively simple. The gasometer is first brought under reduced pressure by evacuating it. Then a weighed amount of the solvent whose vapor is to be tested is placed in a small flask and attached to the inlet valve. The latter is opened, and the inrushing air passes just above the surface of the solvent. By the time the chamber has reached atmospheric pressure, all the solvent has been swept in. Compressed air from a high-pressure cylinder is then admitted into the gasometer, where it mixes with the solvent vapor. When the desired amount of air has been let in, as registered by the gauge, the supply is shut off. A simple calculation then tells just what concentration of vapor and air exists in terms of parts per million. The maximum operating pressure of the gasometer is 150 pounds.

Opening the needle valve releases a stream of vapor-laden air, which is sufficient for a long series of calibrations. By starting off with a heavy concentration of solvent vapor, alternately calibrating instruments, and then admitting more air to dilute the residual moisture, it is easy to obtain a series of different known concentrations.

Only compressed air which is pure is used in the gasometer, and the bureau's laboratory personnel keeps close check on the manner in which it is supplied. This method of using compressed air has saved the bureau many hours of tedious work and has played an important role in assuring adequate health and safety factors in the Detroit workers' environment.

R. H. O.

Materials for Gas Turbine Service.—Great strides have been made in the development of suitable materials for gas turbine service within a comparatively limited time.

Selection of promising materials has been based on creep and rupture test results. Development of materials in use requires a knowledge of all properties of each material and extensive laboratory testing, including behavior under vibration or stress or temperature variation, heat shock, notch sensitivity, corrosion and erosion by hot gases, ductility and stability.

One difficulty which required a vast amount of testing to destruction was obtaining satisfactory large forgings of austenitic materials. This problem was best solved by the development of the so-called "composite" wheel. This consists of a central or hub and disk portion of ferritic low-temperature steel and an outer or rim portion of high-temperature austenitic material solidly bonded together by a suitably controlled weld, using austenitic weld rod. This design has an additional advantage, in that the hub forging can include the shaft extension, thus eliminating the necessity of welding on a shaft extension.

Despite the fact that much progress already has been made, extensive additional data are needed to develop new materials and to understand the laws governing high temperature material performance. A great deal must be done, too, to develop controls needed to ensure that materials always will possess the most desirable properties.

R. H. O.

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Vol. 245 JUNE, 1948 No. 6

METEOROLOGICAL CONDITIONS ACCOMPANYING MIRAGES IN THE SALT LAKE DESERT.

BY

RONALD L. IVES.[1]

ABSTRACT.

Meteorological conditions commonly or invariably accompanying mirages in the Salt Lake Desert are here described, and the descriptions illustrated with photographs and instrumental records, where such are available and pertinent. Extended observation indicates that mirages are a standard and predictable phenomenon in the Salt Lake Desert. Less extended field studies show that similar meteorological conditions accompany mirages in all other American deserts.

INTRODUCTION.

Annoying, serious, or tragic difficulties with mirages and similar optical phenomena in the Salt Lake Desert have been recorded by many field workers since October 1, 1776, when the expedition led by Silvestre Veléz de Escalante noted a "lake" in the distance, hastened their steps, and "found that what we had thought to be water was in some places salt, in others saltpeter, and in still others tequesquite (caliche)" (1).[2]

General appearance and behavior of mirages in the Salt Lake Desert were clearly understood by scouts, stage-drivers, and (some) military men at least as early as 1866.[3] Problems of desert observation were noted by G. K. Gilbert during field studies prior to 1885; partial solutions of these problems as they affect geological observations (2) and aeronautical operations (3) have been published recently. Humphreys' general summary of atmospheric optics (4), originally based on data available prior to 1920, but subsequently augmented, contains specific references to the Salt Lake Desert.

[1] Department of Geography, Indiana University, Bloomington, Ind.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

[3] The writer is indebted to the late William H. Jackson for lucid descriptions of mirages seen by him in 1866 near Fish Springs (Fig. 2) on the old stage road from Salt Lake City to Sacramento.

Field observations here reported were made during the years 1942 to 1946, inclusive, while the writer was stationed at Dugway, Utah, and assigned to problems of chemical meteorology.

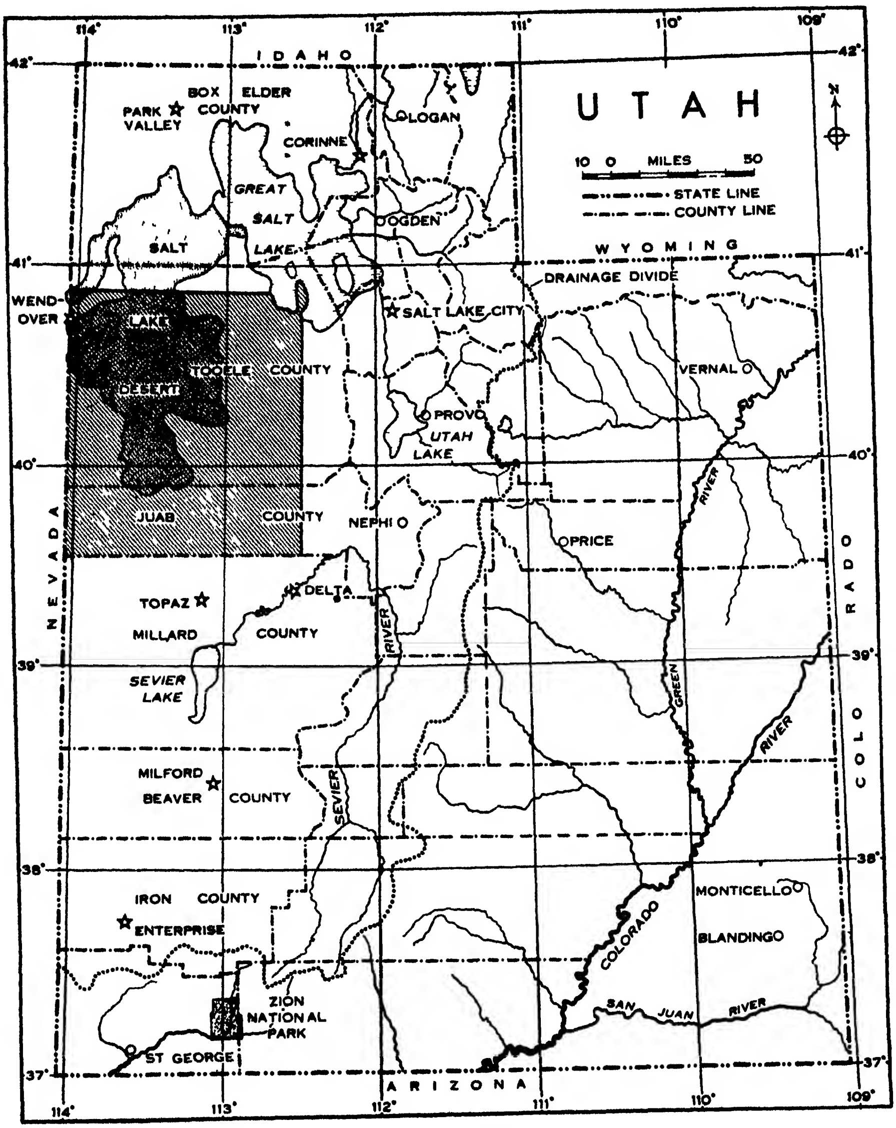

FIG. 1. Index map, showing general location of the Salt Lake Desert with respect to major hydrographic and political divisions of Utah. Intensive field studies were conducted in the southern part of this desert (section lined); numerous field reconnaissances, augmented by reports from cooperative weather observers and airways stations (indicated by stars), indicate that conditions noted in the main area also prevail in the named western counties of Utah.

GEOGRAPHIC SUMMARY

The Salt Lake Desert is an area of ancient lake beds and fault-block mountains located west and south of Great Salt Lake, in Utah (Fig. 1). About half of the area is occupied by salt deposits, which

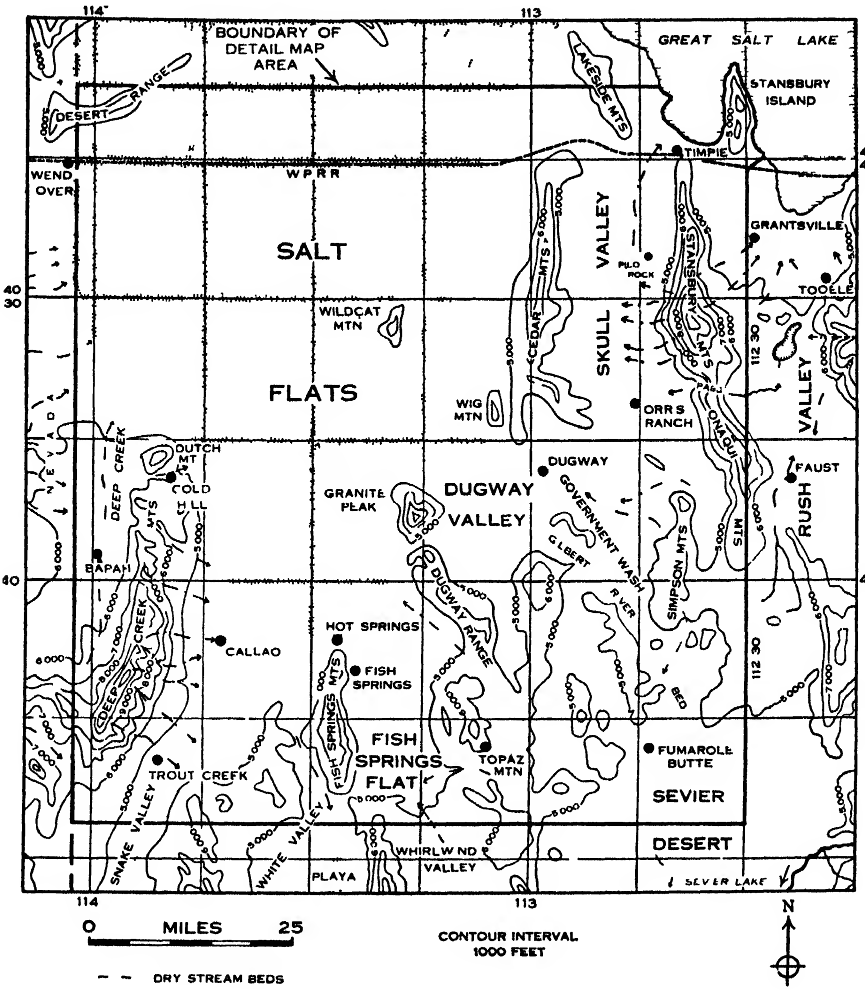

FIG 2 Topographic map of area of intensive study Elevation of Great Salt Lake is approximately 4200 ft, the Salt Flats are about 25 ft higher.

comprise one of the largest level surfaces in the world, and also one of the most barren; a third of the area is floored with lake-bed clays, in places covered by recent wash from bordering highlands; the remainder consists of desert mountains, many of them "inselbergs" Place names

and topographic features pertinent to this study are shown in Fig. 2 (5).[4]

CLIMATE.

The Salt Lake Desert is a region of climatic extremes. At many flatland stations, the highest summer temperature is more than 120° F. above the lowest winter temperature. In these same locations, annual

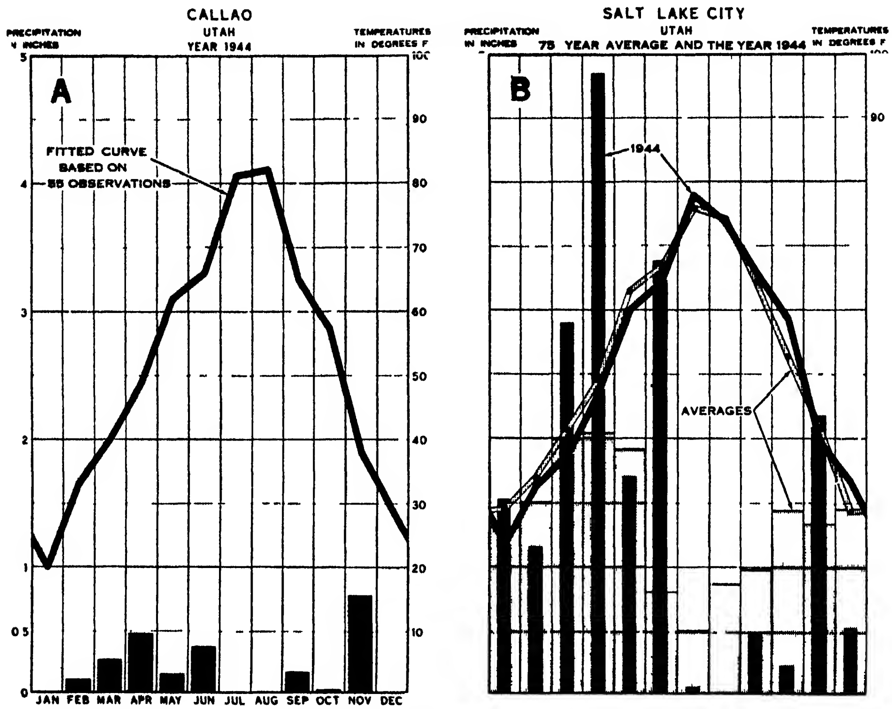

FIG. 3. Climatic charts for the Salt Lake Desert. The 1944 record for Callao is shown at A, monthly mean temperature being shown by the curve, monthly total rainfall by the bar scale. The 1944 climatic record for Salt Lake City is superposed on the 75-yr average in B

range of monthly mean temperatures is 60° or more, and daily range exceeds 20° on many days each year. In contrast, monthly mean temperatures are similar in magnitude and trend at all flatland stations, and annual variations are slight. A typical desert climatic chart constitutes Fig. 3 (*A*); comparison of annual and long-term data is shown in Fig. 3 (*B*).

Rainfall in this region is scanty, and so erratic in its areal and sea-

[4] Map modified from the U. S. C. and G. S. *Salt Lake City* sectional aeronautical map, much civil, and all military, culture being omitted. Local inquiry as to present roads and military restrictions is advised before entering this area. Geographic and geologic features of the region are competently described in ref. 5.

sonal distribution that 1-yr. records give little indication of the actual amount or distribution. Long-term records show that rainfall has a major spring maximum, a minor fall maximum, and a summer minimum. At most desert stations, the rainfall recorded on some days exceeds the total precipitation for some years. A 1-yr. record of rainfall at a desert station is shown in Fig. 3(*A*); comparison of rainfall for the same year with the 75-yr. average, at Salt Lake City, is shown in Fig. 3(*B*).

Dryness of this region at all seasons is caused by the barrier effect of surrounding mountains. An appreciable part of the incoming air is from the north Pacific. The lower strata of this incoming air, in crossing the Cascades and Sierras, are forced upward and chilled, so that some of the contained water vapor is condensed, and falls as rain or snow on the west flanks of the ranges. In winter, inflow of air is inhibited by atmospheric subsidence over the Great Basin ("The Great Basin High"), surface manifestation of which is a lateral outflow of cold air. Only strong disturbances can enter the region, at low levels, in winter.

In summer, the same area is occupied by a thermal low, in which heated air ascends, carrying with it evaporated moisture. This air mixes with the upper winds (above 15,000 ft.) before condensation takes place, and is carried eastward. In consequence, much moisture evaporated in the Salt Lake Desert is removed from the area, and eventually increases the humidity of areas far to the east. Potential evaporation, at many places in the Salt Lake Desert, is five or more times the actual precipitation.

Average aeronautical visibility (visual range) in the Salt Lake Desert is not only good, but is considerably better than might be concluded from study of the observational data, which, until recently, placed all visibilities of over ten miles in a single class. Dense fogs of as much as ten days' duration are not uncommon in winter, during atmospheric subsidences related to "Great Basin High" regimes, but their statistical effect is offset, in large part, by "runs" of twenty to forty days in summer, when, over the desert, "there isn't a cloud in the sky all day." [5]

VISIBILITY.

Desert visibility, like most other desert phenomena, is characterized by extremes. Visibility changes in clear or only slightly cloudy desert weather follow a definite, but not always simple, pattern. Maximum visibility commonly occurs at, or very shortly after, sunrise, at which time stratigraphic and structural details in mountains 90 miles distant may be plainly visible to the unaided eye. Visibility then declines slowly, reaching a minimum at about 3 p.m. (sun time), when visibilities of from ten to twenty miles are commonly reported. Visibility improves

[5] See ref. 3. Summaries of atmospheric conditions in this area, by months and years, as far back as 1872, are contained in "Climatological Data—Utah Section" published monthly, with annual summaries, by the U. S. Weather Bureau.

slowly thereafter, but at sunset has not regained its morning value. Sunset visibilities of from forty to sixty miles are commonly reported. When no marked change takes place in air-mass characteristics (*as measured locally*), successive morning visibility maxima normally show a definite, but small, decline.[6]

Within the limitations of current theories and available instruments, the day-to-day decline in visibility within a single air-mass qualitatively and quantitatively parallels the rate of increase of *luftplanktons* in the lower strata of this air mass. This increase in *luftplanktons* is in part due to convectionally raised dust, a natural desert phenomenon, and in part caused by local smelting operations, notably at Salt Lake City, Provo (Fig. 1) and Tooele (Fig. 2) (6).

Field studies, however, show no simple relation between the diurnal cycle of visibility changes and the *luftplankton* content of the lower atmosphere, particularly at very low relative humidities.[7] Definite qualitative and quantitative relations were noted between diurnal changes in the reciprocal of visibility and:

(*a*) Surface temperature,
(*b*) Reciprocal of relative humidity,
(*c*) Lapse rate in the surface stratum,
(*d*) Fluctuations in this lapse rate,
(*e*) Magnitude of convective activity, and
(*f*) Magnitude of shimmer, or "optical haze."

All of these variables are interdependent, within a single air mass, and all are manifestations, usually with a time lag (of about 2 hr.), of diurnal changes in incident solar radiation. Provided maximum relative humidity is low, effect of changes in the temperature of the air as a whole, within the range of temperatures normally encountered in meteorological work, on the optical properties of the air, is slight. Under desert (and most other) conditions, when specific humidity remains constant, relative humidity declines as air temperature increases, and *vice-versa*.[8] When maximum relative humidity is low, effect of relative humidity, and of changes therein, on visibility, is slight.

Field measurements of lapse rates in the Dugway area show that

[6] This "decay of visibility" on successive days, in the same air mass, is competently detailed in ref. 3, p. 185. Similar occurrences have been noted by the writer in the Ajo area, Arizona, and in the Alpine Basin, Texas.

[7] Relative humidities below 10 per cent are not uncommon in the desert at the time of maximum surface temperature (usually 2 p.m. sun time). Although values of 3 and 4 per cent have been recorded at Dugway in a number of instances, accuracy of these values is not guaranteed, because of shortcomings of psychrometric instruments and tables at high altitudes and very low humidities.

[8] Specific humidity is defined as the mass of water vapor in a given mass of moist air, and is usually expressed as grams of water per kilogram of moist air.

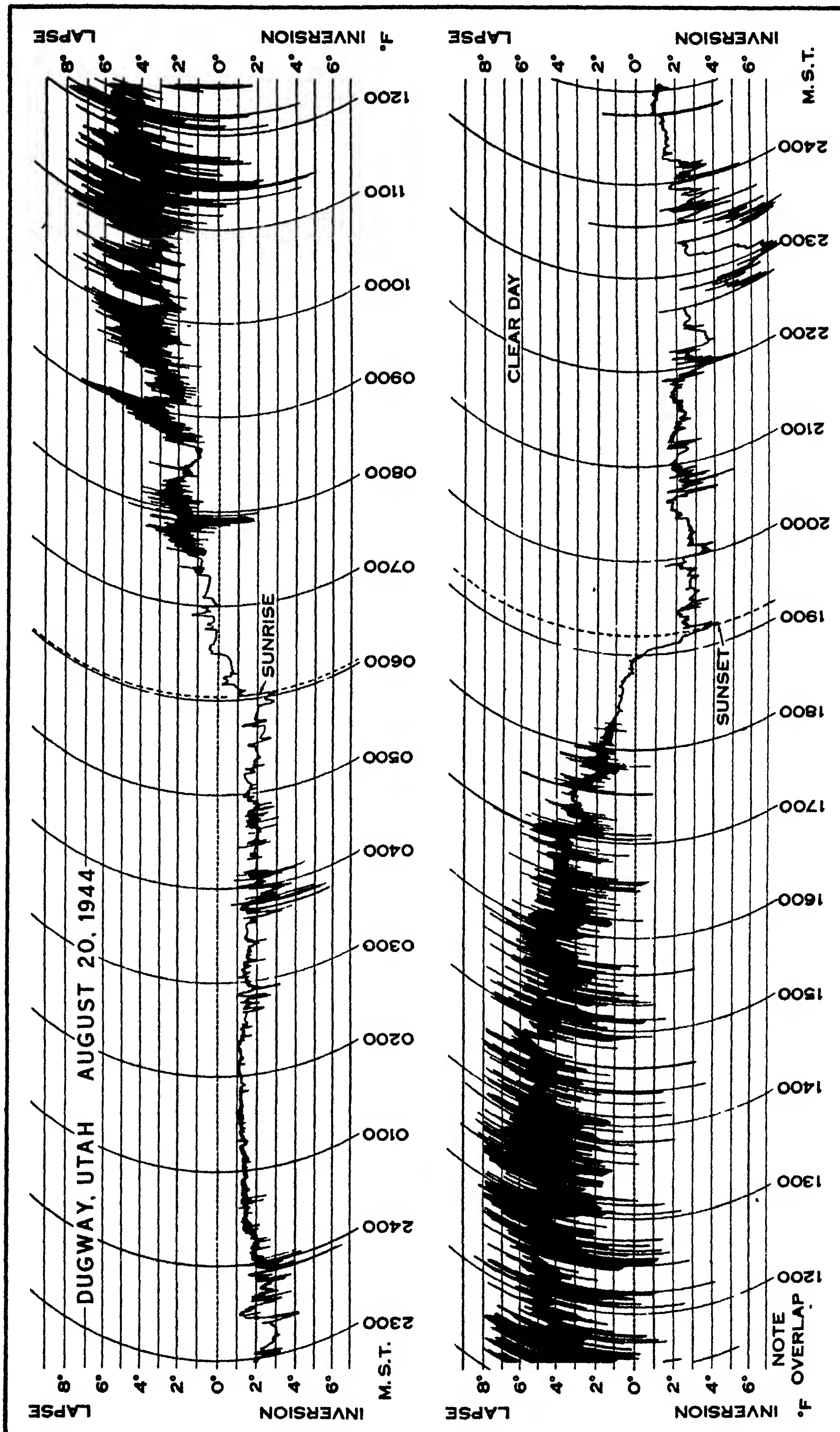

FIG. 4. One-day record of changes in lapse rate at Dugway, Utah. Lower thermocouple was 25 cm. above the ground; upper couple, 2 m. above the surface.

average lapse rate, in clear weather, increases from somewhat negative (slight inversion) at sunrise to strongly positive during the hottest part of the day, then declines, becoming negative shortly before sunset. A sample record of lapse changes, made with shielded fine-wire thermocouples and a high-speed electronic recorder, comprises Fig. 4. Note here the number and magnitude of the short-term fluctuations.[9] If the same instrument is arranged with both thermocouples at the same height above the ground, lateral temperature differences will be recorded. A trace of these resembles the deviations from the 10-min. mean values of Fig. 4.[10]

When a distant small target is observed telescopically, its lateral and vertical "dance rate" is approximately proportional, in frequency

FIG. 5. Typical "oasis and palm tree" mirage, photographed near Wendover, Utah

and magnitude, to the changes in either vertical or lateral temperature difference.

Other tests having shown that the rate of decline in the concentration of a gas released at ground level is roughly proportional to the lapse rate, it appears that lapse rate is a good measure of convective activity (not a new discovery), that shimmer is directly proportional to short-term variability in lapse rate, and that diurnal changes in desert visibility bear an approximate inverse relation to shimmer, and hence to short-term lapse rate fluctuations. These relationships are sufficiently consistent to permit visibility forecasts with an accuracy of slightly more than 80 per cent.

[9] Lapse rate is the rate of temperature decline with altitude above the surface, commonly expressed in either degrees Fahr. per 1000 ft., or degrees Cent. per kilometer. A negative lapse rate is commonly called an inversion.

[10] Development of these differential temperature recorders is largely the work of Dr. S. W. Grinnell, now of Stanford University. Constructional and operating data are contained in ref. 7.

INFERIOR MIRAGE.

Introduction.

Travelers' tales, exploration accounts, and some serious scientific reports from desert regions contain descriptions of interesting and elusive oases, surrounded by palm trees, even in areas where the palm is not indigenous, and sometimes populated by dancing houris and other atypical desert fauna. The subjective portions of mirage reports, although usually real to those who recount them, are beyond the scope of this paper.[11] Objective nature of the typical "oasis and palm tree" mirage report is indicated by the fact that it can be photographed. A typical example, near Wendover, Utah, comprises Fig. 5.[12] Although inferior mirages of many varieties have been reported, two general

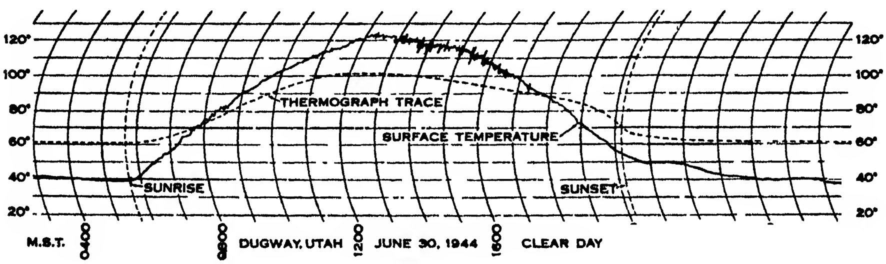

FIG. 6. Twenty-four hour record of temperatures at ground surface and at 5 ft. above the surface, during typical mirage weather.

classifications, based on the thermal state of the lower atmosphere, include the objective portions of all known instances.

Lapse Mirage.

The lapse mirage occurs when the air close to the ground is intensely heated relative to that a few inches or feet above the effective surface. This classification includes the familiar "water on the highway" mirage as well as the common "oasis and palm trees" type (Fig. 5).

Field measurements indicate that the minimum lapse permitting an inferior mirage, in large flat areas in clear sunshiny weather, is about 5° F. in the first foot above a completely barren surface (salt flats). Where vegetation is present, this lapse must occur in the foot above the effective surface, which, in the case of grasslands, is approximately two-

[11] Clinical thirst is normally accompanied by physical desiccation, disturbed body chemistry, and considerable fever. These are commonly augmented by the physiological effects of exhaustion and starvation. In consequence, accounts of things seen during arduous desert journeys are somewhat distorted by the effects of disordered perceptions and by misinterpretations of perceived stimuli.

[12] Additional photographs of mirages are contained in ref. 8 (see opposite p. 144) and ref. 9 (see opposite p. 480).

thirds the height of the grass above the ground. Similar corrections are necessary for areas of slight topographic relief. Because of the "chimney effect" of great relief, mirages are normally absent from badlands and other areas of considerable relief, in which extensive heated air strata cannot normally develop.

Characteristic temperature measurements, during "mirage weather" in the Salt Lake Desert, are shown in Fig. 6. Here, surface tempera-

FIG. 7. Onset of mirage conditions in Skull Valley on December 23, 1945, 30 min. after end of storm. Mountains at extreme right are on Stansbury Island; pinnacle at right is Pilot Rock (Fig. 2), about 8 miles from the camera; mountains at left are part of the Lakeside Range, about 15 miles from the camera. Note small mirage between camera and Pilot Rock; and thick diffusing stratum (shimmer layer) between camera and Lakeside Mountains. Dark line on horizon in center is the surface of Great Salt Lake, possibly accompanied by mirages.

tures, measured with a fine-wire thermocouple and recorded electronically (solid line), are compared to air temperatures recorded on a standard thermograph, 5 ft. above the surface, in a Weather Bureau shelter (dashed line). Notable here is the morning change from inversion to lapse, the correspondence of the afternoon "jitters" in the surface temperature curve with the period of maximum shimmer, and the late afternoon change from lapse to inversion.

Weak mirages appeared in salt flat areas northwest of Dugway (Fig.

2) about 35 min. after sunrise on this date; and strong mirages appeared quite suddenly over clay flats (sparse vegetation) at about 8:30 a.m. Mirages remained over all level areas until about 6:15 p.m., when those over the clay flats dissolved quite rapidly. Some of the mirages over barren salt flats persisted until a few minutes after sunset.

During the hottest portion of the day, tongues of mirage extended upward from the flats onto the lower slopes of some of the pediments fringing the mountains. Where field measurements were possible, it was found that these tongues of mirage coincided with updrafts of extremely hot dry air ("anabats"); and that, in general, they persisted only when the updraft was both thin and much warmer than the air directly above it.[13]

Conditions of lighting and atmosphere are normally unfavorable for effective photography of the onset of mirage conditions. In the winter season, however, during the clearing immediately following a storm, such photography is possible. One such example comprises Fig. 7. Here, 30 min. after the cloud cover "broke" over the end of the valley, a fairly thick diffusing stratum (left) had already developed, and a weak mirage was present on the clay flats in the center of the valley. Within an hour, the lower slopes of the Lakeside Mountains, and all of Stansbury Island had "greyed out"; and the mirage was considerably larger, producing an appearance of water from the horizon (actual water) to the base of the pediment, from which the photograph was taken.

Observed lapse mirages over water require about the same lapse condition as those over land, but this need not be produced by insolation, and frequently is not. Because of the high specific heat of water, relative to air, requisite lapse conditions are seldom attained during these parts of the year when average mean temperatures are increasing (late winter, spring, and early summer); but such conditions are present at most times while average mean temperatures are declining (late summer, fall, and early winter). Because extreme lapse may exist over water even at night, "moonlight mirages" are theoretically possible. Field observations disclosed many nights of distinctly abnormal visibility, but no clear-cut instances of nocturnal mirages.

Forecasting of over-water mirages on the basis of lapse rates appears justifiable, but confirmation of these forecasts was found difficult in many instances, as it was not found possible to distinguish between a distant weak mirage over water, and a distant haze layer viewed through shimmer. In general, mirages may be expected over water in most instances when the air a few inches above the water surface is five or more degrees (F.) warmer than that 5 ft. above the water; and are almost invariably absent when the air 5 ft. above the water is at nearly the

[13] Although based on measurements of less than 10 per cent of observed "mirage tongues" in any given day, this appears to be a standard condition, and has been observed also on inclined rock surfaces in the Red Desert of Wyoming, and on the "Flatirons" near Boulder, Colorado.

same temperature as the water. When lapse over water is between the above limits, over-water visibility is usually poor, but definite identification of mirages is not possible.[14]

As might be expected, any disturbance which locally decreases the lapse rate also locally disturbs any mirage which may be present. Chief natural disturbances are cloud shadows and dust devils (10). Artificial disturbances include extensive vehicular traffic, explosions, irrigation, and artificial refrigeration. When the disturbance ceases, the mirage usually "heals" in a matter of a few minutes; and when the cause of lapse rate decline is migratory (as a cloud shadow), the "hole" created by it in the mirage is also migratory.

Inversion, Subsidence, or Ponding Mirages.

Similar in appearance and behavior to the lapse mirage, but present only in extremely cold weather, and rarely even then, is the inversion mirage, which appears as a sheet of water in a topographic "low."

Atmospheric conditions accompanying such a mirage are shown diagrammatically in Fig. 8. Although there appears to be no theoret-

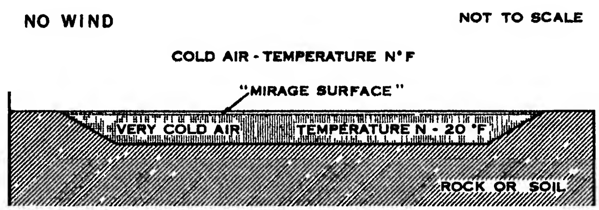

FIG. 8. Atmospheric conditions accompanying inversion mirages.

ical reason why inversion (also known as subsidence or ponding) mirages should not be present whenever very cold air drains into a depression ("ponds"), this type of mirage has only been observed when general air temperatures were far below zero, and the ponded air was twenty or more degrees colder.

These mirages have been observed only at "false dawn," when there was no wind. The boundary between the cold air (above) and the very cold air (below) is quite sharp, one field measurement showing a difference of 15° F. in 1 in. (vertically). Inversion mirages dissipate immediately at sunrise, as does the accompanying extreme inversion, and are destroyed by relatively minor mechanical disturbances, such as a wind of less than 3 mph., the turbulence produced by driving a jeep through them, or the explosion of a half stick of dynamite. Unlike lapse mirages, inversion mirages do not reform rapidly if disturbed.

[14] These statements are based on conditions at Great Salt Lake. Almost identical conditions prevail over the northern part of the Gulf of California, where mirages over the land are most common in summer, and are replaced by mirages over the water in winter.

Field investigations disclose that inversion mirages are formed only in blind basins in the paths of strong air drainages ("katabats") during windless subzero weather, and are not always found even under these rather restrictive conditions.

Rare Types.

At various heights over desert terrain, usually within 2000 ft. of the surface, sheets of "water" are occasionally noted from descending aircraft, or are observed from adjacent mountains. These have the same general appearance as the familiar inferior mirage which commonly is accompanied by steep lapse. In one extreme example, airport operations were disrupted because the entire field appeared submerged, except for the top of the control tower, when observed from a plane coming in for a landing. All airport features were clearly visible from a greater height, as well as from a considerably lesser height.

A few observations indicate that a change in lapse rate occurs at approximately the level of the "water," and suggest that this is simply an elevated inferior mirage.[16] Field observations from mountainsides disclose that these "elevated mirages" are visible through only a very narrow vertical range (such as 15 ft.), and that they are usually visible for only a short time (such as 30 min.) at any given elevation. Slow vertical migration is strongly suspected, but was not conclusively demonstrated by field evidence.

SUPERIOR MIRAGE.

Superior mirages, usually consisting of one or more inverted images of a distant object (which may be below the horizon); in many instances alternating, in "paper doll" fashion, with erect images; are not frequently reported from the Salt Lake Desert. Although superior mirages occur less frequently than inferior mirages, a large part of their apparent rarity is due to non-observation; the sites from which they are most commonly visible being visited rarely.

Relatively simple superior mirages, usually consisting of single inverted images of section houses and similar structures along the railroad at the north end of Skull Valley (*Map*, Fig. 2: *View*, Fig. 7), are occasionally seen, most frequently in early morning, from points ten or fifteen miles upvalley (south). Images are usually distorted, poorly defined, positionally unsteady, and chromatically altered (red in the object appears as brown in the image; white as bluish-green). Life of a superior mirage in this area is seldom as much as an hour. Frequency of occurrence is approximately twice a month; most common time of occurrence is between false dawn and one hour after sunrise. In no

[16] These lapse rate changes take place in too short a vertical distance to be reported with any hope of correctness by any standard radiosonde. Mountain observations, likewise, are subject to some or much error because of mountainside air circulations.

observed or reported instance has a superior mirage coexisted with an inferior mirage in this area, although both have occurred on the same day.

More complicated superior mirages are seen relatively frequently from the north base of Granite Peak (Fig. 2), usually to the north and northwest. Most common mirage of this type is an image of the train going across the desert upside-down. In most instances, the image is multiple, with vertical "paper doll" duplication. Positional unsteadiness, with *jump* being about four times *weave*, is common, as is vertical expansion. Such superior mirages are most common between false dawn and one hour after sunrise, and may be accompanied, just before disappearance, by a nearby inferior mirage.

Moving bands of lights, sometimes in as many as six vertical rows, usually unevenly spaced, are occasionally seen at night in this location. These are probably the same phenomenon, although it was not found possible, even by use of a theodolite, to determine whether the images were erect or inverted.

Rarely (perhaps three times a year), a complicated mirage topography is seen northeast of Wildcat Mountain (Fig. 2) by observers at Granite Peak. This, locally called "Guthman's Towers," is substantially identical to the mirage topography reported by Scoresby from the Davis Strait area.[16] This is invariably an early morning phenomenon, usually vanishing at sunrise. The real topography responsible for this mirage is a series of low hills near Knolls station, south of the railroad (Fig. 2) and near the east edge of the Salt Flats.

Vertical duplication of truck headlights eight or ten miles from the observer was noted in early evening at many points in the Salt Lake Desert, and caused considerable embarrassment to various Military Police charged with traffic counts. About half an hour after five sets of headlights were noted on the road, one truck would pass the M.P. station.

Meteorological investigations disclosed that a superior mirage was invariably accompanied by a definite change in the lapse rate at no great height above the surface. Simple superior mirages were commonly accompanied by a temperature inversion less than 500 ft. above the surface: complex superior mirages were usually accompanied by a complex thermal stratification of the air below the 500-ft. level. Appearance and disappearance of the superior mirage coincided approximately in time with that of the inversion. In general, the mirage and the thermal stratification of the lower atmosphere were of about equal complexity, although limitations of the instruments used preclude any layer-for-layer and image-for-image correlation.

It is notable that inversions, in this area, set in just before sunset, in clear weather, at low levels, and increase in elevation and complexity until shortly after sunrise, when they are dissipated by the upward

[16] See ref. 4, p. 473, Fig. 166. See also included bibliography.

growth of a lapse layer, produced by solar heating of the ground surface and of the air adjacent to it.

FATA MORGANA.

Although Fata Morgana mirages, such as are fairly common in the Straits of Messina, Toyama Bay, Hecate Strait, and the Gulf of Peñas,[17] are very infrequently reported from the environs of Great Salt Lake, many apparently occur there unseen.

Several mirages of this general type were observed from the north-

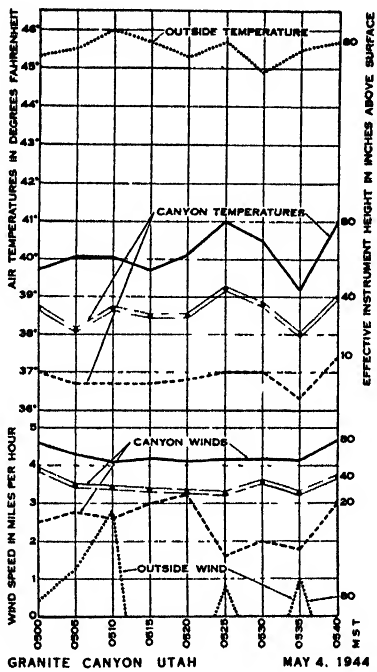

FIG. 9. Wind and temperature conditions within a canyon air drainage.

west shore of Stansbury Island (Fig. 2), and consisted of weirdly-distorted images of mountain terrain, probably adjacent ranges (Stansbury and Lakeside Mountains), although certain identification was impossible.

In contrast to mirages of other types, Fata Morgana images change from minute to minute, and not infrequently have peripheral color fringes, much like those surrounding Brocken Spectres. Complicating the general picture, in many instances, are wisps of "steam fog" rising from the lake water.

[17] Respectively between Italy and Sicily, on the northwest coast of Honshu, on the west coast of British Columbia, and on the Chilean coast between Chiloe Island and the Straits of Magellan.

All observed Fata Morgana displays occurred in early morning, when general air temperatures were not only low, but considerably lower than the temperature of the lake water, when wind speed was very low or zero, and when air drainages from adjacent mountains were strong, and somewhat colder than the static air above the lake.

These conditions produce, at the lake shore, a "two-layer" situation in the lower atmosphere, similar to that commonly accompanying simple mirages. A short distance offshore, this changes to a "three-layer" condition, as basal air is heated by contact with the relatively warm lake water. When this occurs, the basal air stratum, in contact with the lake water, is in unstable equilibrium, and tends to rise. This density readjustment produces convective disturbances largely confined to the coldest air stratum present. Initial conditions within this stratum are shown in Fig. 9. As its base is warmed, the thermal stratification is locally disrupted, producing chaotic thermal distribution (and hence density distribution) within it.

It is notable that substantially identical atmospheric conditions were present in most instances when Fata Morgana mirages were observed in other locations. Although it seems probable that the Fata Morgana can be predicted from a slight modification of the standard weather forecast, the number of joint observations to date is too small to justify establishment of a definite forecasting procedure.

SUMMARY AND CONCLUSIONS.

The foregoing observations indicate that in the Salt Lake Desert, and probably elsewhere, each type of mirage is accompanied by a definite pattern of thermal distribution in the lower atmosphere. These thermal conditions can now be predicted by standard weather forecasting methods, or simple and straightforward modifications thereof.

Density distributions produced in the lower atmosphere by these thermal conditions are in every instance compatible with those called for by current general theories of mirage genesis.[18]

A small amount of additional research, using instruments now extant, although not as yet in general use, should not only refine current theories of mirage formation, permitting quantitative descriptions, but may well lead to the development of dependable procedures for predicting, to a high degree of accuracy, at least 24 hours in advance, the visibility conditions at any place on the earth's surface.

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[18] Salient parts of these theories are more than a half century old, and are, in general, strengthened by recent increases in observational data. For summary of general theories, see ref. 4, pp. 467–475; for detailed descriptions of the various thermal states of the atmosphere, and of the changes therein, see ref. 10, pp. 50–137.

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Pneumatic Barker Cuts Log Waste (*Compressed Air Magazine*, Vol. 53, No. 3).—A significant contribution to the conservation of forest products has apparently been made by the Weyerhaeuser Timber Company, in the State of Washington, with its new air-operated log barker.

Logs intended for processing into plywood have generally been turned in a modified engine lathe while a cutting tool removed the bark. This treatment does the job satisfactorily, but it takes away with the bark around 5 per cent of the wood. Aside from this direct loss of wood, the bared outer surface of the log is torn to some extent and must be smoothed before continuous plywood sheets can be obtained by the subsequent "peeling" operation with a cutting tool. In recent years, use has been made of a hydraulic method that loosens the bark by means of high-pressure water jets. The Weyerhaeuser development represents an effort to accomplish the same result in a simpler and less expensive way.

The pneumatic equipment removes the bark by the compressive and shearing action of a rotating wheel that is pressed tight against the surface of the log as the latter turns in a giant lathe. Pressure is obtained by mounting the wheel on the end of a piston that works in an air cylinder. The apparatus is controlled by one man, who rides a carriage that moves the length of the log as the bark is progressively removed.

The debarker is designed on the simple principle that less force is required to crush bark than to crush wood. When the applied force is sufficient, the bark separates at its point of contact with the cambium, which lies between the bark and the wood, thus producing wood-free green bark as one product and bark-free logs as the other. This is accomplished with the inexpensive equipment at low operating cost. Air pressure of about 50 psi. is used, and in the Weyerhaeuser plant is drawn from the regular supply lines extending to various parts of the mill. The average barking time for a log 8 ft. long and 40 in. in diameter is $1\frac{3}{4}$ min., which is about the same as with previous facilities.

Because of the clean removal of the bark it is possible to utilize the entire log. In the case of fir logs, which are about 12 per cent bark, the latter was formerly considered of little or no commercial value. Now it is converted into ingredients that enter into the manufacture of plastics, insectides, magnesite flooring, rubber compounds, and many other products. It is processed in the Weyerhaeuser Silvacon plant at Longview, Wash., where a huge plywood plant is likewise located. From the bark is also obtained glue extender, of which large amounts are used in fabricating plywood. Thus the new equipment provides more wood for plywood veneer, and the bark contributes its natural properties to help make the plywood durable and weather-proof.

The pneumatic barker represents another forward step in the program undertaken to eliminate the waste that formerly characterized the timber-working industry. Sawdust, wood refuse, bark, and mill ends, that once accumulated in great piles and for which there was no market, are now being converted into valuable by-products by research engineers.

The new barker is the result of years of experimentation by the Weyerhaeuser research staff under the direction of C. C. Heritage, the firm's technical adviser. Patents have been applied for, and the company intends to make the process available to the lumber industry generally. Indications are that it will be applied mainly for debarking large logs.

R. H. O.

APPLICATIONS OF TENSOR ANALYSIS TO ELASTICITY AND PIEZOELECTRICITY.*

BY

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The subject of elasticity has been covered very completely in the standard English texts, for example, Love (1),[2] Sokolnikoff (2), Timoshenko (3). All of them at least mention tensor notation in passing, and one or two use a form of it in their mathematical analysis, but none presents the subject in a consistent development in terms of tensor analysis.

The only book which actually uses the tensor calculus as the analytical tool in the development of the mathematical theory of elasticity is "Les Tenseurs en Méchanique et en Élasticité," by Brillouin (4), and this book is not available in English.

Most developments on this subject are rather complex, involving the introduction of a number of related metric tensors. These could undoubtedly better be combined to produce a simpler over-all formulation. The tendency to use a number of interdependent functions in place of one in this type of analysis is a weakness, since it reduces the generality and simplicity of form which are two of the main reasons for using tensors. No doubt much of the prejudice against the use of tensors, which is so prevalent among mathematical physicists, derives from this source.

In the subject of piezoelectricity, the tensor notation has been used to a limited extent by Lawson (5), and Atanasoff and Hart (6).

Except for Brillouin (4), none of the authors mentioned above has used more than the notation alone. Although the Einstein summation convention is used to good purpose by most, no distinction is made between covariance and contravariance. Thus it can be seen that the idea of using tensor analysis in the development of elasticity is not a new one, but it should at the same time be obvious that in order to show its true effectiveness as a tool in this field, something more thorough and consistent is to be desired. This paper, I hope, will be a step in that direction.

A relatively small amount of work has been done in the mathematical analysis of piezoelectricity, and I believe that the use of tensors in this practically untouched field can be very fruitful indeed.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

In all cases throughout the paper, only small deformations will be considered. The theory of finite deformations is beyond the scope of this investigation, and is not applicable in the piezoelectric effect. By a small deformation is meant a deformation such that Hooke's Law involves no second order terms, that is, the transformation can be considered affine.

The tensor notation and the methods of analysis are taken from Schouten and Struik (7). The rationalized mks. system of units is used throughout this paper.

The most important sources of material used in the development of elasticity are Love (1), and Sokolnikoff (2). The material on piezoelectricity is largely from Cady (8), Heising (9), Lawson (5), and Atanasoff and Hart (6).

In general, this paper presents stress, strain, and their mathematical relationships in a coherent tensor development. It then states the fundamental concepts of piezoelectricity in the same notation. These two major topics are related to Newtonian mechanics in the development of a generalized set of equations of natural motion of piezoelectric media. It is shown that such a set of equations, though presently impossible of complete and general solution, can be made, by discriminating selection of boundary conditions, to offer adequate solutions of practical or theoretical problems of harmonic motion of such media.

I. ELASTICITY.

Stress.

Stress can be shown to be a bivalent contravariant tensor, defined by the equation,

$$dF_i = g_{ij} T^{jk} da_k,$$

where dF_i = force (newton) acting on the element of area da_k,
g_{ij} = metric coefficients,
T^{jk} = stress tensor (newton m^{-2}) caused by dF_i,
da_k = element of area (m^2).

In rectangular cartesian coordinates, stress reduces to a force per unit area, whose components are

$$T^{11} = \frac{dF_1}{da_1}, \qquad T^{22} = \frac{dF_2}{da_2}, \qquad T^{33} = \frac{dF_3}{da_3},$$

$$T^{12} = \frac{dF_1}{da_2}, \qquad T^{21} = \frac{dF_2}{da_1}, \qquad T^{31} = \frac{dF_3}{da_1},$$

$$T^{13} = \frac{dF_1}{da_3}, \qquad T^{23} = \frac{dF_2}{da_3}, \qquad T^{32} = \frac{dF_3}{da_2}.$$

In order to confine the discussion of elasticity to relative displacements, we must place certain limiting conditions upon it. The two

conditions immediately applicable to stress are that the system undergoing the stress be in equilibrium for both moment and force.

Let us first consider the consequences of the condition of force equilibrium. A postulate of the tensor calculus states that if a tensor is transported parallel to itself, the transformation can be expressed in the form:

$$T^{jk}(P') = T^{jk}(P) + \nabla_l T^{jk} dx^l, \tag{1}$$

where the point coordinates are x^l. It is evident from this that the difference between stress components on opposite faces of an elemental volume is

$$- \nabla_l T^{jk} dx^l. \tag{2}$$

The net force creating this stress must by definition be

$$dF_i = - g_{ij} \nabla_l T^{jk} dx^l da_k. \tag{3}$$

Since T^{jk} is defined as acting on the k face, distance between the opposite faces under discussion is dx^k. If the coordinate system is orthogonal, then Eq. 3 can be rewritten as

$$dF_i = - g_{ij} \nabla_k T^{jk} d\tau, \tag{4}$$

where $d\tau$ is an element of volume (m^3). If a body force acts throughout the volume, then if A_i/τ be defined as the force per unit volume (newton m^{-3}),

$$dF_i = A_i - g_{ij} \nabla_k T^{jk} d\tau. \tag{5}$$

In conditions of force equilibrium, the total force is zero, or

$$g_{ij} \nabla_k T^{jk} - \frac{A_i}{\tau} = 0. \tag{6}$$

This reduces, in the case of rectangular cartesian coordinates, to

$$\frac{\partial T^{ik}}{\partial x^k} - \frac{A_i}{\tau} = 0, \tag{6a}$$

which, when expanded, is the set of force equilibrium conditions commonly used.

The condition that the system be in a state of moment equilibrium requires that the stress tensor be symmetrical. This can be shown in the following manner.

Moment is a univalent covariant vector defined by the equation,

$$M_i = \frac{g^{ij}}{\sqrt{g}} \oint_s e_{ijk} dF_l dx^k, \tag{7}$$

where dx^k = an incremental moment arm in an axial direction,
dF_l = an incremental force,

$$g = \text{determinant of } g_{ij}, \text{ and}$$

$$e_{ijk} = \begin{cases} 1, & \text{if } ijk \text{ is a positive permutation of 123,} \\ -1, & \text{if } ijk \text{ is a negative permutation of 123,} \\ 0, & \text{if } ijk \text{ has any two indices equal.} \end{cases}$$

In equilibrium,

$$M_i = 0.$$

Substituting for dF_l from Eq. 1,

$$\begin{aligned} 0 &= g^{lj} \oint_S e_{ijk} dx^k g_{lm} T^{mn} da_n \\ &= \overset{j}{\underset{m}{\delta}} \oint_S e_{ijk} dx^k T^{mn} da_n. \end{aligned} \tag{8}$$

Application of Gauss's Theorem,

$$\oint_S v^i da_i = \int_V \frac{1}{\sqrt{g}} \frac{\partial(\sqrt{g} v^i)}{\partial x^i} d\tau$$

(where S is the closed surface bounding the volume V), to Eqs. 7 and 8, yields

$$\begin{aligned} M_i &= \overset{j}{\underset{m}{\delta}} \int_V \frac{1}{\sqrt{g}} \frac{\partial \sqrt{g}(e_{ijk} dx^k T^{mn})}{\partial x^n} d\tau \\ &= \overset{j}{\underset{m}{\delta}} \int_V \frac{1}{\sqrt{g}} \Big[\sqrt{g} e_{ijk} T^{mn} \frac{\partial dx^k}{\partial x^n} + \sqrt{g} e_{ijk} \frac{\partial T^{mn}}{\partial x^n} dx^k \\ &\qquad + e_{ijk} dx^k T^{mn} \frac{\partial \sqrt{g}}{\partial x^n} \Big] d\tau. \end{aligned} \tag{9}$$

Since the volume V is not generally zero,

$$\sqrt{g} \overset{j}{\underset{m}{\delta}} e_{ijk} T^{mn} \frac{\partial dx^k}{\partial x^n} + \sqrt{g} \overset{j}{\underset{m}{\delta}} e_{ijk} dx^k \frac{\partial T^{mn}}{\partial x^n} + \overset{j}{\underset{m}{\delta}} e_{ijk} dx^k T^{mn} \frac{\partial \sqrt{g}}{\partial x^n} = 0. \tag{10}$$

But in force equilibrium conditions, by Eq. 6,

$$\frac{\partial T^{mn}}{\partial x^n} = 0$$

(if higher order terms are neglected and body forces equal zero). Therefore

$$\sqrt{g} \overset{j}{\underset{m}{\delta}} e_{ijk} T^{mn} \frac{\partial dx^k}{\partial x^n} + \overset{j}{\underset{m}{\delta}} e_{ijk} dx^k T^{mn} \frac{\partial \sqrt{g}}{\partial x^n} = 0. \tag{11}$$

In orthogonal coordinate systems

$$\frac{\partial dx^k}{\partial x^n} = \begin{cases} 1, & \text{for } n = k, \\ 0, & \text{for } n \neq k. \end{cases}$$

Thus we may state

$$e_{ijk}T^{jk} = 0,$$

or

$$T^{jk} = T^{kj}. \tag{12}$$

But tensor symmetry is an invariant property under coordinate transformations and therefore Eq. 12 holds in general if there are no unbalanced external forces.

Strain.

If surface and volume forces are applied to a deformable body, its parts undergo relative displacements which increase until equilibrium is reestablished by the internal forces. The changes in relative positions of the parts of a body under stress are called strains.

Under the influence of applied force, a point P is displaced to a point P'. Call the displacement vector $s^i(P)$. In the same deformation, a nearby point, $\overset{(1)}{P}$, is displaced to a point $\overset{(1')}{P}$. Its displacement vector is $s^i(\overset{(1)}{P})$. The relative displacement of the two points is then

$$s^i(P) - s^i(\overset{(1)}{P}).$$

If we now apply the postulate of parallel transport, we observe that

$$s^i(P) - s^i(\overset{(1)}{P}) = \nabla_k s^i(P)dx^k = \Delta s^i. \tag{13}$$

Expanding, we obtain:

$$\Delta s^i = \nabla_1 s^i(P)dx^1 + \nabla_2 s^i(P)dx^2 + \nabla_3 s^i(P)dx^3, \tag{14}$$

or,

$$\Delta s^i = \frac{\partial s^i}{\partial x^1}dx^1 + \frac{\partial s^i}{\partial x^2}dx^2 + \frac{\partial s^i}{\partial x^3}dx^3 + \Gamma^i_{l1}s^l + \Gamma^i_{l2}s^l + \Gamma^i_{l3}s^l. \tag{15}$$

The terms $\dfrac{\partial s^i}{\partial x^j}$ are called deformation components.

Let us define two tensors S_{ij} and R_{ij} in the following way:

$$S_{ij} = \frac{1}{2}\left(g_{ik}\frac{\partial s^k}{\partial x^j} + g_{jk}\frac{\partial s^k}{\partial x^i}\right), \tag{16}$$

and

$$R_{ij} = \frac{1}{2}\left(g_{ik}\frac{\partial s^k}{\partial x^j} - g_{jk}\frac{\partial s^k}{\partial x^i}\right). \tag{17}$$

It is now evident that

$$R_{ij} + S_{ij} = g_{ik}\frac{\partial s^k}{\partial x^j}. \tag{18}$$

Thus the sum of the two tensors is precisely a measure of the deforma-

tion components already defined. If the higher order Christoffel functions are neglected, a justifiable step in a small deformation, the transformation can be described completely in terms of the tensors S_{ij} and R_{ij}.

Further investigation will disclose that the tensor R_{ij} is related to that portion of the deformation which is a local translation or rotation of a segment of the deformed body, but not a relative displacement. If we consider only conditions of equilibrium for such local rotation and translation, the R_{ij} terms vanish.

The S_{ij} terms, which define only that portion of the deformation which is a relative displacement, are called strains. Thus S_{ij} is known as the *strain tensor*.

Although no extension of the idea will be attempted in this paper, it is interesting to note that both stress and strain, in any practical problem, include both symmetric and antisymmetric components. If moment equilibrium of a volume element is not maintained, that is, rotation is permitted, stress can be broken into symmetric and antisymmetric components.

To maintain point-to-point continuity of the transformation when a volume element is strained, a certain amount of rotation and translation must take place in addition to pure relative displacements. In this case, the antisymmetric portion of the strain, R_{ij}, does not vanish.

Thus it is apparent that in any practical consideration, although Hooke's Law is satisfied, there must be another relation linking the antisymmetric portions of stress and strain. This condition results in the equations of compatibility which will be considered in the next section.

A physical interpretation may be placed upon strain in the following way: It can be shown that for $i \neq j$, S_{ij} is a measure of one half the angular deformation produced by an applied stress; and that the diagonal terms of the strain matrix are the relative changes in length.

Stress-Strain Relations.

1. Hooke's Law.

Hooke has shown experimentally that small deformations of an elastic body can be described by an affine relationship between stress and strain. In tensor notation this may be expressed as

$$T^{ij} = c^{ijkl}S_{kl}, \tag{19}$$

with the inverse relation,

$$S_{kl} = b_{klij}T^{ij}. \tag{20}$$

It is evident from these two equations that

$$c^{ijkl}b_{klmn} = \overset{ij}{\underset{mn}{\delta}}, \tag{21}$$

where $\overset{ij}{\underset{mn}{\delta}}$ is the quadruply indexed Kroniker Delta. The elastic constants, c^{ijkl}, exhibit symmetry in indices such that

$$c^{ijkl} = c^{jikl} = c^{ijlk} = c^{jilk} = c^{klij} = c^{lkij} = c^{lkji} = c^{klji}.$$

2. Equations of Compatibility.

From the points of view both of the physical and the mathematical aspects of displacements, it would be desirable that s^i, the displacement vector, be continuous and single valued throughout the deformation. This will put restrictions upon the values of S_{ij}, due to the fact that pure deformation with no translation of the various portions of the body would produce point-to-point discontinuities through the body. The equations stating these restrictions are called the equations of compatibility.

One method of stating the displacement equation is

$$s^j(\overset{(1)}{P}) = s^j(P) + \Delta s^j = s^j(P) + \int_P^{\overset{(1)}{P}} ds^j$$

$$= s^j(P) + g^{ji} \int_P^{\overset{(1)}{P}} S_{ik} dx^k + g^{ji} \int_P^{\overset{(1)}{P}} R_{ik} dx^k. \quad (22)$$

If the last member of this equation be integrated by parts,

$$\int_P^{\overset{(1)}{P}} R_{ik} dx^k = R_{ik}(\overset{(1)}{x^k} - x^k) - \int_P^{\overset{(1)}{P}} (\overset{(1)}{x^k} - x^k) \frac{\partial R_{ik}}{\partial x^l} dx^l. \quad (23)$$

It is a simple matter to show that

$$\frac{\partial R_{ik}}{\partial x^l} = \frac{\partial S_{il}}{\partial x^k} - \frac{\partial S_{kl}}{\partial x^i}. \quad (24)$$

Substituting back into the displacement equation, we obtain

$$s^j(P) = s^j(P) + g^{ji} R_{ik}(\overset{(1)}{x^k} - x^k)$$

$$+ g^{ji} \int_P^{\overset{(1)}{P}} \left[S_{il} - (\overset{(1)}{x^k} - x^k) \left(\frac{\partial S_{il}}{\partial x^k} - \frac{\partial S_{kl}}{\partial x^i} \right) \right] dx^l. \quad (25)$$

Since we wish the displacement, $s^j(\overset{(1)}{P})$, to be independent of the path of integration, the argument under the integral must be an exact differential. The necessary and sufficient condition is

$$\frac{\partial}{\partial x^j}\left[S_{il} - (\overset{(1)}{x^k} - x^k) \left(\frac{\partial S_{il}}{\partial x^k} - \frac{\partial S_{kl}}{\partial x^i} \right) \right]$$

$$- \frac{\partial}{\partial x^l}\left[S_{ij} - (\overset{(1)}{x^k} - x^k) \left(\frac{\partial S_{ij}}{\partial x^k} - \frac{\partial S_{kj}}{\partial x^i} \right) \right] = 0. \quad (26)$$

In orthogonal coordinate systems this equation reduces to

$$\frac{\partial^2 S_{ij}}{\partial x^k \partial x^l} - \frac{\partial^2 S_{kj}}{\partial x^i \partial x^l} - \frac{\partial^2 S_{il}}{\partial x^k \partial x^j} + \frac{\partial^2 S_{kl}}{\partial x^i \partial x^j} = 0. \tag{27}$$

Since i, j, k, l can take on only three values, two at least must be equal. The non-trivial conditions $i = j$, $i = l$, $j = k$, or $k = l$, all yield the same equation:

$$\frac{\partial^2 S_{ii}}{\partial x^k \partial x^l} - \frac{\partial^2 S_{ki}}{\partial x^i \partial x^l} + \frac{\partial^2 S_{il}}{\partial x^k \partial x^i} - \frac{\partial^2 S_{kl}}{(\partial x^i)^2}. \tag{28}$$

The six non-identical equations obtainable from the above relation are known as the equations of compatibility.

Elastic Symmetry.

It is evident that, in the most general case, Hooke's Law can be expressed by two sets of nine equations, each equation containing nine terms.

In many cases, however, particularly those involving crystalline substances, the materials under consideration exhibit some degree of inherent structural symmetry. Such symmetry simplifies the stress-strain relations to an extent dependent upon the degree of symmetry. Tensor algebra can be used to obtain the simplified relations with a minimum of difficulty. The method is to determine all independent coordinate transformations with respect to which the elastic constants of the medium are invariant, to compare the relations obtained, and to simplify.

Thus *in a medium exhibiting orthonormal symmetry*, there are two independent transformations; these are any two of the three simple reflections:

$$\begin{aligned} x^{1'} &= -x^1, \\ x^{2'} &= x^2, \\ x^{3'} &= x^3, \end{aligned} \qquad \begin{aligned} x^{1'} &= x^1, \\ x^{2'} &= x^2, \\ x^{3'} &= -x^3, \end{aligned} \qquad \text{and} \qquad \begin{aligned} x^{1'} &= x^1, \\ x^{2'} &= -x^2, \\ x^{3'} &= x^3. \end{aligned}$$

Hooke's Law can be expressed as follows:

$$\begin{aligned} T^{11} &= c^{1111}S_{11} + c^{1122}S_{22} + c^{1133}S_{33}, \\ T^{22} &= c^{2211}S_{11} + c^{2222}S_{22} + c^{2233}S_{33}, \\ T^{33} &= c^{3311}S_{11} + c^{3322}S_{22} + c^{3333}S_{33}, \\ T^{23} &= T^{32} = 2c^{2323}S_{23}, \\ T^{13} &= T^{31} = 2c^{1313}S_{13}, \\ T^{12} &= T^{21} = 2c^{1212}S_{12}. \end{aligned}$$

In a homogeneous isotropic medium, the above reflections, plus three independent rotations, will yield all the relations necessary for the derivation of the equations characterizing an isotropic medium.

Three possible rotations are:

$$x^{1'} = x^1, \qquad x^{1'} = x^2, \qquad x^{1'} = x^1\frac{\sqrt{2}}{2} + x^2\frac{\sqrt{2}}{2},$$

$$x^{2'} = x^3, \qquad x^{2'} = -x^1, \quad \text{and} \quad x^{2'} = -x^1\frac{\sqrt{2}}{2} + x^2\frac{\sqrt{2}}{2},$$

$$x^{3'} = -x^2, \quad x^{3'} = x^3, \qquad x^{3'} = x^3.$$

In this case Hooke's Law reduces to

$$\begin{aligned}
T^{11} &= c^{1111}S_{11} + c^{1122}(S_{22} + S_{33}),\\
T^{22} &= c^{1111}S_{22} + c^{1122}(S_{11} + S_{33}),\\
T^{33} &= c^{1111}S_{33} + c^{1122}(S_{11} + S_{22}),\\
T^{23} &= T^{32} = 2c^{2323}S_{23},\\
T^{13} &= T^{31} = 2c^{2323}S_{13},\\
T^{12} &= T^{21} = 2c^{2323}S_{12},
\end{aligned}$$

where

$$c^{2323} = \tfrac{1}{2}(c^{1111} - c^{1122}).$$

If the medium is homogeneous and isotropic, let us consider the values of the elastic constants throughout the body to be as follows:

$$\mu = c^{2323} \qquad \text{and} \qquad \lambda = c^{1122}.$$

Now it is easy to obtain the relations

$$T^{ij} = \lambda(S_{11} + S_{22} + S_{33}) + 2\mu S_{ij} \qquad (i = j) \tag{29a}$$

and

$$T^{ij} = 2\mu S_{ij} \qquad (i \neq j). \tag{29b}$$

These are the familiar equations relating stress and strain in a homogeneous isotropic medium.

Similar equations may be obtained for the strain terms.

$$S_{ij} = \frac{T^{ij}}{2\mu} - \frac{\lambda}{2\mu(3\lambda + 2\mu)}(T^{11} + T^{22} + T^{33}) \qquad (i = j), \tag{30a}$$

$$S_{ij} = \frac{T^{ij}}{2\mu} \qquad (i \neq j). \tag{30b}$$

Simple Examples in Elasticity.

We are now in a position to derive the relation between the c^{ijkl} and commonly used moduli of elasticity, and to observe their use in simple cases of elastic deformations.

Example I.—Consider a circular cylinder whose axis is in the x^1 direction, acted upon by a longitudinal stress only. Then $T^{11} = T$, and all other stresses are zero. Thus

$$S_{11} = \frac{T}{2\mu} - \frac{\lambda T}{2\mu(3\lambda + 2\mu)} = \frac{(\lambda + \mu)T}{\mu(3\lambda + 2\mu)},$$

$$S_{22} = S_{33} = \frac{-\lambda T}{2\mu(3\lambda + 2\mu)}.$$

$$S_{ij} = 0 \qquad (i \neq j).$$

Thus

$$\frac{S_{22}}{S_{11}} = \frac{S_{33}}{S_{11}} = \frac{-\lambda}{2(\lambda + \mu)}.$$

Let us define a term σ such that

$$\sigma \equiv \frac{\lambda}{2(\lambda + \mu)}, \qquad (31)$$

and a term E, such that

$$E \equiv \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}. \qquad (32)$$

Then

$$S_{11} = \frac{T}{E},$$

and

$$S_{22} = S_{33} = \frac{-\sigma T}{E} = -\sigma S_{11}.$$

Thus

$$E = \frac{T}{S_{11}}.$$

E is known as *Young's Modulus* and is the ratio of tensile stress to the extension produced by the stress. Also,

$$\sigma = \left|\frac{S_{22}}{S_{11}}\right| = \left|\frac{S_{33}}{S_{11}}\right|,$$

known as *Poisson's Ratio*, shows the ratio of the contraction perpendicular to the length, to the extension along the length of the cylinder.

The constants λ and μ, already introduced, are known as the *Lamé constants*. They satisfy the relations,

$$\lambda = \frac{E\sigma}{(1 + \sigma)(1 - 2\sigma)} \qquad (33)$$

and

$$\mu = \frac{E}{2(1 + \sigma)}. \qquad (34)$$

Example II.—Consider next the problem of a parallelepiped acted upon by a pure shear, such that

$$T^{23} = T^{32} = T$$

and

$$T^{11} = T^{22} = T^{33} = T^{12} = T^{21} = T^{13} = T^{31} = 0.$$

Then

$$S_{23} = S_{32} = \frac{T}{2\mu}$$

and

$$\mu = \frac{T}{2S_{32}}.$$

Recalling the fact that S_{32} is a measure of the angular deformation, it is evident, then, that μ is the ratio of shear stress $T^{ij}(i \neq j)$ to the changes in angle, $2S_{ij}(i \neq j)$. μ is called the *modulus of rigidity*, or *shear modulus*.

Example III.—In a body under hydrostatic pressure there is no shear stress, and all compressional stresses are equal. Thus

$$T^{11} = T^{22} = T^{33} = -\rho$$

and

$$T^{ij} = 0 \qquad (i \neq j).$$

Then

$$S_{11} = S_{22} = S_{33} = \frac{-\rho}{3\lambda + 2\mu}$$

and

$$S_{ij} = 0 \qquad (i \neq j).$$

From this,

$$S_{11} + S_{22} + S_{33} = \frac{-\rho}{\lambda + \frac{2}{3}\mu}.$$

Let us define a term K such that

$$K = \lambda + \tfrac{2}{3}\mu. \tag{35a}$$

It can be seen also then that

$$K = \frac{E}{3(1 - 2\sigma)}. \tag{35b}$$

Then

$$K = \frac{-\rho}{S_{11} + S_{22} + S_{33}}$$

and is thus the ratio of compressional stress to the sum of the relative changes in length in the principal directions, or simply, to the change in volume or bulk. K is called the *modulus of compression*, or *bulk modulus*.

It will be observed that K is always positive, thus yielding the condition,

$$\sigma < \tfrac{1}{2}.$$

We may now write the equations for Hooke's Law in a homogeneous isotropic medium in the form:

$$S_{11} = \frac{1}{E}[T^{11} - \sigma(T^{22} + T^{33})],$$
$$S_{22} = \frac{1}{E}[T^{22} - \sigma(T^{33} + T^{11})], \qquad (36a)$$
$$S_{33} = \frac{1}{E}[T^{33} - \sigma(T^{11} + T^{22})],$$

and

$$S_{ij} = \frac{1 + \sigma}{E} T^{ij} \qquad (i \neq j). \qquad (36b)$$

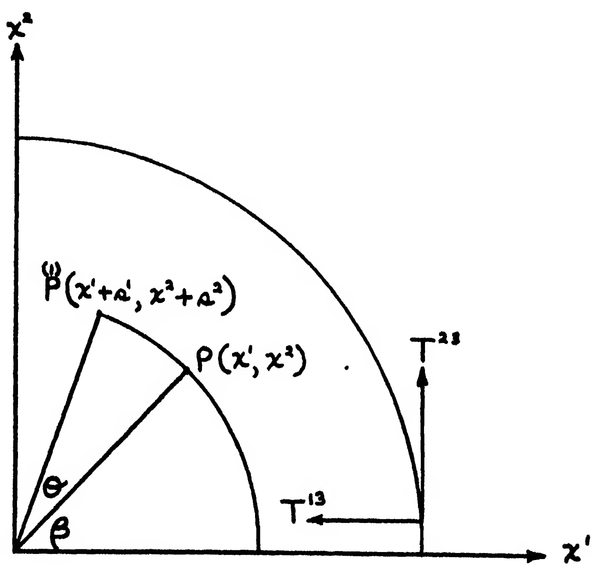

FIG. 1.

Example IV.—Finally, let us consider simple torsion of a circular shaft. We must assume that angular rotation at any cross section will be proportional to the perpendicular distance from the fixed face.

Let the length of the cylinder lie along the x^3 axis. Let

$$\theta = \alpha x^3,$$

where α is the angular rotation per unit length, and θ is the angle through which the cross section to be considered is rotated.

In a cartesian coordinate system, after deformation, a point $P(x^1, x^2)$ will become $\overset{(1)}{P}(x^1 + s^1, x^2 + s^2)$, as in Fig. 1. Then

$$s^1 = r \cos(\beta + \theta) - r \cos \beta = x^1(\cos \theta - 1) - x^2 \sin \theta$$

and

$$s^2 = r \sin(\beta + \theta) - r \sin \beta = x^1 \sin \theta + x^2(\cos \theta - 1).$$

If θ is small, $s^1 \sim - x^2 \theta$ and $s^2 \sim x^1\theta$. Then

$$s^1 = - \alpha x^2 x^3, \qquad s^2 = \alpha x^1 x^3.$$

From this,

$$\begin{aligned} S_{11} &= 0, \quad S_{22} = 0, \quad S_{33} = 0, \\ S_{12} &= S_{21} = \tfrac{1}{2}(- \alpha x^3 + \alpha x^3) = 0, \\ S_{13} &= S_{31} = \tfrac{1}{2}(- \alpha x^2), \\ S_{23} &= S_{32} = \tfrac{1}{2}(\alpha x^1), \end{aligned}$$

and from Hooke's Law,

$$\begin{aligned} T^{13} &= T^{31} = - \mu \alpha x^2, \\ T^{23} &= T^{32} = \mu \alpha x^1, \end{aligned}$$

and

$$T^{11} = T^{22} = T^{33} = T^{12} = T^{21} = 0.$$

An even simpler method of handling such a problem is provided by transformation to a circular cylinder coordinate system.

$$\begin{aligned} x^{1'} &= r, \\ x^{2'} &= \theta, \\ x^{3'} &= z. \end{aligned}$$

Thus $P(x^{1'}, x^{2'})$ after deformation becomes $\overset{(1)}{P}(x^{1'}, x^{2'} + s^{2'})$, where

$$s^{2'} = \alpha x^{3'}, \qquad s^{1'} = 0, \quad s^{3'} = 0.$$

Then

$$S_{1'1'} = S_{2'2'} = S_{3'3'} = S_{1'2'} = S_{2'1'} = S_{1'3'} = S_{3'1'} = 0$$

and

$$S_{2'3'} = S_{3'2'} = \tfrac{1}{2} g_{2'2'} \alpha = \frac{(x^{1'})^2 \alpha}{2} = \frac{\alpha r^2}{2}.$$

From Hooke's Law,

$$T^{2'3'} = T^{3'2'} = \mu' \alpha (x^{1'})^2 = \mu' \alpha r^2.$$

The solution obtained by the two methods can easily be correlated. In cartesian coordinates

$$\begin{aligned} |T^{ij}| &= \sqrt{g_{ij} g_{kl} T^{ik} T^{jl}} = \sqrt{g_{11} g_{33} (T^{13})^2 + g_{22} g_{33} (T^{23})^2} \\ &= \mu \alpha \sqrt{(x^1)^2 + (x^2)^2} = \mu \alpha r. \end{aligned}$$

In circular cylinder coordinates

$$|T^{i'j'}| = \sqrt{g_{i'j'} g_{k'l'} T^{i'k'} T^{j'l'}} = \sqrt{g_{2'2'} g_{3'3'} (T^{2'3'})^2} = r T^{2'3'} = \mu' \alpha r^3.$$

But

$$\mu' = c^{2'3'2'3'} = \frac{\partial x^{2'}}{\partial x^i} \frac{\partial x^{3'}}{\partial x^j} \frac{\partial x^{2'}}{\partial x^k} \frac{\partial x^{3'}}{\partial x^l} c^{ijkl}.$$

From the transformation equations

$$r = \sqrt{(x^1)^2 + (x^2)^2},$$

$$\theta = \tan^{-1}\frac{x^2}{x^1},$$

$$z = z,$$

it can be shown that

$$\mu' = \frac{\mu}{r^2}.$$

Thus

$$|T^{i'j'}| = \mu\alpha r = |T^{ij}|.$$

II. PIEZOELECTRICITY.

Fundamental Equations.

To clarify the terms used in the *piezoelectric equations* below, let us define certain of them which have not already appeared in the paper.

Electric Field Intensity is expressed by E_i (volts m^{-1}),
Electric Flux Density by D^i (coulomb m^{-2}),
Permittivity by ϵ^{ij} (farad m^{-1}),
Polarization by P^i (coulomb m^{-2}), and
Electric Susceptibility by η^{ij} (farad m^{-1}).

They are related in the following ways:

$$D^i = \epsilon^{ij}E_j, \tag{37a}$$

$$P^i = D^i - \underset{(0)}{\epsilon^{ij}}E_j, \tag{37b}$$

where $\underset{(0)}{\epsilon^{ij}}$ is the permittivity of free space, and

$$\underset{(0)}{\epsilon^{11}} = \underset{(0)}{\epsilon^{22}} = \underset{(0)}{\epsilon^{33}} = 8.854 \times 10^{-12} \text{ farad } m^{-1},$$

$$\underset{(0)}{\epsilon^{ij}} = 0 \quad \text{if} \quad (i \neq j),$$

$$\eta^{ij} = \epsilon^{ij} - \underset{(0)}{\epsilon^{ij}},$$

and hence

$$P^i = \eta^{ij}E_j. \tag{37c}$$

Certain materials manifest a property whereby the existence of mechanical stress or strain is directly associated with the existence of an electric polarization within the material, and with an electric field in the surrounding medium.

It has been found (8) that this property can be adequately de-

scribed in four equations which we shall call the fundamental piezoelectric equations.

$$P^i = \eta^{ij}E_j + e^{ikl}S_{kl}, \tag{I}$$

is the direct effect of polarization caused by a specified applied field and strain.

A similar equation may be written for the direct effect of polarization resulting from an applied field and stress.

$$P^i = \eta^{ij}E_j + d_{kl}{}^iT^{kl}. \tag{II}$$

The converse effect may also be stated in two equations. With both strain and field prescribed,

$$T^{ij} = c^{ijkl}S_{kl} - e^{mij}E_m. \tag{III}$$

With both stress and field prescribed,

$$S_{ij} = b_{ijkl}T^{kl} + d_{ij}{}^mE_m: \tag{IV}$$

The trivalent tensors e^{ijk} and $d_{jk}{}^i$ are known respectively as the *piezoelectric stress coefficient* and the *piezoelectric strain coefficient.*

The stress T^{ij} in Eq. III above is the total external stress caused by the strain S_{kl} and field E_m. However, an external stress is only indirectly caused by E_m, which, since it acts as a body force, causes an internal stress. The body, due to its rigidity, establishes an equal and opposite external stress which is thus a portion of the measurable stress.

The most important piezoelectric substances are crystalline in nature. The condition which determines in which direction piezoelectric effects appear, is physical asymmetry of axes. That is, any axis along which a piezoelectric effect appears must be such that if the crystal is rotated 180° about an axis normal to the given axis, the rotated crystal cannot be superimposed on the unrotated crystal by a pure translation. Thus the axis considered has a distinct directionality, and piezoelectric constants will be zero except along such axes.

Piezoelectricity in Quartz.

All the applications of piezoelectricity to be considered in this paper will be to quartz crystals. The piezoelectric stress and strain coefficients reduce to the following matrices.

For e^{ijk}:

$$e^{1jk} = \begin{pmatrix} e^{111} & 0 & 0 \\ 0 & -e^{122} & e^{123} \\ 0 & e^{132} & 0 \end{pmatrix}, \quad e^{2jk} = \begin{pmatrix} 0 & e^{212} & e^{213} \\ e^{221} & 0 & 0 \\ e^{231} & 0 & 0 \end{pmatrix}, \quad e^{3jk} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$e^{111} = -e^{122} = -e^{221} = -e^{212},$$
$$e^{123} = e^{132} = -e^{231} = -e^{213}.$$

For d_{jk}^{i}:

$$d_{jk}^{1} = \begin{pmatrix} d_{11}^{1} & 0 & 0 \\ 0 & d_{22}^{1} & d_{23}^{1} \\ 0 & d_{32}^{1} & 0 \end{pmatrix}, \quad d_{jk}^{2} = \begin{pmatrix} 0 & d_{12}^{2} & d_{13}^{2} \\ d_{21}^{2} & 0 & 0 \\ d_{31}^{2} & 0 & 0 \end{pmatrix}, \quad d_{jk}^{3} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$d_{11}^{1} = -d_{22}^{1} = -d_{12}^{2} = -d_{21}^{2},$$
$$d_{23}^{1} = d_{32}^{1} = -d_{31}^{2} = -d_{13}^{2}.$$

And for c^{ijkl}:

$$c^{11kl} = \begin{pmatrix} c^{1111} & 0 & 0 \\ 0 & c^{1122} & c^{1123} \\ 0 & c^{1132} & c^{1133} \end{pmatrix}, \quad c^{12kl} = \begin{pmatrix} 0 & c^{1212} & c^{1213} \\ c^{1221} & 0 & 0 \\ c^{1231} & 0 & 0 \end{pmatrix},$$

$$c^{13kl} = \begin{pmatrix} 0 & c^{1312} & c^{1313} \\ c^{1321} & 0 & 0 \\ c^{1331} & 0 & 0 \end{pmatrix}, \quad c^{22kl} = \begin{pmatrix} c^{2211} & 0 & 0 \\ 0 & c^{2222} & c^{2223} \\ 0 & c^{2232} & c^{2233} \end{pmatrix},$$

$$c^{21kl} = \begin{pmatrix} 0 & c^{2112} & c^{2113} \\ c^{2121} & 0 & 0 \\ c^{2131} & 0 & 0 \end{pmatrix}, \quad c^{23kl} = \begin{pmatrix} c^{2311} & 0 & 0 \\ 0 & c^{2322} & c^{2323} \\ 0 & c^{2332} & 0 \end{pmatrix},$$

$$c^{31kl} = \begin{pmatrix} 0 & c^{3112} & c^{3113} \\ c^{3121} & 0 & 0 \\ c^{3131} & 0 & 0 \end{pmatrix}, \quad c^{32kl} = \begin{pmatrix} c^{3211} & 0 & 0 \\ 0 & c^{3222} & c^{3223} \\ 0 & c^{3232} & 0 \end{pmatrix},$$

$$c^{33kl} = \begin{pmatrix} c^{3311} & 0 & 0 \\ 0 & c^{3322} & 0 \\ 0 & 0 & c^{3333} \end{pmatrix},$$

where

$$\begin{aligned}
c^{1111} &= c^{2222}, \\
c^{2323} &= c^{3232} = c^{3223} = c^{1313} = c^{1331} = c^{3113} = c^{3131} = c^{2332}, \\
c^{1122} &= c^{2211}, \\
c^{1133} &= c^{2233} = c^{3311} = c^{3322}, \\
c^{1123} &= c^{1132} = c^{1312} = c^{1321} = c^{3112} = c^{3121} = c^{1213} = c^{1231} = c^{2113} \\
&= c^{2131} = c^{2311} = c^{3211} = -c^{2223} = -c^{2232} = -c^{2322} = -c^{3222}, \\
c^{1212} &= c^{1221} = c^{2112} = c^{2121} = \tfrac{1}{2}(c^{1111} - c^{1122}).
\end{aligned}$$

The only non-vanishing permittivity values are ϵ^{11}, ϵ^{22}, and ϵ^{33} where $\epsilon^{11} = \epsilon^{22}$. Thus electric susceptibility has only three values η^{11}, η^{22}, and η^{33}.

The standard axes in quartz are three Euclidian sets, all possessing a common x^3 or Z-axis. The x^3 axis is ordinarily known as the *optical axis* because of its orientation in the direction of optical activity.

The x^1 axes or X-axes are diametral lines connecting opposite corners of the hexagon formed when an idealized crystal is cut in a plane perpendicular to the m faces (8, p. 26).

The x^2 axes or Y-axes are respectively perpendicular to the three x^1 axes, and are, consequently, perpendicular to the m faces.

The x^1 axes are also known as *electric axes*, and the x^2 axes are known as *mechanical axes*.

Quartz shows the same properties with respect to each of the three sets of Euclidean axes. The constants given above for quartz hold for all three sets of axes; that is, they are invariant to rotations of $\frac{2\pi}{3}$ radians about the x^3 axis.

In actual practice quartz is customarily used in the form of small plates cut in specific orientations with respect to the fundamental sets of axes. The purpose of these various "cuts," as they are called, are diverse, usually having to do with the characteristics under temperature change of the cuts selected. Thus, cuts such as the AT, BT, and GT produce the least variation in frequency under changes in temperature. Another purpose is to obtain cuts which show a high selectivity of vibrational modes.

Analysis of Motion in Quartz Plates.

In order to set up the equations describing the motion of a piezoelectric body, we shall use the following equations which have appeared previously in this paper:

the stress equation of force equilibrium,

$$F_i = g_{ik}\nabla_j T^{kj}, \tag{6b}$$

where F_i is a body force, or force per unit volume;

the defining equation of the strain tensor,

$$S_{ij} = \tfrac{1}{2} g_{ik}\frac{\partial s^k}{\partial x^j} + g_{jk}\frac{\partial s^k}{\partial x^i}\,; \tag{16}$$

and the third piezoelectric equation,

$$T^{ij} = c^{ij\alpha\beta}S_{\alpha\beta} - e^{\gamma ij}E_\gamma. \tag{III}$$

In addition to these equations we shall use Newton's second law, which can be expressed in the form

$$F_i = g_{ij}\rho\frac{\partial^2 s^j}{\partial t^2}, \tag{38}$$

where F_i is again a body force (newton m^{-3}), ρ is the density (kg m^{-3}) of the medium under consideration, and t is the time (sec.).

In orthogonal affine transformations of cartesian coordinates, Eq. 1 becomes

$$F_i = g_{ik}\frac{\partial T^{kj}}{\partial x^j}. \tag{39}$$

Equations 38 and 39 can be combined to obtain

$$g_{ij}\rho \frac{\partial^2 s^j}{\partial t^2} - g_{ik}\frac{\partial T^{kj}}{\partial x^j} = 0. \tag{40}$$

Since we have already limited this portion of the discussion to orthogonal coordinate systems, we may write Eq. 40 in the form

$$\rho \frac{\partial^2 s^i}{\partial t^2} - \frac{\partial T^{ij}}{\partial x^j} = 0. \tag{41}$$

Using Eq. III and substituting for T^{ij} in Eq. 41, we obtain

$$\rho \frac{\partial^2 s^i}{\partial t^2} - c^{ij\alpha\beta}\frac{\partial S_{\alpha\beta}}{\partial x^j} + e^{\gamma ij}\frac{\partial E_\gamma}{\partial x^j} = 0. \tag{42}$$

We may now substitute for $S_{\alpha\beta}$ in Eq. 42 by making use of Eq. 16. This yields the equation

$$\rho \frac{\partial^2 s^i}{\partial t^2} - \frac{c^{ij\alpha\beta}}{2}\left(g_{\alpha\sigma}\frac{\partial^2 s^\sigma}{\partial x^\beta \partial x^j} + g_{\beta\sigma}\frac{\partial^2 s^\sigma}{\partial x^\alpha \partial x^j}\right) + e^{\gamma ij}\frac{\partial E_\gamma}{\partial x^j} = 0. \tag{43}$$

Using the matrices for the elastic and piezoelectric constants in quartz, we can expand Eq. 43 to the following three equations. For $i = 1$,

$$\begin{aligned}
\rho \frac{\partial^2 s^1}{\partial t^2} = {} & c^{1111}\frac{\partial^2 s^1}{(\partial x^1)^2} + c^{1212}\frac{\partial^2 s^1}{(\partial x^2)^2} + c^{1313}\frac{\partial^2 s^1}{(\partial x^3)^2} + 2c^{1213}\frac{\partial^2 s^1}{\partial x^2 \partial x^3} \\
& + (c^{1122} + c^{1212})\frac{\partial^2 s^2}{\partial x^2 \partial x^1} + (c^{1123} + c^{1312})\frac{\partial^2 s^2}{\partial x^1 \partial x^3} \\
& + (c^{1133} + c^{1313})\frac{\partial^2 s^3}{\partial x^1 \partial x^3} + (c^{1123} + c^{1213})\frac{\partial^2 s^3}{\partial x^1 \partial x^2} \\
& - e^{111}\frac{\partial E_1}{\partial x^1} - e^{213}\frac{\partial F_2}{\partial x^3} - e^{212}\frac{\partial E_2}{\partial x^2}.
\end{aligned} \tag{44}$$

For $i = 2$,

$$\begin{aligned}
\rho \frac{\partial^2 s^2}{\partial t^2} = {} & c^{2112}\frac{\partial^2 s^2}{(\partial x^1)^2} + c^{2222}\frac{\partial^2 s^2}{(\partial x^2)^2} + c^{2323}\frac{\partial^2 s^2}{(\partial x^3)^2} + (c^{2223} + c^{2322})\frac{\partial^2 s^2}{\partial x^2 \partial x^3} \\
& + (c^{2112} + c^{2211})\frac{\partial^2 s^1}{\partial x^1 \partial x^2} + (c^{2311} + c^{2113})\frac{\partial^2 s^1}{\partial x^1 \partial x^3} + c^{2113}\frac{\partial^2 s^3}{(\partial x^1)^2} \\
& + c^{2323}\frac{\partial^2 s^3}{(\partial x^2)^2} + (c^{2233} + c^{2323})\frac{\partial^2 s^3}{\partial x^2 \partial x^3} - e^{122}\frac{\partial E_1}{\partial x^2} \\
& - e^{123}\frac{\partial E_1}{\partial x^3} - e^{221}\frac{\partial E_2}{\partial x^1}.
\end{aligned} \tag{45}$$

For $i = 3$,

$$\frac{\partial^2 s^3}{\partial t^2} = c^{3113}\frac{\partial^2 s^3}{(\partial x^1)^2} + c^{3223}\frac{\partial^2 s^3}{(\partial x^2)^2} + c^{3333}\frac{\partial^2 s^3}{(\partial x^3)^2}$$
$$+ (c^{3211} + c^{3112})\frac{\partial^2 s^1}{\partial x^1 \partial x^2} + (c^{3311} + c^{3113})\frac{\partial^2 s^1}{\partial x^1 \partial x^3}$$
$$+ c^{3112}\frac{\partial^2 s^2}{(\partial x^1)^2} + c^{3222}\frac{\partial^2 s^2}{(\partial x^2)^2} + (c^{3322} + c^{3223})\frac{\partial^2 s^2}{\partial x^2 \partial x^3}$$
$$- e^{132}\frac{\partial E_1}{\partial x^2} - e^{231}\frac{\partial E_2}{\partial x^1}. \quad (46)$$

A set of equations which holds in generalized orthogonal coordinate systems can be developed as follows:

$$g_{ij}\rho\frac{\partial^2 s^j}{\partial t^2} = g_{ij}\nabla_k T^{jk}.$$

This may be rewritten as

$$g_{ii}\left(\rho\frac{\partial^2 s^i}{\partial t^2} - \nabla_j T^{ij}\right) = 0,$$

or

$$\rho\frac{\partial^2 s^i}{\partial t^2} - \nabla_j T^{ij} = 0.$$

Expanding on the covariant derivative, we obtain

$$\rho\frac{\partial^2 s^i}{\partial t^2} - \frac{\partial T^{ij}}{\partial x^j} - \Gamma^i_{jl}T^{lj} - \Gamma^j_{jl}T^{il} = 0.$$

Apply Eqs. 16 and III to obtain

$$\rho\frac{\partial^2 s^i}{\partial t^2} - \frac{c^{ij\alpha\beta}}{2}\left(g_{\alpha\sigma}\frac{\partial^2 s^\sigma}{\partial x^\beta \partial x^j} + g_{\beta\sigma}\frac{\partial^2 s^\sigma}{\partial x^\alpha \partial x^j}\right)$$
$$+ e^{\gamma ij}\frac{\partial E_\gamma}{\partial x^j} - \Gamma^i_{jl}T^{lj} - \Gamma^j_{jl}T^{il} = 0.$$

This last equation is quite general since we have put no restrictions on it as to particular coordinate systems. Thus it includes, among others, the cartesian coordinate system, for which the equations have already been developed.

Applications to Piezoelectric Vibrations.

The generalized equations derived above are far too complex for solution by ordinary methods. It is not the purpose of this paper to enter into any discussion of the complete solution of the system of three differential equations which they present. However, by applying appropriate boundary conditions, we shall see that the standard solutions

for the motion of certain idealized bodies can be obtained from these equations; and we shall attempt to obtain some insight into the methods of applying sophisticated boundary conditions in order to obtain solutions which are very close approximations to actual physical conditions.

In general, those terms involving electric field intensity can be neglected, since at resonance they represent much smaller forces than the elastic stresses.

Example I: Let us first consider the *longitudinal vibration* of an infinitely thin rod, which can be expressed by the following boundary conditions:

$$s^1 = f(x^1, t)$$

and

$$s^2 = s^3 = 0.$$

Applying these conditions to Eqs. 44, 45 and 46 above, the only non-trivial expression remaining is

$$\rho \frac{\partial^2 s^1}{\partial t^2} = c^{1111} \frac{\partial^2 s^1}{(\partial x^1)^2},$$

which defines the wave velocity of the motion in the x^1 direction as

$$V_1 = \sqrt{\frac{c^{1111}}{\rho}}.$$

To solve for the frequency of oscillation, we assume that

$$s^1 = g(x^1)h(t).$$

Then

$$\frac{\rho}{h(t)} \frac{\partial^2 h(t)}{\partial t^2} = \frac{c^{1111}}{g(x^1)} \frac{\partial^2 g(x^1)}{(\partial x^1)^2},$$

$$\frac{\partial^2 h(t)}{\partial t^2} = -\frac{m^2}{\rho} h(t),$$

and

$$\frac{\partial^2 g(x^1)}{(\partial x^1)^2} = -\frac{m^2}{c^{1111}} g(x^1).$$

The solutions of these equations are

$$h(t) = A \sin \sqrt{\frac{m^2}{\rho}}\, t + B \cos \sqrt{\frac{m^2}{\rho}}\, t$$

and

$$g(x^1) = c \sin \sqrt{\frac{m^2}{c^{1111}}}\, x^1 + D \cos \sqrt{\frac{m^2}{c^{1111}}}\, x^1.$$

Thus

$$s^1 = \left(A \sin \frac{m}{\sqrt{\rho}} t + B \cos \frac{m}{\sqrt{\rho}} t \right)\left(C \sin \frac{m}{\sqrt{c^{1111}}} x^1 + D \cos \frac{m}{\sqrt{c^{1111}}} x^1 \right).$$

Now let us apply the boundary condition that the rod be clamped at one end.

$$x^1 = 0,$$
$$s^1 = 0.$$

Then

$$AD \sin \frac{m}{\sqrt{\rho}} t + BD \cos \frac{m}{\sqrt{\rho}} t = 0.$$

Either

$$D = 0,$$

or

$$\tan \frac{m}{\sqrt{\rho}} t = -\frac{B}{A}.$$

The latter case is of less practical interest than the first. Thus

$$s^1 = C\left(A \sin \frac{m}{\sqrt{\rho}} t + B \cos \frac{m}{\sqrt{\rho}} t \right) \sin \frac{m}{\sqrt{c^{1111}}} x^1.$$

Let us assume also that at $t = 0$, $s^1 = 0$. This requires that $B = 0$, and hence that

$$s^1 = AC \sin \frac{m}{\sqrt{\rho}} t \sin \frac{m}{\sqrt{c^{1111}}} x^1.$$

If the other end of the rod is an antinode, then at $x^1 = L$,

$$\frac{\partial s^1}{\partial x^1} = 0.$$

This yields the equation

$$\cos \frac{m}{\sqrt{c^{1111}}} L = 0.$$

But

$$\frac{m}{\sqrt{\rho}} = 2\pi f.$$

Then

$$2\pi f \sqrt{\frac{\rho}{c^{1111}}} L = \frac{(2n + 1)\pi}{2}$$

and

$$f = \frac{(2n + 1)}{L} \sqrt{\frac{c^{1111}}{\rho}}.$$

Example II: The pure shear problem, with boundary conditions

$$s^1 = f(x^2)g(t)$$

and

$$s^2 = s^3 = 0,$$

yields the differential equation

$$\rho \frac{\partial^2 s^1}{\partial t^2} = c^{1212} \frac{\partial^2 s^1}{(\partial x^2)^2}.$$

If the body is assumed to be clamped at both x^2 faces, and a time boundary is set, that is, $s^1 = 0$ at $x^2 = \pm b/2$ and $s^1 = 0$ at $t = 0$, the solution becomes

$$f = \frac{n}{b}\sqrt{\frac{c^{1212}}{\rho}},$$

where b is the thickness of the plate.

Example III: A more complex problem and one more applicable to actual conditions uses the boundary conditions

$$s^1 = l(x^1, x^2, x^3, t) \tag{47}$$

and

$$s^2 = s^3 = 0.$$

Equations 44, 45, and 46 above reduce to

$$\rho \frac{\partial^2 s^1}{\partial t^2} = c^{1111} \frac{\partial^2 s^1}{(\partial x^1)^2} + c^{1212} \frac{\partial^2 s^1}{(\partial x^2)^2} + c^{1313} \frac{\partial^2 s^1}{(\partial x^3)^2} + 2c^{1213} \frac{\partial^2 s^1}{\partial x^2 \partial x^3}, \tag{48}$$

$$0 = (c^{2112} + c^{2211}) \frac{\partial^2 s^1}{\partial x^1 \partial x^2} + (c^{2311} + c^{2113}) \frac{\partial^2 s^1}{\partial x^1 \partial x^3}, \tag{49}$$

and

$$0 = (c^{3211} + c^{3112}) \frac{\partial^2 s^1}{\partial x^1 \partial x^2} + (c^{3311} + c^{3113}) \frac{\partial^2 s^1}{\partial x^1 \partial x^3}. \tag{50}$$

Equations 49 and 50 can be combined to show that

$$\frac{\partial^2 s^1}{\partial x^1 \partial x^2} = 0 \tag{51}$$

and

$$\frac{\partial^2 s^1}{\partial x^1 \partial x^3} = 0. \tag{52}$$

Integrate Eq. 51 with respect to x^2 to obtain

$$\frac{\partial s^1}{\partial x^1} = m(x^1, x^3, t). \tag{53}$$

Integrate Eq. 52 with respect to x^3 to obtain

$$\frac{\partial s^1}{\partial x^1} = n(x^1, x^2, t). \tag{54}$$

Thus

$$m(x^1, x^3, t) = n(x^1, x^2, t). \tag{55}$$

But Eq. 55 requires that x^2 and x^3 appear only as constants in Eqs. 53 and 54. Therefore

$$\frac{\partial s^1}{\partial x^1} = h(x^1, t). \tag{56}$$

Integrating with respect to x^1, we obtain

$$s^1 = \int h(x^1, t)dx^1 + g(x^2, x^3, t), \tag{57}$$

which may be written

$$s^1 = f(x^1, t) + g(x^2, x^3, t). \tag{58}$$

Assume now that both f and g are sinusoidal with respect to time.

$$f(x^1, t) = F(x^1) \sin \omega t. \tag{59}$$

$$g(x^2, x^3, t) = G(x^2, x^3) \sin \omega t. \tag{60}$$

Applying Eqs. 59 and 60 to Eq. 48, we obtain

$$- \rho\omega^2[F(x^1) + (Gx^2, x^3)] = c^{1111}\frac{\partial^2 F(x^1)}{(\partial x^1)^2} + c^{1212}\frac{\partial^2 G(x^2, x^3)}{(\partial x^2)^2} + c^{1313}\frac{\partial^2 G(x^2, x^3)}{(\partial x^3)^2} + 2c^{1213}\frac{\partial^2 G(x^2, x^3)}{\partial x^2 \partial x^3}, \tag{61}$$

which may be rewritten in the form

$$c^{1111}\frac{\partial^2 F(x^1)}{(\partial x^1)^2} + \rho\omega^2 F(x^1) + c^{1212}\frac{\partial^2 G(x^2, x^3)}{(\partial x^2)^2} + c^{1313}\frac{\partial^2 G(x^2, x^3)}{(\partial x^3)^2} + 2c^{1213}\frac{\partial^2 G(x^2, x^3)}{\partial x^2 \partial x^3} + \rho\omega^2 G(x^2, x^3) = 0. \tag{62}$$

Since all terms in Eq. 62 are functions alone of x^1 or of the combination of x^2 and x^3, we may separate Eq. 62 into the following two equations:

$$\frac{c^{1111}\partial^2 F(x^1)}{(\partial x^1)^2} + \rho\omega^2 F(x^1) = - m^2 \tag{63}$$

and

$$\frac{c^{1212}\partial^2 G(x^2, x^3)}{(\partial x^2)^2} + c^{1313}\frac{\partial^2 G(x^2, x^3)}{(\partial x^3)^2} + 2c^{1213}\frac{\partial^2 G(x^2, x^3)}{\partial x^2 \partial x^3} + \rho\omega^2 G(x^2, x^3) = m^2, \tag{64}$$

where m^2 is an arbitrary constant.

Assume that $G(x^2, x^3)$ can be written in the form

$$G(x^2, x^3) = A(x^2)B(x^3). \tag{65}$$

If we substitute Eq. 65 into Eq. 64 and divide through by $G(x^2, x^3)$, we obtain

$$\frac{c^{1212}}{A(x^2)}\frac{\partial^2 A(x^2)}{(\partial x^2)^2} + \frac{c^{1313}}{B(x^3)}\frac{\partial^2 B(x^3)}{(\partial x^3)^2} + \rho\omega^2$$

$$= \frac{-2c^{1213}}{A(x^2)B(x^3)}\left[\frac{\partial A(x^2)}{\partial x^2}\frac{\partial B(x^3)}{\partial x^3} - \frac{m^2}{2c^{1213}}\right]. \quad (66)$$

The solution of Eq. 63 is

$$F(x^1) = T \sin\sqrt{\frac{\rho\omega^2}{c^{1111}}}\,x^1 + U\cos\sqrt{\frac{\rho\omega^2}{c^{1111}}}\,x^1 - \frac{m^2}{\rho\omega^2}. \quad (67)$$

Equation 66 has the general form

$$A(x) + B(y) + C = M(x)N(y) + R(x)\mathfrak{Z}(y), \quad (68)$$

where C is a constant. Differentiating with respect to x in Eq. 66, we obtain

$$\frac{\partial A(x)}{\partial x} = N(y)\frac{\partial M(x)}{\partial x} + \mathfrak{Z}(y)\frac{\partial R(x)}{\partial x}. \quad (69)$$

It is evident that in order that Eq. 69 be satisfied, either $N(y)$ and $\mathfrak{Z}(y)$ are constants or $R(x)$ and $M(x)$ are constants, or both. In other words, with the initial assumptions of the problem, only one shear motion is possible.

Let us assume

$$\frac{m^2}{B(x^3)} = k, \quad (70)$$

where k is a constant. Equation 66 reduces to

$$\frac{c^{1212}}{A(x^2)}\frac{\partial^2 A(x^2)}{(\partial x^2)^2} + \rho\omega^2 = \frac{k}{A(x^2)} \quad (71)$$

or

$$c^{1212}\frac{\partial^2 A(x^2)}{(\partial x^2)^2} + \rho\omega^2 A(x^2) = k. \quad (72)$$

The solution of Eq. 72 is similar to that of Eq. 63.

$$A(x^2) = V^1 \sin\sqrt{\frac{\rho\omega^2}{c^{1212}}}\,x^2 + W^1\cos\sqrt{\frac{\rho\omega^2}{c^{1212}}}\,x^2 + \frac{k}{\rho\omega^2}. \quad (73)$$

Multiply Eq. 73 by $B(x^3) = \frac{m^2}{k}$ to obtain

$$G(x^2, x^3) = V \sin\sqrt{\frac{\rho\omega^2}{c^{1212}}}\,x^2 + W\cos\sqrt{\frac{\rho\omega^2}{c^{1212}}}\,x^2 + \frac{m^2}{\rho\omega^2}. \quad (74)$$

The displacement s^1, can now be expressed as

$$s^1 = \left[T \sin \sqrt{\frac{\rho\omega^2}{c^{1111}}}\, x^1 + U \cos \sqrt{\frac{\rho\omega^2}{c^{1111}}}\, x^1 \right.$$
$$\left. + V \sin \sqrt{\frac{\rho\omega^2}{c^{1212}}}\, x^2 + W \cos \sqrt{\frac{\rho\omega^2}{c^{1212}}}\, x^2 \right] \sin \omega t. \quad (75)$$

To solve for frequency we must know the dimensions of the plate and the method of mounting. Let us assume the dimensions to be as follows:

length $= l$ in the x^1 direction,
width $= b$ in the x^3 direction,
thickness $= u$ in the x^2 direction,

with the origin of the coordinate system located at a corner of the crystal.

Probably the most common method of mounting crystal plates is by clamping at all corners. Thus $s^1 = 0$, at each of the following points:

$(0, 0, 0)$, $(0, u, 0)$, $(l, 0, 0)$, $(l, u, 0)$,
$(l, 0, b)$, (l, u, b), $(0, 0, b)$, $(0, u, b)$.

Applying these to Eq. 75, we obtain the following equations:

$$U = -V, \quad (76)$$

$$U = \frac{T \sin \sqrt{\frac{\rho\omega^2}{c^{1111}}}\, l}{1 - \cos \sqrt{\frac{\rho\omega^2}{c^{1111}}}\, l}, \quad (77)$$

and

$$W = \frac{V \sin \sqrt{\frac{\rho\omega^2}{c^{1212}}}\, u}{1 - \cos \sqrt{\frac{\rho\omega^2}{c^{1212}}}\, u}. \quad (78)$$

As a final assumption to obtain the frequency of oscillation, consider the motion to be symmetrical about a vertical filament through the geometrical center of the rectangular crystal. That is, let

$$s^1 = 0 \quad \text{at} \quad \left(\frac{l}{2}, \frac{u}{2}, x^3\right). \quad (79)$$

Applying Eqs. 76, 77, 78, and 79 to Eq. 75, we obtain

$$\cos \frac{u}{2} \sqrt{\frac{\rho\omega^2}{c^{1212}}} = \cos \frac{l}{2} \sqrt{\frac{\rho\omega^2}{c^{1111}}}. \quad (80)$$

From Eq. 80 we may state that

$$2n\pi \pm \frac{u}{2}\sqrt{\frac{\rho\omega^2}{c^{1212}}} = \frac{l}{2}\sqrt{\frac{\rho\omega^2}{c^{1111}}}, \tag{81}$$

where n is any integer. Thus

$$f = \frac{n}{\frac{l}{2}\sqrt{\frac{\rho}{c^{1111}}} \mp \frac{u}{2}\sqrt{\frac{\rho}{c^{1212}}}}. \tag{82}$$

It is evident then, that by careful selection of boundary conditions, Eqs. 44, 45, and 46 can be simplified to provide solutions for almost any desired modes of oscillation.

III. CONCLUSIONS.

This paper has considered a systematic tensor development of elasticity, and a statement in tensor terms of piezoelectric theory. Simple applications have been made to all of the elementary elastic deformation problems. A set of equations of motion has been set up by the combination of the elastic and piezoelectric equations previously derived, with Newton's laws of motion. Specific boundary conditions have been placed upon these differential equations to obtain solutions for displacement and frequency of natural harmonic motion of bodies with particular dimensions and specific constraining conditions.

The possibility of further applications and extensions of the work begun here is evident. The methods are the application of different and perhaps more sophisticated boundary conditions. Such development is not directly an application of the tensor theory, but falls more in the realm of the solution of partial differential equations. The quantity of empirical data being collected on these topics today is adequate evidence of its practical value.

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STRAIN ENERGY IN GREATLY DEFORMED ELASTIC OR INELASTIC ANISOTROPIC ENGINEERING METALS.

BY

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SUMMARY.

The equilibrating stresses on an *orthogonal* element in a *strained* body induce normal strains defined as ***relative displacement per unit length of deformed body.*** This vector "straining"-displacement must be a point function for continuity of the solid. The work done by the forces on the faces of an orthogonal element of unit volume in the loaded body during a differential increase of deformation is found. Integration gives the total work done on the element from the initial unstrained state up to the current state of deformation.

1. INTRODUCTION.

The idea of using strain energy in the study of stress-strain appears to be due to Poncelet (**1**, p. 534) [2] in 1830 studying the effect of impactive simple tension. Saint Venant from about 1850 gave final form to the classical equations which consider only differential displacements and strains in isotropic elastic materials and it was then a short step to utilize Poncelet's idea with the general stress-strain equations. Kelvin (**2**) broadened the energy conception by introducing the theorem of minimum strain energy. Saint Venant recognized the severe physical limitations imposed by the theory utilizing differentially small strains and displacements and evolved a theory (**1**, p. 864), but he was not successful in removing the restrictions if we can accept Todhunter's comment that the expressions apply only when the strains are small although the rates of change of displacement with position can be large (**1**, p. 865). The same comment will apply to Filon's large strains theory (**3**) which considers the co-ordinate point as referring to the deformed body, whereas in Saint Venant's method the co-ordinate point is in the unstrained body. Saint Venant's and Filon's theories lead to non-linear strain-displacement expressions so that the solution of particular elastic problems is mathematically complicated as seen in Seth's paper (**19**). Murnaghan in a recent paper (**4**) accepted Filon's expressions but then concentrated attention on the elastic strain energy function using a power series having one more term than in the classical small strain theory. Sokolnikoff states (**20**) that it is difficult to give a geometrical interpretation to such strains as evolved in the theories of Saint Venant, Filon and Murnaghan. For this same reason the present writer has difficulty in seeing if stress-strain compatibility has been achieved in these works.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

The expressions for energy presented in the present paper follow from the stress-strain-displacement theories due to the writer and now awaiting publication (5,6,7,8,9,10). These general ideas follow on from a theory giving stress-strain compatibility for isotropic metals yielded under small strains and displacements (11). The group of theories removes the classical restrictions by allowing the analysis of any finite displacements and strains in elastic or inelastic anisotropic engineering

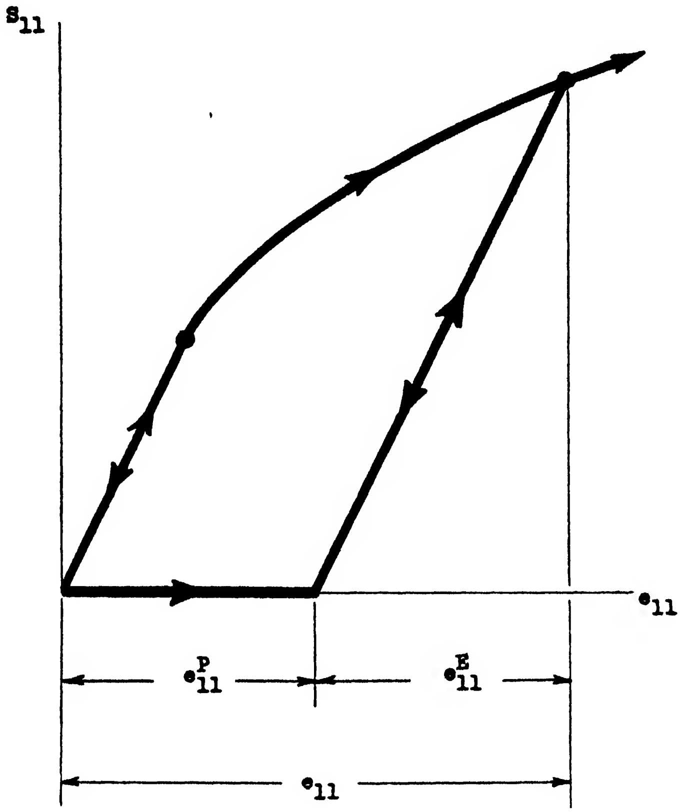

FIG. 1. Elastic and plastic components of inelastic strains in the one-stress system.

metals while still retaining linear forms in the equations. In view of delay in the publication of these theories some space has been taken up in the present paper by an abbreviated recapitulation of the ideas involved.

The degree of generality aimed at has led to more complicated notation than is usual when only the slightly strained isotropic elastic solid is considered. Vector methods were selected as most convenient to retain the physical picture and to give the simplest expressions. It is hoped the reader will be patient with the subscript notation used to ensure brevity.

2. STRESS-STRAIN COMPATIBILITY (10, 8).

Consider an *orthogonal* element in a loaded one-stress test piece. The load is applied in the direction $\mathbf{c}_1$ of an orthogonal system of unit vectors $\mathbf{c}_1$, $\mathbf{c}_2$, $\mathbf{c}_3$ and induces the "true" normal stress S_{11}.

Now define the "true" normal strain

$$e = \frac{\text{Stretched length} - \text{Initial length}}{\text{Stretched length}}. \tag{2.1}$$

Experiment shows that a strained metal is stable well into the yielded range as pointed out by Brix (1, p. 462) in 1837 and Taylor and Quinney (12) in 1932. Further, in the one-stress test the inelastic strains can be split into elastic and plastic components as in Fig. 1.

$$e_{11} = e_{11}^E + e_{11}^P. \tag{2.2}$$

Hence regard each component as accompanied by an appropriate transverse strain in the usual *static* equilibrium fashion so that the strains in the directions $\mathbf{c}_2$, $\mathbf{c}_3$ are respectively

$$\begin{aligned} e_{22,11} &= -q_{22,11}e_{11}^E - p_{22,11}e_{11}^P, \\ e_{33,11} &= -q_{33,11}e_{11}^E - p_{33,11}e_{11}^P, \end{aligned} \tag{2.3}$$

where using true strains the q's are "true" Poisson elastic transverse contraction ratios while the p's are the "true" plastic transverse contraction ratios (11,13) appropriate to the various directions to allow for anisotropy of the stress-strain parameters.

Now consider an *orthogonal* element in equilibrium under a complex true normal stress system S_{11}, S_{22}, S_{33} for which the unit reference vectors can be any orthogonal set taking the directions appropriate to the co-ordinate system to which the deformed body is referred. Suppose as the basic hypothesis for the succeeding theory that in the engineering metals considered *a normal stress is accompanied by a normal strain in the direction of the stress and by appropriate orthogonally transverse contraction effects.* Then in the absence of any experimental evidence to the contrary it is hypothesized that effects shown in (2.2) and (2.3) can be superimposed to give the total strain in direction $\mathbf{c}_i$ as

$$\begin{gathered} e_{ii} = \epsilon_{ii} + \epsilon_{jj,ii} + \epsilon_{kk,ii}, \\ i = 1, 2, 3, \qquad j = 2, 3, 1, \qquad k = 3, 1, 2, \end{gathered} \tag{2.4}$$

where ϵ_{ii} is the "partial" strain (11) that would operate in the direction $\mathbf{c}_i$ in the absence of transverse effects from the other two orthogonal directions $\mathbf{c}_j$, $\mathbf{c}_k$.

Now consider an *orthogonal* element in equilibrium under only a complex shear stress system S_{ij}, S_{ji}, where $i = 1, 2, 3, j = 2, 3, 1$. Resolve the shear stresses into components $T_{ij}/\sqrt{2}$, $-T_{ji}/\sqrt{2}$ in the direc-

tions c_{ij}, c_{ji} respectively of planes bisecting the right angle between the planes on which the shear stresses act as in Fig. 2. When these stresses are transformed into normal stresses acting on areas normal to the c_{ij}, c_{ji} then they have values T_{ij}, $-T_{ji}$ of the same arithmetical values as the shear stresses S_{ij}, S_{ji} as was first pointed out by Saint Venant (1). It is convenient to call these normal stresses T_{ij}, T_{ji} the "auxiliary" normal stresses of the shear stresses S_{ij}, S_{ji}. Compatible with the auxiliary normal stresses will be "auxiliary" normal strains e_{ij}, e_{ji} in

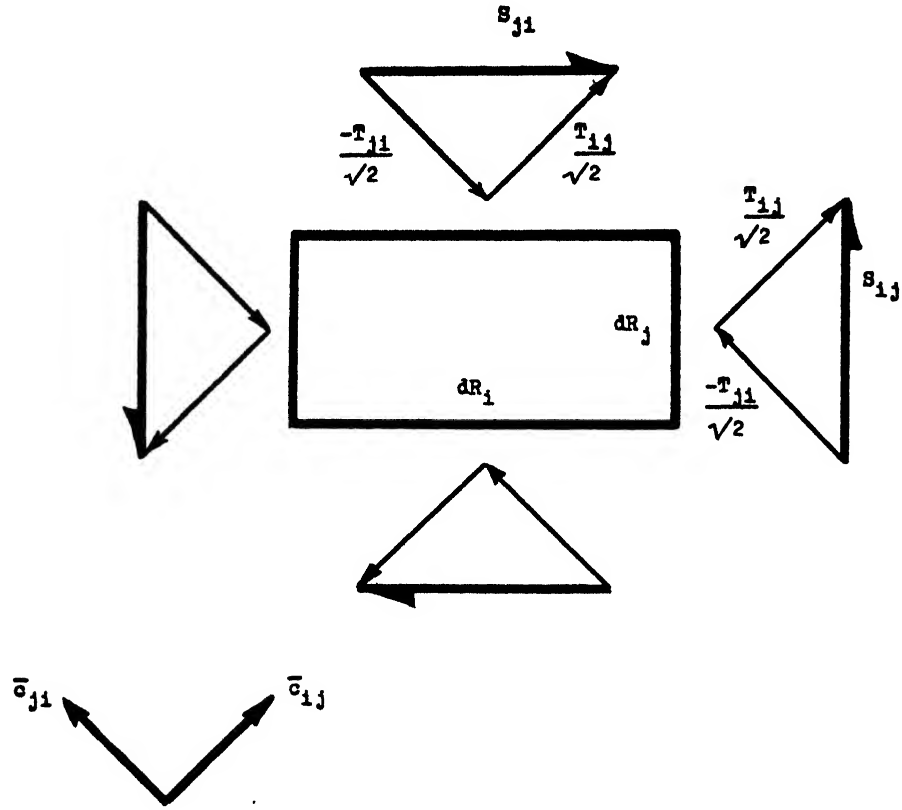

FIG. 2. Components of shear stresses acting in the directions bisecting the right angle between the planes.

the directions c_{ij}, c_{ji} respectively. Following precisely the same argument as previously we have

$$\left.\begin{aligned} e_{ij} &= \epsilon_{ij} + \epsilon_{ji,ij}, \\ e_{ji} &= \epsilon_{ji} + \epsilon_{ij,ji}, \\ e_{ij,kk} &= \epsilon_{ij,kk} + \epsilon_{ji,kk}, \end{aligned}\right\} \tag{2.5}$$

where the last term is the transverse strain in direction c_k normal to the plane of c_{ij}, c_{ji}.

If now a complex system of both normal and shear stresses operates

on an *orthogonal* element then it is hypothesized that the strains in Eqs. 2.4 and 2.5 must occur simultaneously for stress-strain compatability.

Now consider the elastic and plastic components of the inelastic strains. Suppose the progressive straining of a body is referred in general to a curvilinear co-ordinates system in which the orthogonal unit reference vectors $\mathbf{c}_i$ take local directions at each point considered. Then the current *point* of the deformed body at each instantaneous position is associated with an orthogonal element kept in equilibrium by

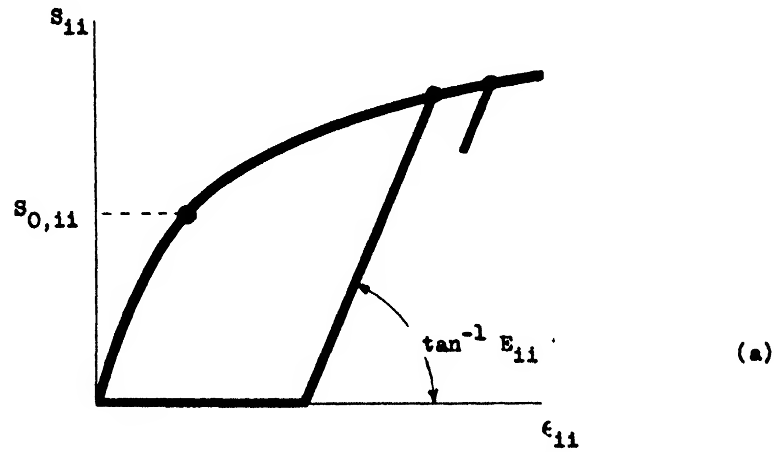

FIG. 3(*a*). Typical stress S_{ii} plotted against the corresponding "partial" strain ϵ_{ii}.

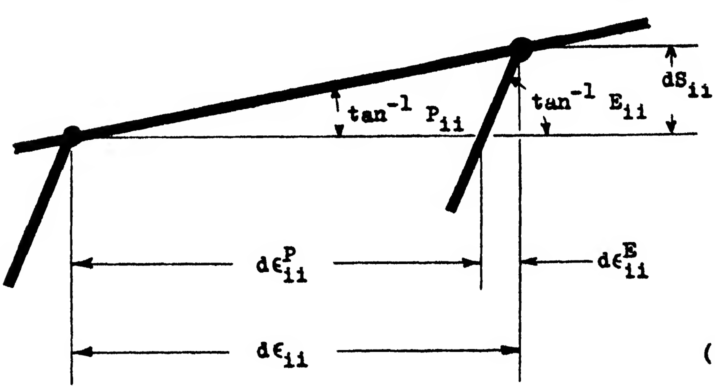

FIG. 3(*b*).

the instantaneous stresses S_{ii}, S_{ij} operating on its faces. In general *the orientation of the orthogonal element will change as the current point moves from point-to-point and if of a given size it will not always contain the same particles of the body.* As the straining proceeds the curve of the typical stress S_{ii} plotted against the corresponding "partial" strain ϵ_{ii} will have something the form shown in Fig. 3(*a*). At some instant the metal about the point will yield and the typical stress on the element will

have the value $S_{0,ii}$. From the definition of elastic strain as that part recoverable by release of load and the physical fact that Young's elastic modulus is approximately linear during this change then

$$\epsilon_{ii}{}^E = \frac{S_{ii}}{E_{ii}}. \tag{2.6}$$

But by definition the plastic part of the inelastic strain is the non-recoverable part with release of load. Hence considering Fig. 3(*b*)

$$\begin{aligned} d\epsilon_{ii}{}^P &= d\epsilon_{ii} - d\epsilon_{ii}{}^E \\ &= \left(\frac{1}{P_{ii}} - \frac{1}{E_{ii}}\right) dS_{ii} \end{aligned}$$

and integration gives

$$\epsilon_{ii}{}^P = \int_{S_{0\,ii}}^{S_{ii}} \left(\frac{1}{P_{ii}} - \frac{1}{E_{ii}}\right) dS_{ii} = I_{ii} \quad \text{say.} \tag{2.7}$$

Notice that an expression such as (2.7) considered for the auxiliary stresses T_{ij} will be referred to the shear stresses S_{ij} in virtue of the arithmetical equality of these stresses. However the parameters P_{ij}, E_{ij} correspond to directions c_{ij} in the plane bisecting the right angle between planes on which operate the shear stresses S_{ij}, S_{ji}. Substitute such expressions as (2.6), (2.7) in (2.4), (2.5) noting (2.2), (2.3) to give

$$\left.\begin{aligned} e_{ii} &= \frac{S_{ii}}{E_{ii}} - q_{jj\,ii}\frac{S_{jj}}{E_{jj}} - q_{kk,ii}\frac{S_{kk}}{E_{kk}} + I_{ii} - p_{jj\,ii}I_{jj} - p_{kk\,ii}I_{kk}, \\ e_{ij} &= \frac{S_{ij}}{E_{ij}} + q_{ji,ij}\frac{S_{ji}}{E_{ji}} + I_{ij} + p_{ji,ij}I_{ji}, \\ e_{ji} &= -\frac{S_{ji}}{E_{ji}} - q_{ij,ji}\frac{S_{ij}}{E_{ij}} - I_{ji} - p_{ij,ji}I_{ij}, \\ e_{ij,kk} &= -q_{ij,kk}\frac{S_{ij}}{E_{ij}} + q_{ji,kk}\frac{S_{ji}}{E_{ji}} - p_{ij\,kk}I_{ij} + p_{ji,kk}I_{ji}, \\ &\qquad i = 1, 2, 3, \qquad j = 2, 3, 1, \qquad k = 3, 1, 2. \end{aligned}\right\} \tag{2.8}$$

Note that in an isotropic metal for which the tensile and compressive parameters are equal $e_{ij,kk}$ is zero and that e_{ij} and e_{ji} are equal but of opposite sign.

3. STRAIN-DISPLACEMENT COMPATIBILITY (7, 8, 9).

An *orthogonal* element in a *deformed* body is maintained in equilibrium by the complex stresses operating on its faces. The discussion of paragraph 2 indicated that in the presence of a stress there must be a strain. Thus due to the true *normal* stresses S_{ii} operating on an or-

thogonal element with edges of length dR_1, dR_2, dR_3 there must be strains e_{ii} operating. But from Eq. 2.1 defining "true" strain the differential relative "normal" displacement of the diagonally opposite corners of the orthogonal element will be

$$d\mathbf{D}^N = \sum_{i=1}^{3} e_{ii} dR_i \mathbf{c}_i. \tag{3.1}$$

The normal stresses T_{12}, T_{21} "auxiliary" to the shear stresses S_{12},

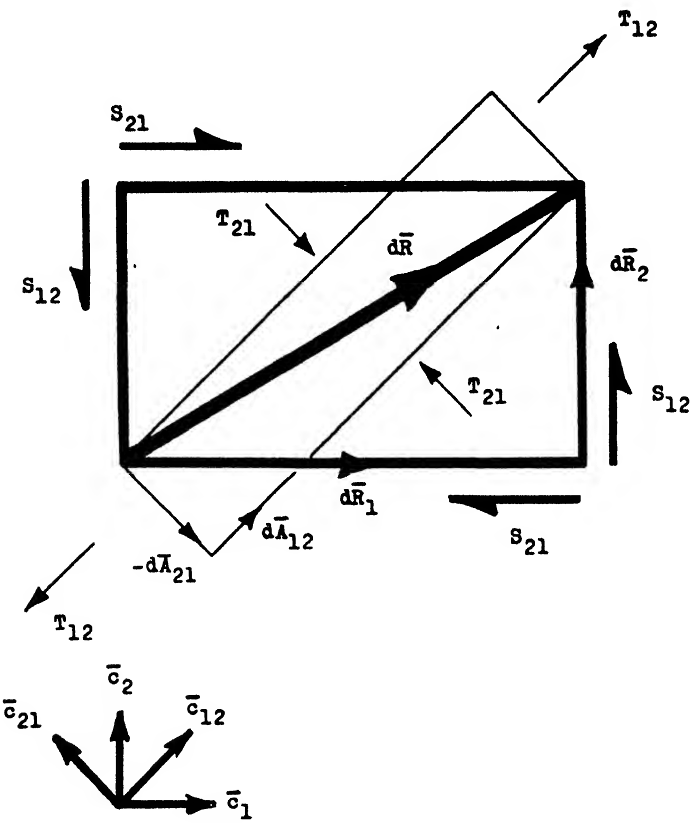

FIG. 4. Normal stresses T_{12}, T_{21} are "auxiliary" to the shear stresses S_{12}, S_{21}.

S_{21} are shown in Fig. 4. As shown T_{12} is a tension and T_{21} is a compression. Consider the *orthogonal* element with edges of length dR_1, dR_2, dR_3. Differential position vector $d\mathbf{R}$ is parallel to the plane of $\mathbf{c}_1$, $\mathbf{c}_2$. Hence from the geometry of Fig. 4

$$d\mathbf{R}_1 + d\mathbf{R}_2 = d\mathbf{R} = d\mathbf{A}_{12} - d\mathbf{A}_{21}, \tag{3.2}$$

where the $d\mathbf{A}$'s are in the directions of the auxiliary normal stresses. Compatible with these auxiliary stresses must be auxiliary strains e_{12}, e_{21}.

From the definition of normal strains and displacements there will be a relative differential *normal* displacement between the ends of $d\mathbf{R}$,

$$e_{12}d\mathbf{A}_{12} - e_{21}d\mathbf{A}_{21} = \tfrac{1}{2}d\mathbf{R}\cdot[(e_{12}+e_{21})(\mathbf{c}_1\mathbf{c}_1+\mathbf{c}_2\mathbf{c}_2) + (e_{12}-e_{21})(\mathbf{c}_1\mathbf{c}_2+\mathbf{c}_2\mathbf{c}_1)] \quad (3.3)$$

after resolution to the $\mathbf{c}_1$, $\mathbf{c}_2$ directions using (3.2). Also in the direction $\mathbf{c}_3$ is a strain $e_{12,33}$ as in (2.5). Compatible with this strain will be a displacement

$$dR_3 e_{12,33}\mathbf{c}_3. \quad (3.4)$$

Hence if now the differential vector $d\mathbf{R}$ is the diagonal of the orthogonal

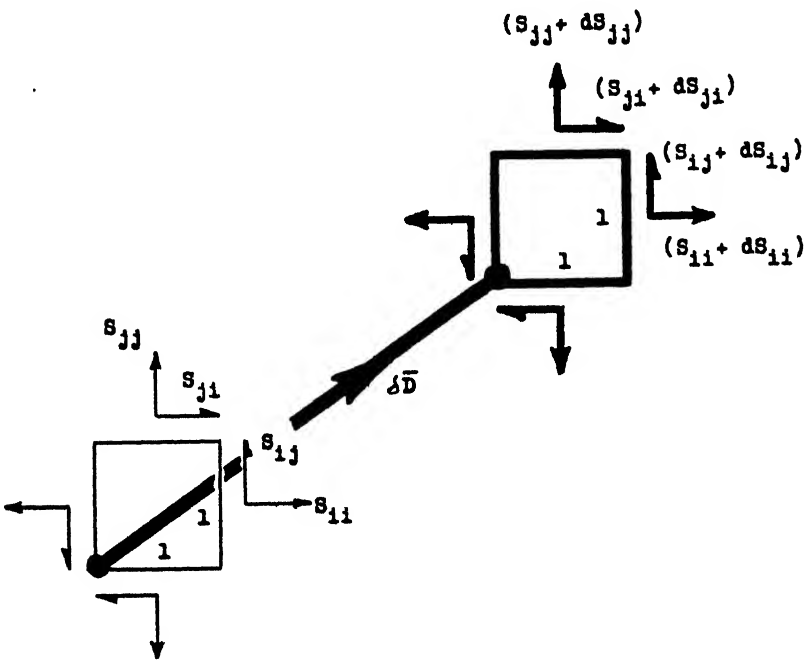

FIG. 5.

element the total differential relative displacement between its ends due to the shear stress system will be found by adding (3.3) and (3.4) and then summing for all directions to give

$$d\mathbf{D}^s = d\mathbf{R}\cdot \sum_{\substack{i=1,2,3\\ j=2,3,1\\ k=3,1,2}} \{\tfrac{1}{2}[(e_{ij}+e_{ji})(\mathbf{c}_i\mathbf{c}_i+\mathbf{c}_j\mathbf{c}_j) + (e_{ij}-e_{ji})(\mathbf{c}_i\mathbf{c}_j+\mathbf{c}_j\mathbf{c}_i)] + e_{ij,kk}\mathbf{c}_k\mathbf{c}_k\}. \quad (3.5)$$

If now a complex stress system S_{ii}, S_{ij} acts on an *orthogonal* element of a *deformed* body, then the strains compatible with the stresses will also be compatible with the straining-displacement $\mathbf{D}$. Hence by adding (3.1) to (3.5) is found the total differential relative displacement be-

tween the ends of the differential position vector $d\mathbf{R}$ diagonal to the orthogonal element

$$d\mathbf{D} = d\mathbf{D}^N + d\mathbf{D}^S. \tag{3.6}$$

But in a solid deformed body the displacement must be a point function to ensure continuity so that

$$d\mathbf{D} = d\mathbf{R}\cdot\nabla\mathbf{D}, \tag{3.7}$$

where ∇ is the vector gradient operator

$$\sum_{i=1}^{3} \mathbf{c}_i \frac{\partial}{\partial R_i}.$$

Hence substituting (3.7), (3.1) and (3.5) in (3.6) and noting that $d\mathbf{R}$ is arbitrary gives

$$\nabla\mathbf{D} = \sum_{\substack{i=1,2,3\\ j=2,3,1\\ k=3,1,2}} \{e_{ii}\mathbf{c}_i\mathbf{c}_i + e_{ij,kk}\mathbf{c}_k\mathbf{c}_k + \tfrac{1}{2}[(e_{ij} + e_{ji})(\mathbf{c}_i\mathbf{c}_i + \mathbf{c}_j\mathbf{c}_j) + (e_{ij} - e_{ji})(\mathbf{c}_i\mathbf{c}_j + \mathbf{c}_j\mathbf{c}_i)]\}. \tag{3.8}$$

This is the displacement-strain compatibility expression for any finite displacements and strains in anisotropic metals. In expansions for any particular orthogonal co-ordinate system it must be remembered that the $\mathbf{c}_i$ have local values appropriate to the current point considered and their rates of change with position calculated if the scalar expressions are to be separated.

The displacement $\mathbf{D}$ is seen as due only to the stresses inducing relative displacements $d\mathbf{D}$ between adjacent points. Thus rigid-body translation and rotation of the overall deformed body must be excluded. This is secured by causing *the axes of the strain ellipsoid at any one arbitrary point in the deformed body to coincide with the corresponding line elements of different curvilinearity in the unstrained body.* The existence of the strain ellipsoid in the case of strains of any finite magnitude as considered here is shown by considering the differential position vector $d\mathbf{R}'$ in the unstrained body corresponding to $d\mathbf{R}$ in the strained body. Geometry gives

$$d\mathbf{R}' = d\mathbf{R} - d\mathbf{D}. \tag{3.9}$$

Substitute for $d\mathbf{D}$ from (3.8) and (3.7) and then write down a similar expression using the conjugate dyadic $\mathbf{D}\nabla$. Form the scalar product of these two expressions and noting that the dyadics are self-conjugate the existence of the strain ellipsoid is proven (**14**, p. 94).

4. DISPLACEMENT-STRESS COMPATIBILITY.

Paragraphs 2 and 3 discussed the conditions for stress-strain and strain-displacement compatibility, respectively. Substituting the appropriate expressions gives the conditions to assure displacement-stress compatibility. This may be expressed

$$\nabla\mathbf{D} = \bar{\phi}, \tag{4.1}$$

where $\bar{\phi}$ is the dyadic found when the strain expressions (2.8) are substituted in (3.8). The dyads are of unit vectors $\mathbf{c}_i$ appropriate to the orthogonal reference system chosen and with coefficients of scalar stresses S_{ii}, S_{ij}.

An interesting result follows from taking the curl of (4.1). By a well-known result for any vector point function (14, p. 122)

$$\text{curl grad } \mathbf{D} = \nabla \times (\nabla\mathbf{D}) = 0. \tag{4.2}$$

This should be compared with Lame's analysis (14, p. 2) showing that the infinitesimal displacement in the classical theory is a biharmonic. Evidently Lame's higher order equation follows from the inclusion of rigid-body displacements whereas in the present theory they are excluded.

Again from (4.1) and (4.2)

$$\text{curl } \bar{\phi} = 0. \tag{4.3}$$

But for rigid body equilibrium of the element the stress dyadic $\bar{\psi}$ and body forces $\mathbf{F}$ obey the well-known expression (14, p. 142)

$$\text{div } \bar{\psi} + m\mathbf{F} = 0, \tag{4.4}$$

where m is the density of metal at the current point. Hence in a particular solution the stress system must obey Eqs. 4.3 and 4.4 simultaneously.

5. STRAIN ENERGY.

The analysis of the general energy case when the co-ordinate system is curvilinear follows the same lines as discussed in this paragraph for a cartesian system except that variations in the unit reference vectors $\mathbf{c}_i$ must be considered. The manipulation is much more complicated than what follows for the cartesian case and since the final expressions derived are of the same *form* except for a further integral term allowing for the way the unit vectors vary during the deformation, then it is not considered worth-while here since brevity is important. The general treatment will be given in a treatise (15) now in preparation.

A point at $\mathbf{R}$ in the deformed body has suffered the vector straining-displacement $\mathbf{D}$. A further differential displacement $\delta\mathbf{D}$ of the point is allowed. Thus Eq. 4.1 with Eq. 3.7 gives

$$d\mathbf{D} = d\mathbf{R}\cdot\bar{\phi}; \tag{5.1}$$

then due to the variation

$$d(\mathbf{D} + \delta\mathbf{D}) = d\mathbf{R}\cdot(\bar{\phi} + \delta\bar{\phi}) \tag{5.2}$$

when the co-ordinate system is cartesian and the element defined about

the current point is always of the same dimensions so that $d\mathbf{R}$ is not varied. Then (5.1) in (5.2) gives

$$d(\delta\mathbf{D}) = d\mathbf{R}\cdot\delta\bar{\phi}. \tag{5.3}$$

Now in particular to simplify the algebra and discussion, choose the orthogonal element with equal edges of unit length so that the true stresses S_{ii}, S_{ij} multiplied by unity give the *forces* operating on the faces of the element. Consider the work done by the normal force S_{ii}

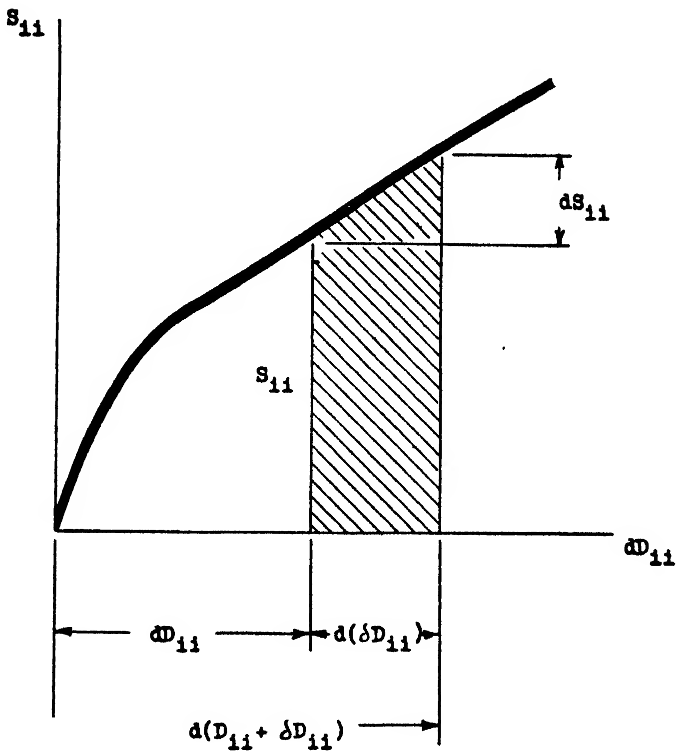

FIG. 6. Complex-stress typical direction.

operating on the face normal to $\mathbf{c}_i$ due to the variation in displacement. From Fig. 6 the work done will be

$$S_{ii}d(\delta D_{ii}).$$

Hence considering the work done on both faces normal to the $\mathbf{c}_i$ direction with the $d\mathbf{R}$ in Eq. 5.3 taking the values $\frac{1}{2}(\mathbf{c}_j + \mathbf{c}_k)$ and $(\mathbf{c}_i + \frac{1}{2}\mathbf{c}_j + \frac{1}{2}\mathbf{c}_k)$ appropriate to the faces the strain energy due to this stress is

$$dU_{ii} = S_{ii}\mathbf{c}_i\cdot\delta\bar{\phi}\cdot\mathbf{c}_i. \qquad (i = 1, 2, 3). \tag{5.4}$$

Similarly for the shear stresses

$$
\begin{aligned}
dU_{ij} &= S_{ij}\mathbf{c}_i \cdot \delta\bar{\phi} \cdot \mathbf{c}_j, \\
dU_{ji} &= S_{ji}\mathbf{c}_j \cdot \delta\bar{\phi} \cdot \mathbf{c}_i.
\end{aligned}
\qquad (5.5)
$$

$$
\begin{aligned}
(i &= 1, 2, 3) \\
(j &= 2, 3, 1).
\end{aligned}
$$

The total differential strain energy is found by summing all such as (5.4) and (5.5) to give

$$
dU = \sum_{\substack{i=1,2,3\\ j=2,3,1}} [S_{ii}\mathbf{c}_i \cdot \delta\bar{\phi} \cdot \mathbf{c}_i + S_{ij}\mathbf{c}_i \cdot \delta\bar{\phi} \cdot \mathbf{c}_j + S_{ji}\mathbf{c}_j \cdot \delta\bar{\phi} \cdot \mathbf{c}_i]. \qquad (5.6)
$$

But in terms of strain variation from Eq. 3.8 since the co-ordinate system is cartesian with the $\mathbf{c}_i$ constant,

$$
\begin{aligned}
\delta\bar{\phi} = \sum_{\substack{i=1,2,3\\ j=2,3,1\\ k=3,1,2}} [\{\delta e_{ii}\mathbf{c}_i\mathbf{c}_i + \delta e_{ij,kk}\mathbf{c}_k\mathbf{c}_k + \tfrac{1}{2}[(\delta e_{ij} + \delta e_{ji})(\mathbf{c}_i\mathbf{c}_i + \mathbf{c}_j\mathbf{c}_j) \\
+ (\delta e_{ij} - \delta e_{ji})(\mathbf{c}_i\mathbf{c}_j + \mathbf{c}_j\mathbf{c}_i)]\}. \qquad (5.7)
\end{aligned}
$$

Substitute (5.7) in (5.6) and forming the scalar products and then integrating between the initial and current deformation condition after noting that S_{ij} equals S_{ji} gives

$$
\begin{aligned}
U = \int \sum_{\substack{i=1,2,3\\ j=2,3,1\\ k=3,1,2}} [S_{ii}de_{ii} + S_{kk}de_{ij\ kk} + \tfrac{1}{2}(S_{ii} + S_{jj})(de_{ij} + de_{ji}) \\
+ S_{ij}(de_{ij} - de_{ji})], \qquad (5.8)
\end{aligned}
$$

in which d can now replace δ for the purpose of the usual notation of integration. This is the general expression for the strain energy per unit volume in any finite deformation. Further changes in the form of the expression depend on the stress-strain relationships and parameters chosen. The differential variations of the strain equations (2.8) can be used to give the general strain energy expression for anisotropic yielded metal. However these are clearly complicated expressions due to variations in the stress-strain parameters and for the present one may consider the simpler but useful case when the metal can be considered as isotropic.

Strain Energy in an Isotropic Metal with Tension and Compression Parameters of Equal Values.

In an isotropic metal with the tension and compression parameters of equal values the auxiliary strains will be of equal values but of opposite sign in virtue of the stresses. Thus

$$
\begin{aligned}
de_{ij} + de_{ji} &= 0, \\
de_{ij,kk} &= 0,
\end{aligned}
\qquad (5.9)
$$

so that (5.8) becomes

$$
U = \int \sum_{\substack{i=1,2,3\\ j=2,3,1}} [S_{ii}de_{ii} + 2S_{ij}de_{ij}]. \qquad (5.10)
$$

The variations in strains de_{ii}, de_{ij} are found from (2.8) without subscripts to the stress-strain parameters. However although the parameters are isotropic at a point they are not in general constant with change of position in the deformed body. However a useful case occurs when the parameters can be regarded as constant.

Isotropic metal with constant stress-strain parameters.

The constancy of Young's modulus E and Poisson's elastic transverse contraction ratio q is usually accepted. The constancy of the inelastic modulus P involves the assumption that yielding begins abruptly as in Fig. 7. This has been shown reasonable within engineer-

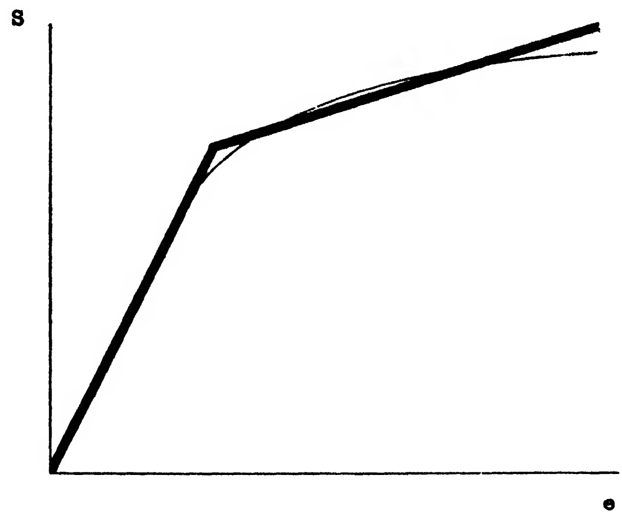

FIG. 7. Stress-strain curve showing the abrupt beginning of yield when the inelastic parameter is constant.

ing accuracy theoretically (11) and experimentally (17, 18). In reference 17 are discussed rectangular rosette readings made on a simple tensile specimen using an optical extensometer described in reference 16. The stress-strain relationships of reference 11 were used to show that even with P constant the shear stress applied to the specimen was zero. This is an elegant check of the theory because the three directions of strain measurement had to be used in the calculation and this involved the assumption of isotropy of the parameters P and p. The constancy of the plastic transverse contraction ratio p is approximately correct as shown in reference 13, once the yielding has passed the transitional range. The value of p as about 0.36 for duralumin was justified up to 40 per cent nominal strain by measurements on photogrid negatives very kindly supplied by the Armour Research Foundation, Illinois, and recently with the optical rectangular rosette extensometer up to 14 per cent nominal strain.

With these constant parameters in Eqs. 2.7, 2.8, we have

$$
\begin{aligned}
de_{ii} &= \frac{1}{P}\,dS_{ii} - \left[\frac{q}{E} + p\left(\frac{1}{P} - \frac{1}{E}\right)\right][dS_{jj} + dS_{kk}],\\
de_{ij} &= \left[\frac{1}{E}(1+q) + \left(\frac{1}{P} - \frac{1}{E}\right)(1+p)\right]dS_{ij}.
\end{aligned} \tag{5.11}
$$

Substitute (5.11) in (5.10) and integrating by parts

$$\int S_{ii}dS_{jj} = S_{ii}S_{jj} - \int S_{jj}dS_{ii}$$

and substituting this expression gives

$$U = \sum_{\substack{i=1,2,3\\ j=2,3,1\\ k=3,1,2}} \left\{ fS_{ii}^2 - gS_{ii}(S_{jj} + S_{kk}) + g\int (S_{jj} + S_{kk})dS_{ii} + hS_{ij}^2 \right\}, \tag{5.12}$$

where

$$
\begin{aligned}
f &= \frac{1}{2P},\\
g &= \frac{q}{E} + p\left(\frac{1}{P} - \frac{1}{E}\right),\\
h &= \frac{1+q}{E} + (1+p)\left(\frac{1}{P} - \frac{1}{E}\right).
\end{aligned}
$$

This is the strain energy per unit volume of inelastic metal with constant parameters for any finite deformation. In particular when the metal is still elastic so that P equals E then for any finite deformation

$$U = \frac{1}{E}\sum_{\substack{i=1,2,3\\ j=2,3,1\\ k=3,1,2}} \left\{ \tfrac{1}{2}S_{ii}^2 - qS_{ii}(S_{jj} + S_{kk}) + q\int (S_{jj} + S_{kk})dS_{ii} + (1+q)S_{ij}^2 \right\}. \tag{5.13}$$

Now in particular suppose the deformation to be one of infinitesimal strains and displacements as in the classical theory so that it is plausible to hypothesize that as load increases on the structure the ratios of the stresses one to another will be constant so that for example

$$
\begin{aligned}
S_{jj} &= X_jS_{ii},\\
S_{kk} &= X_kS_{ii},
\end{aligned}
$$

where the X's are constant, then (5.13) becomes

$$U = \frac{1}{2E}\sum_{\substack{i=1,2,3\\ j=2,3,1\\ k=3,1,2}} [S_{ii}^2 - qS_{ii}(S_{jj} + S_{kk}) + 2(1+q)S_{ij}^2]. \tag{5.14}$$

This is the classical expression for the strain energy of unit volume in an isotropic elastic body suffering infinitesimal strains and displacements as a very special case of the foregoing general results.

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NOTATION.

i, j, k used cyclically to denote three orthogonal directions at the current point of the deformed body.

c unit reference vector with subscripts i, j, k.

R position vector of the current point in the deformed body. Subscripts i, j, k indicate components.

R scalar modulus of **R**.

S "true" stress. Repeated subscripts denote normal stress. Mixed subscripts denote shear stress.

T "true" "auxiliary" stress.

e "true" normal strain. Repeated subscripts such as ii indicate strain due to stress S_{ii} normal to the face of the element. Mixed subscripts ij denote normal strain due to shear stresses S_{ij}, S_{ji}.

E "true" Young's elastic modulus. As an index denotes "elastic" component.

P "true" inelastic modulus. As an index denotes "plastic" component.

q "true" Poisson's elastic transverse contraction ratio.

p "true" plastic transverse contraction ratio.

ϵ "true" "partial" strain.

I plastic "partial" strain integral.

$\mathbf{D}$ "straining"-displacement.

N, S indices denoting normal and shear components respectively.

$\mathbf{A}$ position vector associated with a differential element in paragraph 3.

∇ vector gradient operator.

$\bar{\phi}$ dyadic first used in Eq. 4.1.

$\bar{\psi}$ stress dyadic.

m density.

$\mathbf{F}$ body force.

U work done per unit volume at the current point during the deformation.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

PARTICLE-SIZE DISTRIBUTION IN POWDER METALLURGY.

Advances in the science of powder metallurgy—the art of making objects by pressing and heating metal powders—have long been hindered by the difficulty of obtaining reproducible results in large-scale manufacture. This is due principally to present methods and techniques which introduce numerous variables; as yet not completely understood or controllable. In conjunction with the current program of standardization of test methods and techniques in this field, the National Bureau of Standards undertook an extensive investigation of the conditions contributing to the lack of reproducibility in sieve analyses of metal powders. Such analyses of particle-size distribution are of major importance in powder metallurgy.

Investigations at the Bureau have revealed that atmospheric humidity has a marked effect on the results obtained by sieve analyses of metal powders and that controlled atmospheric conditions during sieve testing of metal powders may, therefore, be necessary when close control of particle size is desired. Increase in humidity tends to increase the weight of the fractions retained on the sieves and decrease the weight of the pan fraction. Differences of as much as 10 per cent between the weight of fractions of powdered iron sieved under high and low humidities have been observed.

In sieve tests of sponge iron, electrolytic iron, electrolytic copper, and nickel, made for the purpose of accumulating supplies of sieved fractions of these powders for other studies, it was found that reproducible results could be obtained only when certain variables were controlled. Significant differences in sieve analysis often were obtained when samples of the sieve powder were sieved at different times with the same sieves. Furthermore, different sets of certified sieves used for the same powder gave variations of considerable magnitude. A contributing factor, in addition to atmospheric humidity, was a cumulative sampling error that resulted from repeated riffle cutting of limited powder supplies.

The effects of these variables were demonstrated by tests on sponge iron. This powder was made from reduced mill scale and consisted of irregular platelike particles. Many of the larger particles were made up of several such plates held together by the oxide of the metal.

* Communicated by the Director.

The sampling procedure included the use of a riffle-type sample splitter to reduce the entire supply of metal powder (50 to 100 lb.) to "sample supplies" which could be stored in one to five 1-pt. Mason jars (3 to 15 lb.).

Bureau tests of the effect of high humidity on the sieving characteristics of sponge iron illustrated the difficulty of reproducible analyses of metal powders. The set of certified U. S. Standard sieves used in these tests, Sieve set 3, included Nos. 100, 140, 200, 230, and 235. One hundred-gram samples taken from the same freshly riffle cut sample supply were sieved for periods of 5, 10, 15, 20, 30, 40, and 60 min. after the following treatments of the powder:

TABLE I.

Sieve tests of sponge iron showing variations due to replacement of sieves.

Average of three tests on 100-g. samples sieved 30 min. with certified sieves.

Sieve No.	With original No. 325 sieve		With new No. 325 sieve (NBS No. 8887)		Difference from original (3) minus (1)	With new No. 325 sieve (NBS No. 8888)		Difference from original (6) minus (1)	Difference between new sieves (6) minus (3)
	Mean	Deviation from mean	Mean	Deviation from mean		Mean	Deviation from mean		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				Percentage, by weight, of sponge iron retained with sieve set 1					
80	Trace		Trace			Trace			
100	Trace		Trace			Trace			
140	7.3	+0 1 to 0.2	7.0	+0.1 to 0 2	−0.3	6.9	+0.1 to 0	−0.4	
200	22.6	± .1	22.4	+ .2 to .4	− .2	22.6	± .1	0	
325	31.2	+ .4 to .5	38.2	± .1	+7.0	41.0	+ .4 to .2	+9.8	+2.8
Pan	38.4	+ .2 to .1	32.0	+ .2 to .1	−6.4	28.9	± .3	−9.5	−3 1
				Percentage, by weight, of sponge iron retained with sieve set 2					
80	Trace		Trace			Trace			
100	Trace		Trace			Trace			
140	4.4	±0.6	5.2	+0.1 to 0	+0.8	5.1	0	+0.7	
200	24.4	± .6	23.5	+ .1 to 0.3	− .9	23.3	±0.1	−1 1	
325	33.0	± .2	38.6	± .3 to .2	+5.6	40.8	+ 1 to 0	+7.8	+2 2
Pan	37.9	± .2	32.2	± .1	−4.7	30.2	± .1	−7.7	- 2.2

Series III.—For each time period three samples were oven dried for 1 hr. at 110°C. prior to sieving.

Series IV.—For each time period three samples were exposed for 64 to 72 hr. to a humid atmosphere in a closed vessel (desiccator) over water, with wicks dipped into the water to increase the evaporating surface.

Series V.—After sieving, the humidified samples of Series IV were mixed, dried for 5 hr. at 110°C., and resieved.

For the humidified samples, Series IV, the amounts of material retained on each sieve were consistently lower than those of the dried material, Series III. For example, differences in the case of the pen fractions ranged from 3.5 to 6.5 per cent of the original weight of the sample. Part of this difference (approximately 25 per cent) was re-

covered when the humidified samples were dried and resieved, as shown by the values obtained for Series V. Values for the individual samples of Series III lay close to the curve of the plotted results, while the values for Series IV and V were more scattered. Similar but smaller differences were obtained for the other sieve fractions.

Two additional series of tests were made using the same sieves and the same procedure as in Series III, but with samples taken from different sample supplies. The differences between each succeeding series, in the order in which they were riffle-cut, amount to about 1 per cent of the original weight of the sample. It is believed that these differences were due chiefly to the loss of fines as dust during riffle cutting.

TABLE II

Average opening of sieves as measured in certification tests

Sieve No.	NBS Certification No.	Average opening, microns*	
		Between warp wires	Between shoot wires
		Original sieve set I	
80	8805	186	174
100	8806	143	148
140	8807	107	102
200	8808	78	75
325	8809	46	46
		Original sieve set II	
80	8889	185	174
100	8890	144	110
140	8891	104	107
200	8892	77	73
325	8893	43	45
		New No. 325 sieves	
325	8887	41	43
325	8888	41	42

* Measurements were made by the Bureau's Metrology Division with an accuracy sufficient to determine whether the openings were within the limits permitted by specification. The errors are probably not in excess of about 1 micron.

Variations of considerable magnitude also were noted when powders were sieved with different sets of certified sieves. The results obtained with sponge iron when replacements of the No. 325 sieve were made in two sieve sets are given in Table I. The tests were made under approximately the same sieving conditions and all samples were taken from the same sample supply.

The fractions retained by the sieves which were not replaced (all sieves larger than No. 325) agreed closely. The new No. 325 sieves retained 7.0 and 9.8 per cent more material, respectively, with sieve set 1; and with sieve set 2, 5.6 and 7.8 per cent. The amounts retained by the two new sieves differed by 2.8 per cent when used with sieve set 1 and 2.2 per cent with sieve set 2.

In connection with these differences it is interesting to compare the

average openings of the several sieves as measured during the certification tests. These measurements are given in Table II. From a plot of the values of Table I against the measurements of the average openings, it is evident that the differences between sieves are considerably less than they would be if the comparison were made on the basis of the nominal opening. The agreement probably would be even closer if the variations in sieve openings could be taken into consideration. Due to manufacturing limitations the dimension tolerences of wire cloth permitted by specifications are necessarily rather wide, particularly for fine sieves. These variations are not fully reflected by the measurement of the average opening.

Further investigations of possible methods of eliminating, controlling, or evaluating the effects of these and other variables encountered in sieve analyses of metal powders are needed, and studies toward this end are under way at the National Bureau of Standards.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING, WEDNESDAY, MAY 19, 1948.

The Stated Monthly Meeting of The Franklin Institute was held on May 19, 1948 in the Lecture Hall of the Institute. The President, Mr. Richard T. Nalle, called the meeting to order at 8:15 p.m. There were approximately 225 persons in attendance.

The President stated that the minutes of the monthly meeting for March had been printed in full in the April issue of the JOURNAL OF THE FRANKLIN INSTITUTE, and if there were no corrections or additions the minutes would stand approved as printed. There was no dissent.

The President then presented an abstract of the annual report of the Board of Managers. He gave a brief resumé of the outstanding events and the progress made by The Franklin Institute in the year 1947. He further stated that the complete annual report would be printed in full in a subsequent issue of the JOURNAL for the benefit of the members of The Franklin Institute.

The report of membership for The Franklin Institute for the month of April was presented as:

Active	26
Associate	16
Student	8

The total Institute membership as of April 30, 1948 was 5329.

Our President then spoke of the Charles Day Lecture Foundation, which he said was established as a memorial to Mr. Charles Day, who was an outstanding industrialist and a distinguished mechanical engineer. He was a lecturer and author of many technical papers on industrial problems of scientific management, and a good friend of The Franklin Institute. Mr. Day was a valued member of the Board of Managers from 1908–1931, and a Vice President from 1920–1923.

The speaker of the evening, Mr. William J. Meinel, President of Heintz Manufacturing Company was then called upon to speak. Mr. Meinel chose for his topic "The Full Responsibility of Management." This paper will be printed in full in the July issue of the JOURNAL. After Mr. Meinel's talk, a very edifying and enlightening session was held, wherein members of the audience questioned Mr. Meinel on various aspects of management. Mr. Meinel's answers were most helpful to everyone.

The meeting was adjourned at 9:45 p.m., with a rising vote of thanks to the speaker of the evening.

HENRY B. ALLEN,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, May 12, 1948.)

HALL OF THE COMMITTEE,
PHILADELPHIA, MAY 12, 1948.

DR. JULIAN W. HILL in the Chair.

The following report was presented for final action:

No. 3194: Ballantine Medal.

This report recommended the award of the Stuart Ballantine Medal to Ray Davis Kell, of Princeton, New Jersey, "In consideration of his outstanding pioneer work in Television;

the adaptation of this means of communication to military needs and for his inventive contributions and leadership in the development of Color Television."

JOHN FRAZER,
Secretary to Committee.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Wednesdays, and Fridays from 9 A.M. until 5 P.M., Thursdays from 2 P.M. until 10 P.M. and Saturdays from 9 A.M. until 12 noon.

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Vibration and Sound, by Philip M. Morse. Second edition, 469 pages, 16 × 23 cm., tables and drawings. New York, McGraw-Hill Book Co., 1948. Price, $5.50.

Theory of Propellers, by Theodore Theodorsen. 164 pages, 16 × 23 cm., drawings, tables, and charts. New York, McGraw-Hill Book Co., 1948. Price, $6.00.

Photoelasticity, by Max Mark Frocht. Volume 2, 505 pages, 16 × 23 cm., tables, drawings and illustrations. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $10.00.

Growth of Plants, by William Crocker. 459 pages, 16 × 23 cm., tables, drawings, and illustrations. New York, Reinhold Publishing Corp., 1948. Price, $10.00.

The Genius of Industrial Research, by D. H. Killeffer. 263 pages, 15 × 23 cm., drawings. New York, Reinhold Publishing Corp., 1948. Price, $4.50.

Fundamentals of Statistics, by J. B. Scarborough and R. W. Wagner. 145 pages, 16 × 24 cm., tables and drawings. Boston, Ginn & Co., 1948. Price, $2.40.

The Radio Amateur's Beam Pointer Guide, by John F. Rider. 30 pages, 21 × 28 cm., illustrations. New York, John F. Rider Publisher, Inc., 1948. Price, $1.00 (Paper).

Preparation and Characteristics of Solid Luminescent Materials, Symposium held at Cornell University, October 24–26, 1946, sponsored by The Division of Electron Optics of The American Physical Society, published under the auspices of The National Research Council. 459 pages, 15 × 22 cm., drawings and illustrations. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $5.00.

BOOK REVIEWS.

ENERGY UNLIMITED. THE ELECTRON AND ATOM IN EVERYDAY LIFE, by Harry M. Davis. 273 pages, plates, 14 × 22 cm. New York, Murray Hill Books, Inc., 1947. Price, $4.00.

Mr. Davis, a newspaper and magazine writer on scientific matters, has brought together in this book some of the articles which he wrote for the *New York Times Sunday Magazine* and *Popular Science Monthly*. These contribute to ten of the seventeen chapters. Writing lucidly and interestingly for the average reader, Mr. Davis discloses the exciting panorama that is the result of man's new discoveries in the field of nuclear physics and electronics. Technicalities are held to the minimum, and the definitions of terms that cannot be avoided are often repeated throughout the text.

In the first five chapters the history and elementary background of the more recent advances in physicial science are given. Chapters Six to Ten deal in more detail with the application of modern electronics, while Chapters Eleven and Twelve show the relationship of the electronic and nuclear fields, the latter being then taken up in the remaining five chapters.

The treatment of the material in this book is comprehensive to the extent that the reader, whether he be equipped with a varying degree of technical background or not, will gain a unified outlook on the achievements and problems of physical sciences that face the world at large today.

HENRY N. MICHAEL.

NEUTRON EFFECTS ON ANIMALS, by the Staff of the Biochemical Research Foundation. 198 pages, illustrations, 15 × 23 cm. Baltimore, The Williams & Wilkins Co., 1947. Price, $3.00.

With the atomic age upon us, knowledge of the effects of atomic energy becomes important. The worker in atomic energy plants needs to be protected from radiation, and, if he should be exposed, he must receive adequate remedial and curative measures. The Biochemical Research

Foundation, which for fifteen years has been studying various radiation effects, has lately been investigating the radiation effect of the neutron on cells.

The present volume, essentially preliminary in nature, is a report of these experiments on the effects of neutrons on animals and has been published to permit others to secure a general perspective of the problem.

Dr. McDonald offers first a general statement concerning radiation effects on cells. In this he points out that while neutron bombardment and X-radiation may produce similar results, there is no proof that they cause these results by a similar mechanism. Also "the most significant fact in neutron bombardment is its effect upon growing or immature tissue." Once the mechanism of this action is understood, the possibility arises that similar conditions may be produced in cancer cells, which are relatively immature, rapidly-dividing cells. The hope of thus producing a stoppage of cancer cell growth is an alluring goal.

The second chapter compares the physical properties of fast neutrons with other types of radiation while the third chapter describes the general techniques employed in the neutron irradiation procedure. The subsequent chapters are devoted to individual reports on the different experiments. Various subjects were used including rats, chickens, rabbits, dogs, corn and certain bacteria. The scope of the work is best indicated by some chapter titles: Relation between Neutron Dose and the Mortality, Body Weight, and Hematology of White Rats; Effects of Neutrons on Early Root Development of Zea Mays; Studies of Proteolytic Enzymes in Bone Marrow; Electrophoresis of the Plasma of Dogs Irradiated with Neutrons.

This is a report of significant work in a new field, which should prove a stimulus to further study. It is gratifying to know, as a prefatory note states, that considerable progress has already been made in explaining the *mechanism of radiation* on mammalian cells, findings which it was not possible to insert in the present volume.

G. E. PETTENGILL.

COMPOSITION AND PICTURES, by Eleanor Parke Custis. 224 pages, illustrations, 22 × 29 cm. Boston, American Photographic Publishing Co., 1947. Price, $6.00.

A photographer or an artist may be truly artistic and may have a natural instinct for making pictures. But no matter how strong this instinct is, it is more workable when backed by actual knowledge. In her book Miss Custis aims at presenting the fundamental rules of composition which will enable the artist to govern his perhaps undisciplined instincts. The language of the book is simple and direct, and the points of discussion are profusely illustrated with the author's own work and with that of many others.

HENRY N. MICHAEL.

UNDERSTANDING VECTORS AND PHASE, by John F. Rider and Seymour D. Uslan. 153 pages, illustrations, 13 × 19 cm. New York, John F. Rider Publisher, Inc., 1947. Price, $0.99 (paper).

This short text is intended for the man who has not had an engineering background, yet is thrown in contact with both vectors and phase in his efforts to keep abreast of electronic developments. This means the radio serviceman, the radio technician, the radio amateur.

Those who have a strong desire to know more about the field in which they are engaged, and who have a general appreciation of radio principles, will find the information contained in this booklet of much value to their understanding of articles in radio engineering publications and textbooks.

HENRY N. MICHAEL.

FACTUAL COMMUNICATION: A HANDBOOK OF AMERICAN ENGLISH, by L. O. Guthrie. 448 pages, illustrations, 15 × 22 cm. New York, Macmillan Co., 1948. Price, $3.50.

In this textbook on American English, the author has stressed the practical uses of English in articles, talks, business letters, and reports. The first part deals with different types of spoken and written communication. To guide the student further in his composition, the second part introduces pertinent theories and suggests procedure for finding information. A comprehensive section on grammar and rules forms the concluding part entitled "Handbook."

Since most of the examples use scientific and technical vocabulary, this text will be especially useful with students in technical institutions.

G. E. PETTENGILL.

FUNDAMENTALS OF PHOTOGRAPHIC THEORY, by T. H. James and George C. Higgins. 286 pages, illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1948. Price, $3.50.

This book gives a general account of the theory of the photographic process, based on the fundamental chemical and physical concepts. Thus a specialist's knowledge of chemistry and physics is not required, although their basic knowledge on the part of the reader is presupposed.

Only the black-and-white photographic processes which involve the use of silver salts are considered here. The field of color photography is not treated specifically, since every major color process involves primarily a black-and-white process.

The comprehensive nature of the book, in so far as black-and-white photography is concerned, should make it appealing both to the intelligent amateur and to the specialist.

HENRY N. MICHAEL.

INTRODUCTION TO MODERN PHYSICS, by F. K. Richtmyer and E. H. Kennard. Fourth edition, 759 pages, illustrations, 16 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1947. Price, $6.00.

The fourth edition of this extensively used text-book preserves the aims of the first edition of 1928, namely, "to present such a discussion of the origin, development, and present status of some of the more important concepts of physics, as will give the student the correct perspective of the growth and present trend of physics as a whole." In pursuing this aim the author has not given a collected bibliography, nor a list of problems and suggested topics for study. Instead, references, in most cases to original sources, have been given at the appropriate points in the text.

After the death of Dr. Richtmyer, Dr. Kennard assumed the task of revising the book. Thus the third edition of 1942 presented major changes. The historical introduction was abbreviated. The chapters on electromagnetism, the nuclear atom, and the X-rays were rewritten and new chapters on relativity and on cosmic ray phenomena inserted.

The present edition (1947) records the more significant advances of the last five years. It also reflects on further changes in perspective on the physical scene.

The notable changes are evidenced in Chapters XI (The Nucleus) and XII (Cosmic Rays), where extensive additions have been made. Chapter VIII, on many-electron atoms, has been rewritten and simplified. The entire discussion proceeds in terms of quantum states rather than of wave functions.

HENRY N. MICHAEL.

MODERN TIMBER DESIGN, by Howard J. Hansen. Second edition, 312 pages, illustrations, 15 × 23 cm. New York, John Wiley and Sons, Inc., 1948. Price, $4.50.

The second edition of "Modern Timber Design" is brought up-to-date to conform with the specifications for working stresses which were adopted since the first publication in 1943. These specifications increased the working stresses by 20 per cent. Thus the revision is quite extensive, since the above-mentioned modification affected the material in practically every chapter. Also, some new material not found in the first edition was added. This new material deals with the testing methods for wood and factors affecting the strength of wood.

The text is divided among eleven chapters. A list of headings will indicate its comprehensiveness: I. Characteristics and Properties of Wood; II. Mechanical and Related Properties; III. Grading Rules and Working Stresses; IV. Mechanics of Wood; V. Fastenings; VI. Beams and Columns; VII. Timber Decks and Bridges; IX. Glued Laminated Construction; X. Plywood; and XI. Decay, Wood-Destroying Organisms, and Preservatives.

The Appendix, among others, contains a list of definitions of terms used in describing standard grades for lumber.

Since this text-book also contains all the basic information needed by the designing engineer, it recommends itself as a guide and reference for practicing engineers.

HENRY N. MICHAEL.

PRINCIPLES OF FOOD FREEZING, by Willis A. Gortner and others. 281 pages, illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1948. Price, $3.75

The treatment of foods prior to freezing, the care and operation of the freezing equipment, the temperature of storage, and the thawing and cooking procedures used in the kitchen are all important factors in determining the nutritive value of the product as it reaches the table. The proper procedures are not difficult but the extent to which they are understood and applied will determine the success of the consumer experiments now being made. This timely book deals with all the aspects involved—from the raw materials to the finished product. It is technical in the sense that it explains the biochemistry and nutrition in freezing, storage, and cooking processes, and deals with engineering principles involved in the construction and operation of the equipment. In this connection numerous references are given. The food packer and locker operator will profit from this information. However, the book should prove of equal value to the intelligent homemaker, who although not interested in the chemical or engineering bases, may want to be able to select the proper equipment and carry out the cooking processes which will insure the best results. For this latter use the information is presented in easily understandable language.

HENRY N. MICHAEL.

PROBLÈME GÉNÉRAL DE LA STABILITÉ DU MOUVEMENT, by M. A. Liapounoff. Pages 203–474. 23 × 15 cm. Princeton, Princeton University Press, 1947. Price, $3.50.

This publication, which forms Number 17 of the *Annals of Mathematics Studies*, is a photolithoprint reproduction from the *Annales de la Faculté des Sciences de Toulouse*, Second Series, Vol. 9 (1907). This latter was itself a translation from the original Russian issued in 1892 by the Societé Mathématique de Kharkow. The author treats of methods employed in investigations of the properties of movement and of equilibrium, known as stability and instability. Two different cases are considered, one for constant movements and the other for periodic.

The *Annals* is performing a valuable service in making this study more widely available.

G. E. PETTENGILL.

MATHEMATICAL TABLE MAKERS, PORTRAITS, PAINTINGS, BUSTS, MONUMENTS, BIO-BIBLIOGRAPHICAL NOTES, by Raymond Clare Archibald. 82 pages, portraits, 17 × 24 cm. New York, Scripta Mathematica, 1948. Price, $2.00.

In the daily use of mathematical tables, probably few give much thought to the men who made these tables. This slender volume should stimulate such an interest for it is a helpful guide to the portraiture and literature of fifty-three mathematical table makers. The entry for each individual gives his full name, dates, and a brief biographical note. This is followed by a list of references to original portraits, busts, monuments, or to reproductions of them. Lists of biographical material not accompanied by illustrations, and lists of published tables, are also given.

Portraits are included for twenty of the men discussed, including most of the living ones. The text, reprinted from *Scripta Mathematica* in a revised and enlarged version, will prove of even more value in this form.

G. E. PETTENGILL.

THE CORROSION HANDBOOK, edited by Herbert H. Uhlig. 1188 pages, illustrations, tables, diagrams, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1948. Price, $12.00.

Sponsored by the Electrochemical Society and written by 103 contributors, including many of the outstanding figures in corrosion research, this volume will undoubtedly rank as one of the most important technical books of the year. It brings between two covers a summary of

corrosion information covering all fields and representing a cross section of scientific data and industrial experience.

In compiling the volume, emphasis has been placed on quantitative information. Actual corrosion rates have been given, with exposure conditions defined as precisely as possible. To facilitate comparisons and to establish standard units, most data have been transposed to inches penetration per year (ipy) or milligrams per square decimeter per day (mdd).

After a brief section on corrosion theory, the corrosion of the various metals and alloys in liquid media, in the atmosphere and in gases is discussed. This includes under each metal such topics as acids, alkalies, salt solutions, velocity, effect of mechanical factors (stress and vibration), organic compounds, and atmospheric corrosion. Although not strictly corrosion, the deterioration of various nonmetallic materials and semi-metals is also treated, because of their frequent use under similar conditions.

In the third section special topics are considered such as corrosion by sea water, by soils, by lubricants, and the effects of mechanical factors on corrosion. Section four deals with high-temperature corrosion of the more important metals with the addition of carbon and graphite, and refractories. The next two sections present tables of high-temperature and chemical-resistant materials. The latter appears of great practical value for it evaluates the resistance of twenty-five groups of materials to twelve different groups of chemicals. The materials are rated in three classes in accordance with their suitability for use under varying conditions.

The problem of corrosion protection is divided into three major parts: metallic coatings, inorganic coatings and organic coatings. Corrosion testing, both in the laboratory and in the field, is the subject of the eighth section. Miscellaneous information and general tables are gathered in a concluding section.

Numerous tables and diagrams and ample reference to the literature on corrosion are features of the book. It should be noted, too, that many data previously unpublished have also been included. A comprehensive index insures ready access to the contents.

The volume goes a long way toward making readily available the present knowledge of corrosion. It also reveals gaps in that knowledge, which, it is to be hoped, will stimulate further experimental work. In view of the high cost of depreciation and the ever-increasing necessity of preserving our raw materials, this volume should be readily available to anyone working with metals where corrosion is a factor.

G. E. PETTENGILL.

DETOXICATION MECHANISMS. THE METABOLISM OF DRUGS AND ALLIED ORGANIC COMPOUNDS, by R. Tecwyn Williams. 288 pages, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1948. Price, $5.50.

Present knowledge of the metabolic fate of even simple organic molecules is often incomplete. However, since in recent years synthetic organic compounds have been used increasingly in the treatment of various diseases, what happens to these compounds in the body has become an important question. The object of this book is to present the available material, so that a working hypothesis may be advanced.

In the past most published material concerned itself with qualitative analysis, without reference to the quantitative aspects. It is probable that in such cases other undetected metabolites have formed.

Dr. Williams' treatise covers the metabolism of the following compounds: aliphatic compounds and cyclohexane derivatives; aromatic hydrocarbons; halogenated aromatic hydrocarbons; phenols; aromatic alcohols, ethers, aldehydes, ketones, and amides; aromatic acids; organic cyanides; aromatic nitro, amino, and azo compounds; sulphones, and sulphonic acids, and sulphonamides; terpenes and camphors; heterocyclic compounds; and organic compounds of arsenic. The concluding chapter deals with the hypotheses prevalent in the past and formulates the criteria upon which a working, probably non-unitary, hypothesis must be based.

This book should prove of high value to the student and particularly to the research man who is interested in the behavior of drugs in the animal body.

HENRY N. MICHAEL.

STRESS ANALYSIS AND DESIGN OF ELEMENTARY STRUCTURES, by James H. Cissel. Second edition, 419 pages, illustrations, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1948. Price, $5.00.

This textbook presents the basic principles and procedures related to stress analysis and design of simple structure. It is intended for use in those courses of study in engineering other than civil, which include a section on structures. Thus, fundamental, useful and practical material was selected, to provide a self-contained working book which eliminates the need for auxiliary handbooks.

The 1948 edition retains in Part I essentially the same material with the exception of Chapter 2, where up-to-date information on live loads is given. However, Part II, which covers the design of simple structures, is completely rewritten and is consistent with the most recent specifications for designing structures. A new chapter on light-gage steel construction is added. Each chapter offers data for problems as an aid to class instruction.

HENRY N. MICHAEL.

HYDRAULICS, by Horace W. King and others. Fifth edition, 351 pages, illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1948. Price, $4.00.

Twenty-six years have elapsed since this book was first published. Then, as now, it was intended as an elementary text for the engineering student. In the 1948 revision the material is rearranged and clarified, and recent accepted developments, both theoretical and practical, are integrated. This new material deals with the variation of hydrostatic pressure with altitude in a compressible fluid, flow through gates and over dams, flow of liquids in pipes and pipe networks, and resistance offered to motion of objects through fluids. An entirely new chapter on hydraulic similitude and dimensional analysis covers the fundamentals upon which the increasingly important field of hydraulic model testing is based.

The unchanged popularity of this textbook throughout a quarter century testifies to its thoroughness and usefulness.

HENRY N. MICHAEL.

STEEL AND ITS HEAT TREATMENT, by D. K. Bullens. Fifth edition, Vols. I and II, 489 and 293 pages, respectively, illustrations, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1948. Price: Vol. I, $6.00; Vol. II, $4.00.

The fifth edition of this work, which first appeared in 1915, continues in its principles of combining theory and practice for use in conjunction with observation of steel behavior.

Since 1938, the date of the fourth edition of this work, many new methods in the treatment of steel were developed, particularly during the war. These developments, hitherto published fragmentarily, are sifted, and the especially illuminating ones are gathered in an arrangement which aids the practical heat treater.

The present edition is to appear in three volumes, two of which have already been published and are considered here.

Volume I covers the definitions, terminology and fundamental concepts underlying the discussion of heat treatment. It points out the limitations and uses of tests applied to determine suitability of steel for heat treatment, and the suitability of the end-product for engineering use. The principles governing the various types of heat treatment, both of carbon and alloy steel, are thoroughly investigated.

In Vol. II these principles are applied and the tools of heat treatment are described. These are the furnaces, salt baths, controlled atmospheres, quenching media, etc. It also covers such items as prevention of cracking and distortion, conditioning of steel for machining, welding, and the heat treatment of steel castings, malleable iron, and cast iron.

H. N. MICHAEL.

INVENTIONS, PATENTS AND MONOPOLY, by Peter Meinhardt. 352 pages, 14 × 22 cm. London, Stevens & Sons, Ltd., 1946. Price, 25 s.

The author states that Great Britain was the first country in the world where patents

were issued to inventors. The British Patent System undoubtedly has contributed to making the country one of the greatest industrial nations and the law has been adopted with modifications in more than a hundred other countries. In the latter respect it is a part of English law carried with British colonists and used as a basis in the newly settled countries. This volume, written and produced in Great Britain, is primarily given to that country. The larger part of it is devoted to British patent law and practice. This part is mainly descriptive and quite simple, outlining the requirements facing one applying for a patent and the patentee who wants to enforce his rights against an infringer. Other sections in this part deal with the maintenance of the patent, tax liabilities, and patent litigation. Of great general interest to those outside of Great Britain are other parts of the book. The first part dealing with the characteristics of inventors and inventions is in this class and is quite novel. Circumstances, psychology, economics, practical matters, rewards, motives, etc., are all touched in a very human way. In another part of the book, the abuse of the patent monopoly is a topic of a more general interest and a timely subject here in the United States as well as in Great Britain. Charging excessive prices for patented articles, and prejudicing trade or industry by the refusal to grant licenses are given consideration. The last part of the book on suggestions for patent law reform again refers to the British situation.

The book will be of interest to all persons, here and abroad, who desire a knowledge and understanding of inventions and patents.

R. H. OPPERMANN.

BROADCAST OPERATORS HANDBOOK, by Harold E. Ennes. 265 pages, illustrations, 14 × 21 cm. New York, John F. Rider Publisher, Inc., 1947. Price, $3.30.

Although there are numerous books on radio equipment and several on program arrangement, there has been little written on the practical operation of broadcast equipment to insure successful transmission of programs. This volume by the staff engineer of station WIRE aims to fill this gap in the literature. The first four parts are devoted to operating practice in the control room, the master control, remote controls and the transmitter. Among the many points touched on are control of loudness, operation of turntables, location of the microphone for various types of performances, and pick-ups of different programs outside the studio.

The fifth part is devoted to emergency shut-downs with a suggested program of preventive maintenance. This includes both a comprehensive schedule of points to check and instructions on how to effect the maintenance carefully and adequately. The author differentiates six types of operation—feel, inspect, tighten, clean, adjust and lubricate—and discusses their application to the various types of equipment. A final part mentions some of the technical equipment used and offers suggestions on developing a broadcast studio and selecting broadcaster transmitter location.

A practical book by a practicing engineer, this volume should prove helpful to the newcomer in broadcast station operation, as well as a source of valuable hints for the more experienced man.

G. E. PETTENGILL.

JET PROPULSION PROGRESS. THE DEVELOPMENT OF AIRCRAFT GAS TURBINES, by Leslie E. Neville and Nathaniel F. Silsbee. 232 pages, illustrations, 15 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1948. Price, $3.50.

A major revolution in aviation is taking place with the introduction of gas turbines in jet-propelled airplanes. In the present volume the authors have given an outline of the principal developments in jet propulsion, though there has been no attempt to write a complete history, or to mention many designs, patents and experimental projects. As the authors point out, this latter has been well done by G. Geoffrey Smith.

After a brief introductory chapter on the different types of gas turbines, the work of the Germans is reviewed. The jet-propelled plane never played a decisive role in World War II, since the Messerschmitt Me-262 was the only jet plane which was truly operational. However, the German plans were well advanced and it is readily apparent now that if they had had

another year or even less the situation for the Allies would have been critical. The discussion of these German types is of considerable interest.

Experiments were likewise begun in Great Britain at an early date and an experimental jet fighter, the E28/39 flew in 1941. Although subsequent to the first German test plane, this is considered "the first *successful* airplane using the gas-turbine jet-propulsion engine."

From the British contributions the authors turn to consideration of U. S. Army work on the gas turbine and then to the U. S. Navy research. In each of the four chapters a narrative account is given of the developments that took place, with a section of technical data on some of the leading engines.

The role of the NACA in heading up government research is also discussed. Chapter seven considers some of the technical problems which still remain and the final chapter offers conjectures on the part the gas turbine engine is to play in aviation tomorrow. A chronology, a glossary, and a bibliography, which despite its heading "1941 to July 1944" includes items up to 1947, furnish useful auxiliary material.

The book is an interesting, concise report on the gas-turbine engine in aviation, suitable for those who wish to know more about its general development and problems without recourse to an exhaustive study.

G. E. PETTENGILL.

A TEXT-BOOK OF PRACTICAL ORGANIC CHEMISTRY, by Arthur I. Vogel. 1012 pages, illustrations, tables, 16 × 26 cm. New York, Longmans, Green & Co., Inc., 1948. Price, 42s.

This one-thousand page textbook is intended to meet the requirements of the student throughout the whole of his undergraduate training in chemistry. It also introduces research methods in organic chemistry and thus may serve as an intermediate reference for practicing chemists. Preparations and qualitative analysis are dealt with. A short theoretical introduction precedes each of the 600 preparations. The scope of the book may be indicated, in a general way, by a list of its chapter headings: Theory of General Technique; Experimental Technique; Aliphatic Compounds; Aromatic Compounds; Some Heterocyclic and Alicyclic Compounds; Miscellaneous Reactions; Organic Reagents in Inorganic and Organic Chemistry; Dyestuffs, Indicators and Related Compounds; Some Physiologically Active Compounds; Synthetic Polymers; Qualitative Organic Analysis. Considerable detail is given in the sections which are of particular interest to the beginner. This should provide him with a secure foundation of sound experimental technique.

H. N. MICHAEL.

A NEW NOTATION AND ENUMERATION SYSTEM FOR ORGANIC COMPOUNDS, by G. Malcolm Dyson. 63 pages, diagrams, 15 × 25 cm. London, Longmans, Green & Co., 1947. Price, $1.75.

The increasing difficulties in classification and indexing of organic compounds has led to the development of this new cipher system by Dr. Dyson. Although first formulated in 1944, it has not been published until quite extensive tests had been made with it. These tests included the ciphering of the Ring Index and five volumes of Beilstein, which resulted in no entry that did not give a distinctive cipher.

The presentation of the system is prefaced by a discussion of some of the difficulties of existing nomenclature. This new cipher employs capital letters, numerals and certain other symbols and is linear in its application. The principles of the cipher are outlined fully with examples of the various types of structure to be found and the method of ciphering them.

Enumeration is dependent upon the order of citation in the cipher for which specific and definite rules are given. Indexing of the ciphers is done by following the same order of operations used in determining enumeration, with one exception. Numerous examples of the application of the cipher are given including three hundred consecutive entries from the Ring Index, which have been ciphered and arranged in order.

One of the main features of the cipher is the fact that it lends itself to mechanical manipu-

lation. This includes computation of the molecular formula, indexing of the ciphers and searching for compounds of a predetermined structural characteristic. This adaptability to punch-card use seems one of the strongest points in its favor. What its ultimate fate will be is problematical, but it merits the consideration of chemists everywhere as an attempt to introduce simplification into organic nomenclature.

G. E. PETTENGILL.

ELECTRIC POWER TRANSMISSION, by M. P. Weinbach. 362 pages, illustrations, 16 × 24 cm. New York, Macmillan Co., 1948. Price, $5.50.

In his book, Dr. Weinbach deals with the electrical problem of transmission only. The description of generating equipment, voltage control, the mechanical suspension of the line, switching apparatus, lightning protection, etc., have been purposely omitted.

The ten chapters of this work comprehensively analyze the problem at hand. A list of chapter headings will indicate the extent of the material treated: Circuit Properties of Transmission Lines; Transmission Line Formulas; Applications of Transmission Lines; Line with Transformers; Voltage Control of Transmission Systems; Steady State Power Limits; Faulted Transmission Systems; Transient Stability; System Instability.

The rational treatment of the physical phenomena involved and of the mathematics needed for their solution makes this book highly useful to the student and practicing engineer.

H. N. MICHAEL.

PRINCIPLES OF JET PROPULSION AND GAS TURBINES, by M. J. Zucrow. 563 pages, illustrations, diagrams, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1948. Price, $6.50.

Dr. Zucrow's book is based on lecture notes used during the war in his course on jet propulsion, given at the University of California. Although observance of military security confines the book to basic principles, the attainment of the main objective, which is to present the fundamental theory of jet-propulsion engines and gas-turbine power plants, is not impaired. The book is written primarily for the student.

The first six chapters, dealing with the fundamental principles, nomenclature and energy relationships for fluids, gas thermodynamics, and airplane performance calculations, provide a firm basis for the later discussion of the continuous combustion gas turbine, the basic types of air compressors, the axial-flow turbine, the combustion chamber, the turbojet engine, and the rocket. The text is completed with a chapter on high-temperature metallurgy.

H. N. MICHAEL.

LOW-PRESSURE LAMINATING OF PLASTICS, by J. S. Hicks assisted by R. J. Francis. 162 pages, illustrations, 15 × 23 cm. New York, Reinhold Publishing Corp., 1947. Price, $4.50.

Low-pressure lamination of plastics is a field which holds promise, particularly in satisfactory accomplishments in an economical way. Manufacturing advantages involve less capital outlay, as lighter weight equipment is required. The terminology in the field of plastics is in a state of flux, but in this book, low-pressure laminating of plastics is meant to include the field of molded laminates or contoured parts as well as flat sheets, whether one ply of filler is used or several laminae of different reinforcing materials are used. Only a few of the thermosetting resins fall into this class where the curing cycle is at low pressures up to 100 psi. The book gives special attention to pressures of 0 to 15 psi.

The treatment begins with the design and use of molds, discussing materials which can be used, methods used in making them, and how separating compounds or mold lubricants are employed. The subject of what resin to use is next taken up, covering recent additions to the field of useful resins for this work. Means of hastening the curing rate with catalysts, and curing under low pressures such as evacuation of air from the envelope surrounding the mold are subsequent topics. Considerable space is devoted to reinforcements for plastics and sandwich structures, the latter meaning two sheets of laminated reinforced plastic cured with a filler or core material between them. A low density core combined with a high strength face layer gives the greatest strength to lightest weight ratio of various materials. A brief chapter on properties of plastics gives short but pointed definitions of the various properties. The latter

part of the book is devoted to descriptive illustrations of laminating—tank cover, boat deck, and milk bottle case—to show many of the versatile handling properties of low pressure laminating; joining and machining of plastics; and a discussion of product analysis involving engineering and cost principles. Appendices give an outline for laboratory experiments, a patent index, and a subject index.

Those interested in this field will find this little book of practical value.

R. H. OPPERMANN.

ELECTRONIC CIRCUITS AND TUBES, by the Electronics Training Staff at the Cruft Laboratory, Harvard University. 948 pages, illustrations, 15 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1947. Price, $7.50.

The text presented in this book was developed from the lecture notes prepared for the wartime intensified courses in electronics given to Army and Navy personnel. The material is fundamental and, therefore, should not be viewed as applicable to wartime training only. Engineering school students with the background of mathematics through calculus, of electricity and magnetism, will find the subject and its subdivisions comprehensively covered.

H. N. MICHAEL.

PHYSICS FOR STUDENTS OF SCIENCE AND ENGINEERING, by William H. Michener. 646 pages, illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1947. Price, $4.25.

This textbook is specifically written for those undergraduate students who expect to apply their knowledge of physics to science and engineering. By dealing comprehensively with the fundamental divisions of physics—mechanics, heat, wave motion and sound, light, electricity, and magnetism—Dr. Michener ably guides the student toward this goal.

The presentation is clear and pedagogically sound. The stress is on application. Many original problems are included and are carefully coordinated with the text.

H. N. MICHAEL.

SOIL MECHANICS IN ENGINEERING PRACTICE, by Karl Terzaghi and Ralph B. Beck. 566 pages, illustrations, 16 × 23 cm. New York, John Wiley & Sons, Inc., 1948. Price, $5.50.

This engineering textbook is divided into three parts dealing respectively with physical properties of soils, theoretical soil mechanics, and problems of design and construction.

The first two parts are relatively short ones, occupying together less than half of the volume, but they contain all that engineering students and the average engineer need to know about soil mechanics proper at the present time.

A short resumé of the material included in the first part would cover the physical and mechanical properties of homogeneous specimens of undisturbed and remolded soils. It deals with those properties which serve as criteria for distinguishing different soils, as well as describing them adequately. The behavior of soil masses during and after construction is also discussed.

The second part of the book introduces the reader to the theories required for solving problems involving the stability or bearing capacity of soils or the interaction between soil and water.

The third part is the "heart of the book." It is the applied part. Its initial chapter contains a discussion of the properties of natural soil deposits and the methods for investigating them. The following two are concerned with the empirical rules that pertain to the different branches of foundation and earthwork engineering. The final chapter deals with the effect on adjoining structures of such construction operations as excavating and pumping.

Because of their highly specialized nature, the designs for road and airport pavement, and for tunnel engineering are treated only referentially in the appendix.

H. N. MICHAEL.

ELEMENTARY MANUAL OF RADIO PROPAGATION, by Donald H. Menzel. 222 pages, drawings, maps, charts, nomograms, 23 × 28 cm. New York, Prentice-Hall, Inc., 1948. Price, $5.75.

When considering the importance and the expanse of the field, there are altogether too few books on radio wave propagation. Probably this is in a large part due to the ambiguous nature of the subject. Theories of ionospheric phenomena are oftimes unsubstantiated by observational evidence. On the other hand, many instances of point-to-point radio transmission are unpredictable with present-day theories. Exactly what is known and predictable about radio propagation has been collected in this one volume. Dr. Menzel makes no attempt to analyze the accuracy of the information—other than to point out the normally observed limits—nor does the author digress into the involved processes necessary to obtain this working information. The *Elementary Manual of Radio Propagation* is specifically designed for people who use the radio frequencies in communication or broadcasting. It bridges the gap between the research physicist and the radio engineer with the results of practical ionospheric theories that have withstood the test of time.

The *Manual* may be divided into three separate and distinct sections. The first is devoted to the methods of predicting the maximum usable and the lowest usable high frequencies. It will be noted that much of this material has been published previously by the IRPL (National Bureau of Standards) either in their *Radio Propagation Handbook* or in their new *Ionospheric Radio Propagation.* The second section may be said to begin with Chapter Eleven and is primarily a study of the factors involved in ground wave or direct ray transmission. Graphs show the variability in the strength of the received field relative to effective radiated power, antenna height gain and earth conductivity. The third section embraces the radio spectrum above the VHF. The material in this section is exceptionally comprehensive having been drawn from numerous sources, including the M.I.T. Radiation Laboratory Series, Vol. 13 and CRPL-T3.

To anyone familiar with the field of radio wave propagation the fact that one author in one book attempts to be all-inclusive may be rather amazing. Fortunately, however, the results vary from fair to very good. The most faults lie in the first section on sky-wave transmission. In this part of the *Manual* liberal use is made of the IRPL (later CRPL) charts and nomograms. Dr. Menzel performs a great service by reprinting and simplifying the absorption and noise grade maps which are not commonly available. At the same time this excellent intention is partially defeated by the publisher who has made the maps in the *Manual* much smaller than those currently issued monthly with the CRPL D-series Advance MUF Predictions. This necessitates that anyone desiring to use these maps must first redraw a new system of overlays, thereby subjecting his LUHF calculations to twice the normal amount of error. This same section also bears the appearance of having been prepared in great haste. Numerous small printing and proof-reading errors—such as the substitution of KM for KW—are quite common. Care should also have been exercised in coordinating the text with the nomograms in matters of symbols and terminology. The symbol $(Kd)_{sol}$ is used flagrantly in the text, although the standard symbol $\overline{Kd}$ appears on most of the charts and nomograms. No mention is made of the fact that these two are identical.

In contrast, the section of the *Manual* devoted to propagation above the VHF presents a heterogeneous mass of data in a very smooth and orderly fashion. Worksheets, charts and nomograms for calculating the coverage diagrams of VHF to SHF transmitters are given. This is followed by the methods in calculating the field in the shadow zone and then under conditions of sub or superrefraction. The *Manual* ends in a three part Appendix which provides a mathematical introduction to the physical factors that cause the bending of the radio wave.

OLIVER P. FERRELL,
Radio Magazines, Inc.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

ON ENZYMES IN TUMORS.*

By Ernst Waldschmidt-Leitz, Ellice McDonald and Co-workers.

EXPERIMENTAL PART.

I. Pathologic-anatomical Analysis

(work done and reported by Wilhelm C. Hueper)

Of the two transplantable rat tumors used in the foregoing experiments, the one (Philadelphia 1) was a fibrous sarcoma, the other (Walker 256) a cellular carcinoma disposed to early and widespread necrosis. The two tumors differ biologically as well as histologically in various aspects.

Sarcoma Philadelphia 1.

The rate of growth of the tumors injected under the abdominal skin was moderate. Metastases were found only occasionally in the axillary and inguinal lymph nodes when large tumors were present. Smaller tumors were surrounded by a generally well-formed capsule of connective tissue, so that they could be taken out easily without cutting. In most of the larger tumors, the skin and muscles of the abdominal wall were infiltrated with the tumor tissue and grown onto it. The sections of the rather firm tumors were of a light pink, homogeneous, fatty appearance. In older and larger tumors there was always a more or less widespread white, fibrillar central part, which was separated rather indistinctly from the pink periphery and showed many yellow spots. The healthy tumor tissue consisted of diffuse masses or wider and narrower strands of irregularly round to oval, medium-sized tumor cells, which rarely showed division and often fibrillar projections. The surrounding stroma was a fibrous connective tissue, often of a hyalin character in the central, macroscopic whitish part of the tumor and surrounded small groups and thin strands of seemingly atrophic tumor cells. Widespread necroses were generally not present. Smaller foci of necrosis and degeneration were scattered throughout the central part in moderate numbers and were observed especially at the border between the healthy and the fibrous parts of the tumor; they were frequently surrounded and infiltrated by masses of leucocytes in small to moderately large quantities.

Carcinoma Walker 256.

The tumor showed an unusually rapid rate of proliferation; in several

* Continued from p. 453.

animals 6 to 8 weeks after inoculation the tumor was about as heavy as the rat itself. Metastases were observed to be numerous in the axillary and inguinal, but were also present in the retroperitoneal lymph nodes. Taking out these tumors without cutting was for the most part not possible, for the connective tissue capsule was relatively fragile and the tumor very soft. They were often grown fast to the abdominal wall. Ulceration of the tumors through the skin was not unusual in medium-sized and was the rule in large tumors. The healthy tumor tissue appeared pink and homogeneous on section; but even in moderately small tumors necroses of varying extent were the rule, while they were predominant in the larger tumors and constituted the greater part of the tumor, so that often only a very narrow peripheral zone of healthy tumor tissue surrounded the tumor entirely or in part. The central, necrotic part of the tumor was usually a yellowish brown, soft, cheesy, friable mass with irregular, small cavities; these contained a yellow serous fluid, which oozed freely from the section. Reddish, indistinctly outlined areas and hemorrhages were often present, especially at the junction of the necrotic and healthy tumor tissue; they also appeared as striated forms in the peripheral zone.

Histological examination gave the picture of a highly cellular carcinoma, which consisted of dense masses of large, polygonal cells with numerous mitoses. The stroma, which was generally limited, was a loose connective tissue filled with blood vessels. The brown central part was highly edematous, and contained tumor tissue in all states of degeneration and necrosis. Leucocytic infiltrations were usually present in widespread masses and were found particularly in the range of the entirely decomposed tumor; the blood vessels in the necrotic foci and in their neighborhood were often filled with thick masses of leucocytes. In tumors in which surface ulcerations were present a dense and wide leucocytic wall surrounded the cheesy necrotic area extending from the ulcer into the interior. Hemorrhagic extravasations were in abundance.

Since the foregoing work had as its aim the investigation of the quantitative and qualitative relations between certain enzymes present in tumor tissue and in their different components, the histological analysis was made, in so far as was possible, from this point of view. For this purpose, each tumor to be tested for enzymes was weighed and measurements were made of the macroscopically sound and the degenerated portions of the tumor in a section through the center; in addition, the histologic structure of the tumor was determined from a section stained with hematoxylin-phloxin. With the help of data thus obtained, the part expressed as per cent. of parenchyma, of fibrosis and of necrosis of the tumor was estimated. In the macroscopic and microscopic examinations of the tumors the presence of hemorrhage, edema, ulceration and leucocytic infiltration as well as the type of degeneration and necrosis were also noted.

II. Enzymatic Analysis.

1. Proteolytic Enzymes (work done by Arnulf Purr)

TABLE I.—*Proteolytic activity and tumor aging.*

[Tumors or parts of tumors as well as entire organs were cut up with scissors immediately after extirpation and frozen by dipping into liquid nitrogen, then pulverized by crushing with a hammer. This fine material suspended in 10 parts of 90 per cent. or extracted with 10 parts 60 per cent. glycerin; 7.0 cc. of the suspension or extract were treated with H_2S 30 min. at a pH of 7 for activation. 5.0 cc. of 8 per cent. gelatine, 1.0 cc. of *N* acetic acid, total 25.0 cc. at a pH of 4.0 for 24 hr. at 30°. The results are shown of the analysis of 10.0 cc. of the mixture (containing 2.8 cc. of enzyme solution) and signify NH_2 increase in cc. 0.05 *N* KOH. Mixture for the determination of amino-polypeptidase contained 0.25 cc. enzyme solution, 0.244 g. leucyl-di-glycine and 2.0 cc. *N*-ammonia-ammonium chloride buffer of pH 9.5, total 10.0 cc. at pH 7.8, 3 hr. at 30°. Mixture for the determination of dipeptidase contained 0.25 cc. enzyme solution, 0.188 g. leucylglycine and 2.0 cc. *N*-ammonia-ammonium chloride buffer of pH 9.5, total 10.0 cc. pH 7.8, 3 hr. 30°. Results show increase in acidity in cc. 0.05 *N* KOH. P = parenchymal, N = necrotic, F = fibrillar part in per cent.]

Tumor	No.	P	N	F	Cathepsin					
					Glycerin suspension without activation	Glycerin suspension with activation	Glycerin extract without activation	Glycerin extract with activation	Polypept. suspension*	Dipept. suspension*
Ph. 1	P 26	90	0	10	0.42	2.62	0.35	2.80	0.52	0.45
	P 22	70	0	30	1.20	2.40	0.72	2.10	0.40	0.34
	P 7	50	0	50	0.64	1.60	0.32	1.60	0.27	0.25
	P 11	40	5	55	0.65	1.44	0.37	1.44	0.38	0.32
	P 18	40	15	45	0.37	1.44	0.40	1.60	0.60	0.41
	P 4	40	20	40	0.72	1.60	—	—	0.18	0.11
	P 16	35	15	50	0.24	0.74	0.16	0.74	0.50	0.45
	P 12	35	20	45	0.74	1.30	0.48	1.44	0.68	0.55
	P 20	25	55	20	0.29	1.12	0.29	0.98	—	—
W 256	P 19	85	15	0	0.42	1.26	0.28	1.12	0.27	0.22
	P 14	80	15	5	0.37	1.16	0.37	1.40	0.23	0.18
	P 15	75	25	0	0.56	0.87	0.56	1.12	0.28	0.16
	P 17	40	20	40	0.24	0.87	0.16	1.00	0.29	0.20
	P 25	30	55	15	0.42	1.20	0.29	1.01	0.51	0.47
	P 10	20	60	10	0.32	0.74	0.37	0.96	0.55	0.40
	P 27	15	70	10	0.24	0.56	—	—	—	—
	P 24	15	75	10	0.37	0.56	0.32	0.74	0.75	0.67
	P 8	5	90	5	0.56	0.80	0.37	0.74	0.35	0.31
	P 0	0	100	0	0.12	0.24	-	-	0	0
Liver, normal					1.00	1.82	1.26	1.54	0.30	0.25
Liver, diseased	(P 4)				0.32	1.00	0.32	1.10	—	—
	(P 10)				0.65	1.50	0.65	1.40	0.55	0.40
	(P 8)				0.65	1.30	0.48	1.30	0.35	0.31
Musculature, normal					0.12	0.12	0	0	0.08	0.07
					0.15	0.22	0	0	0.06	0.04

* No notable difference was found between the enzyme content of the glycerin suspension and the glycerin extract.

2. Arginase (work done by Leopold Weil)

TABLE II.—*Arginase activity and tumor aging.*

[Preparation of tumors and organs as in Table I, finely ground material suspended (*a*) in 90 per cent. glycerin (1:10), (*b*) 1 hr. in 60 per cent. glycerin (1:10) extract. 5.0 enzyme (tumor) or 0.50 cc. (liver) or 10.0 cc. (musculature) used; for activation 2.0 cc. neutral cysteine-HCl-solution (containing 20 mg. cysteine-HCl) or 0.5 cc. 0.1 *N* $FeSO_4$, with the enzyme 1 hr. at pH 7 at 30°. 10.0 cc. of 1 per cent. arginine carbonate and 5.0 cc. 0.1 mol. glycocol buffer of pH 9.5, 60 min., at 30°. Results shown in cc. 0.02 *N*-H_2SO_4, as is customary.[1] P = parenchymal, F = fibrillar, N = necrotic part in per cent.]

Tumor	No.	N	P	F	Glycerin suspension				Glycerin extract	
					without addition	with Fe	cysteine	cyst. Fe	without addition	with cyst. Fe
Ph. 1	W 17	0	60	40	3.4	4.1	—	7.9	—	—
	W 19	0	50	50	4.2	4.4	—	6.6	—	—
	W 27	0	25	75	2.4	4.3	—	9.6	—	—
	W 13	0	15	85	3.4	6.9	4.7	6.8	2.6	4.9
	W 16	2	75	23	2.0	6.2	—	5.9	—	—
	W 26	20	40	35	4.4	13.0	—	13.3	—	5.5
	K 21	40	25	35	7.7	10.8	7.6	10.8	4.8	11.1
	W 18	60	15	20	6.1	10.5	8.2	14.5	1.9	5.6
	W 29	60	10	25	7.2	11.9	—	14.5	—	—
W 256	W 32	0	—	—	—	—	—	4.5	—	—
	W 11	15	20	65	6.3	10.4	7.3	12.3	2.6	8.4
	W 7	50	40	10	7.3	15.2	8.0	13.0	—	14.7
	W 10	65	25	10	7.2	21.5	11.3	18.2	4.1	5.6
	W 9	70	20	10	5.9	12.3	6.4	13.2	3.2	10.3
	W 28	70	30	0	7.4	12.9	—	19.3	—	—
	P 24	75	15	10	5.3	9.7	—	14.0	—	—
	P 0	100	0	0	—	—	—	17.6	—	—
Liver, normal					26.4	26.9	17.6	25.4	—	—
Liver, diseased (Ph. 1)					15.0	20.7	17.1	22.3	8.8	10.6
					11.4	21.9	—	21.0	9.4	17.0
Musculature, normal					0	—	—	0	—	—
					0	—	—	0	—	—

[1] See E. WALDSCHMIDT-LEITZ, A. SCHARIKOVA AND A. SCHÄFFNER, Hoppe-Seyler's *Zeitschrift für physiologische Chemie*, **214**: 82 (1932–33).

3. Purine Desamidases (work done by Anton Schäffner)

TABLE III.—*Desamidase activity and tumor aging.*

[Tumor Philadelphia 1. Preparation of tumors and organs as in Table I. Suspension of finely ground materials in 10 parts of 90 per cent. or extraction with 10 parts 60 per cent. glycerin. Muscle-adenylic acid or guanine used as substrate. Preparation of substrate and procedure for determinations according to G. Schmidt.[2] 1.0 cc. enzyme solution or enzyme suspension, 5 mg. adenylic acid or 4.4 mg. guanine, 2.0 cc. 0 1 mol. phosphate buffer at pH 5.9 (adenylic acid desamidase) or 10.0 cc. 0.2 mol. borate buffer at pH 8.7 (guanase), total volume 20 cc., 60 min., 30°. The results are expressed as k of the monomolecular reaction for adenylic acid desamidase and as $k = \frac{\text{cc. } 0.0033 \text{ acid}}{t \text{ (minutes)}}$ for the guanine desamidase enzyme. P = parenchymal, F = fibrillar, and N = necrotic part in per cent.]

Tumor No.	P	N	F	Adenylic acid desamidase		Guanase	
				Glycerin suspension	Glycerin extract	Glycerin suspension	Glycerin extract
S 6	75	5	20	0.0024	-	0.048	—
S 9	75	5	20	0.0022	0.0011	0.046	0.042
S 5	65	0	35	0.0024	0.0012	0.043	0.039
S 10	50	0	50	0.0025	0.0011	0.041	0.038
S 7	35	5	60	0.0011	0.0008	0.047	0.040
P 16	35	15	50	0.0013	0.0008	0.026	0.015
S 3	30	30	40	0.0009	0.0005	0.023	0.021
K 17	20	40	40	0.0008	0.0004	0.026	0.026
W 13	15	0	85	0.0013	0.0004	0.028	0.020
Liver, normal				0.0012	0.0012	0.041	0.036
				0.0012	0.0011	0.045	0.038
Liver, diseased				0.0012	0.0011	0.034	0.026
Musculature, normal				0.0120	0.0077	0.005	0.002
				0.0130	0.0080	0.005	0.002

4. Phosphatase (work done by Franz Köhler)

TABLE IV.—*Phosphatase content and tumor aging.*

[Preparation of tumors and organs as in Table I. Suspension of finely ground materials in 10 parts 90 per cent. or extraction with 10 parts 60 per cent. glycerin or removal of fat and desiccation with acetone-ether. 4.0 cc. enzyme suspension or enzyme extract or 60 mg. dry substance used. 10.0 cc. 0.1 mol. Na-glycerophosphate (corresponding to 71.6 mg. P_2O_2) containing 4.0 cc. 0.1 mol. veronal buffer [3] at pH 8.8,[4] total volume 20 cc. 3 hr., 30°. Enzyme activity stopped by addition of 5.0 cc. 5 per cent. trichloracetic acid in 10.0 cc. of the mixture. Measurement of the phosphoric acid split off after filtration was made in 10.0 cc. of the filtrates with strychnine molybdate according to G. Embden.[5] The results are expressed as k (monomol.) and are based on 10.0 cc. of trichloracetic acid—filtrate. P = parenchymal, N = necrotic, and F = fibrillar part in per cent.]

Continued on next page.

[2] G. SCHMIDT, Hoppe-Seyler's *Zeitschrift für physiologische Chemie*, **179**: 256 ff. (1928) (adenylic acid desamidase); **208**: 187 ff. (1932) (guanase).

[3] See L. MICHAELIS, *J. Biol. Chem.*, **87**: 33 (1930).

[4] The addition of magnesium salt proved to be unnecessary.

[5] G. EMBDEN, Hoppe-Seyler's *Zeitschrift für physiologische Chemie*, **113**: 138 (1921).

TABLE IV.—*Continued.*

Tumor	No.	P	N	F	Glycerin suspension	Glycerin extract	Dry substance
Ph. 1	K 24	85	0	15	0.00048	—	0.00035
	K 7	80	5	15	0.00034	0.000250	0.00015
	K 18	50	0	50	0.00021	0.000024	0.00014
	K 8	40	20	40	0.00026	0.000026	0.00013
	K 16	35	0	65	0.00026	0.000060	0.00044
	K 33	35	5	60	0.00024	—	0.00013
	W 15	35	20	45	0.00022	0.000048	0.00031
	K 36	25	0	75	0.00013	—	0.00027
	K 21	25	40	35	0.000069	0.000002	0.00001
W 256	K 22	70	5	25	0.00056	—	—
	K 19	70	15	0	0.00037	—	0.00071
	K 14	50	35	15	0.00019	0.000051	0.00016
	K 25	40	30	0	0.00022	—	0.00018
	K 12	40	45	15	0.000093	0.000054	0.000096
	K 10	40	50	10	0.000096	0.000058	0.000050
	K 11	25	60	15	0.00020	0.000022	0.00012
	K 13	25	60	15	0.00018	0.000044	0.00012
	K 35	20	35	15	0.00011	0.000036	—
	K 34	20	65	10	0.00010	—	0.00031
	K 32	15	80	5	0.000069	0.000049	0.00042
	K 31	5	70	5	0.000069	—	0.00027
Kidney, normal					0.00078	0 00032	0.00070
Kidney, diseased					0.00158	0 000070	0.00051
Musculature, normal					0.000040 0.000074	0.000028 0	0.000033 0.000028

5. Catalase (work done by Franz Köhler)

TABLE V.—*Catalase content and tumor aging.*

[Tumor Philadelphia 1. Preparation of tumors and organs as in Table I. Suspension of finely ground materials in 10 parts 90 per cent. or extraction with 10 parts 60 per cent. glycerin. 1.0 cc. suspension or extract used. Determinations carried out as previously described.[6] Results are expressed as *k* (monomol.) P = parenchymal, F = fibrillar, and N = necrotic part in per cent.]

Tumor No.	N	P	F	Glycerin suspension	Glycerin extract
K 37	0	30	70	0 0037	0 0019
K 16	0	35	65	0.0095	0 0062
K 18	0	50	50	0.0090	0.0044
K 27	5	50	45	0 0024	0
W 26	20	35	40	0.0014	0 0031
K 30	35	15	25	0.0120	0.0049
Liver, normal				4.8*	2 8*
Liver, diseased				5.2*	2.0*
Musculature, normal				0.0042 0.0057	0.0017 0.0035

* Determinations made with 0.025 cc., constant calculated on the basis of 1.00 cc.

[6] E. WALDSCHMIDT-LEITZ, A. SCHARIKOVA AND A. SCHÄFFNER, Hoppe-Seyler's *Zeitschrift ur physiologische Chemie*, **214**: 85 (1932–33).

CURRENT TOPICS.

Dark Hours Improve Fox Fur.—Uncle Remus might have told of how Br'er Fox was like a chrysanthemum.

Such a tale would be truthful according to one of the latest discoveries in scientific fox farming, developed by U. S. Department of Agriculture specialists at the Fur Animal Experiment Station at Saratoga, N. Y. The date of prime quality in fox furs can be advanced a month or more by depriving fox pups of late afternoon sunlight in late summer and early fall.

Years ago, plant scientists of the Department discovered that chrysanthemums could be forced to bloom in advance of their natural schedule by "shortening the day"—that is, by cutting off the sunlight from the plant by shading it with black cloth, or covering it with a box, or moving it into a dark room—on a schedule which corresponded to the shortened daylight hours of autumn, its normal period of bloom. Or bloom could be retarded by giving the plants added hours of light by electricity. Commercial florists made use of this discovery in forcing earlier blooming of late-flowering varieties that were naturally too late for the big football game markets for 'mums.

In somewhat similar experiments, the fox breeding experimenters have found that the "primeness" of fox skins the stage when they are at best market quality—is influenced by light exposure. Fox fur is normally "prime" in December. But the foxes can be deprived of some of the natural daylight by being herded into a darkened shed before sunset in the late summer and early fall—in imitation of what the hours will actually be about two months later. Under such management the fur becomes prime at an earlier date—by late October or early November with Br'er Fox blooming like a chrysanthemum.

Commercial application of the findings is being studied further at the Station.

R. H. O.

Stopping One Crop Pest May Stop Two.—Growing knowledge of nematodes that attack plant roots is helping to clear up some crop ailments and is already modifying farm practices. More developments are bound to come as science reveals the true causes and combinations of causes of low yields.

In some cases nematodes, such as the meadow nematode or the one causing rootknot, are now known to open the way to damage by such other factors as fungi and bacteria. Or nematodes may weaken the roots so as to make the plants extremely susceptible to drought. With some plants—both weeds and crop plants—resistance to nematodes has been found important. Already the killing of nematodes with soil fumigants is becoming a farm practice on some crops and the scientists are widening the field.

Recently studies on the control of cotton wilt have shown that controlling nematodes results also in control of the wilt disease. Experimenters found that even the varieties ordinarily resistant to wilt were badly damaged by the fungus when the roots had previously been injured by the nematodes. The

plants were made vulnerable just as a man with scratches on his hands is vulnerable to many disease organisms, such as tularemia of rabbits and man.

Experimenters using a chemical that kills nematodes have increased yields of fiber greatly by ridding the cotton plants of the pests that assist the wilt, in this case the meadow nematode. After only 8 weeks the plants in fumigated rows were 3 in. taller than the untreated. At harvest there were more bolls, bigger ones, and better lint. Tests on cotton in Alabama showed that even varieties known to be wilt-resistant gave in to it when heavily attacked by nematodes. But when the nematodes were fumigated out, the wilt was cut far down, in some plots almost to the vanishing point. Fumigating with chloropicrin (tear gas) gave similar results on tomatoes suffering from wilt and rootknot nematode attack.

R. H. O.

138,000-Volt High-Pressure Oil-Filled Cable.—Installation of one of the few 138,000-volt high-pressure oil-filled cable systems for power transmission in this country has been completed in San Antonio, Texas.

The new system connects San Antonio's municipal steam-turbine power plant to the outlying Olmos and Grandview substations. These two circuits are approximately $7\frac{1}{2}$ and $3\frac{1}{4}$ miles long, respectively, the Olmos line being among the world's longest.

This type of installation consists of three insulated cables contained in an oil-filled 5-in. pipe. The pipe was shipped to San Antonio in 40-ft. lengths, which were welded to form an air-tight, underground line from the power plant to each of the substations.

The pipe laying progressed by segments of welded pipe up to 1770 ft. long. When such a segment was completed it was pressure tested for leaks, and dried. Cables were then pulled through by winches, and the section was filled with oil.

The line used for pulling the cable through the pipe was blown through by an air compressor unit. In the past, a wire has been pushed through each 40-ft. length of pipe and spliced to the preceding segment before the pipe was welded. The new method used at San Antonio enabled engineers to pull cable into continuous lengths of pipe up to one third of a mile long.

Five 10,000-gallon tank cars of oil were shipped to Texas The special insulating oil was pumped into decontaminated and dried tank cars, and an atmosphere of nitrogen was placed on top of the oil to seal it from the air. This was the first time that insulating oil had been shipped under a controlled gas-pressure seal.

The complete San Antonio line was filled with this oil and held at a nominal pressure of 200 psi. by a permanent pumping unit installed at the power plant ends of the two lines.

R. H. O.

Traffic Congestion Increases. (*Engineering News Record*, Vol. 139, No. 18.)—There are now more cars on the road than there were at the 1941 peak and they are contributing to the biggest traffic jams in history.

The American Society of Planning Officials, commenting on plans by major cities to ease traffic congestion, reports that almost 2.6 million cars and trucks have been produced this year while considerably fewer old ones have been junked.

At Pearl Harbor time, 34,472,145 cars and trucks were licensed according to the Automotive Safety Foundation. At the end of 1946, 33,945,817 vehicles were licensed, to which has been added the bigger part of this year's auto production. Since the bulk of the demand for new cars is still unmet, traffic tangles are expected to get worse before they get better.

Removal of auto travel curbs has increased traffic congestion even more. Present traffic in Milwaukee is 92 per cent greater than during the war, while New York, Chicago, and other cities report increases of 50 per cent or more. Indicating how great the nationwide traffic increases have been, motor vehicles last year consumed 1,457 billion more gallons of gasoline than in 1941.

Not nearly so many cars are being scrapped as before the war. In 1940, 2,350,000 autos ended their service in junkyards. By 1943 that figure had dropped to 920,000. It is estimated that the 1947 total will be only about 1 million.

Traffic Congestion and Masterplans.—Washington, D. C. is among latest additions to the growing list of cities making traffic improvements a central part of long-range development plans. Plans for reducing traffic congestion are basic to masterplans newly drafted or nearing completion in Newark, Pittsburgh, Detroit, Cincinnati, Providence, San Francisco, New York, Chicago, and other major cities.

Parking is the big problem. Washington's proposed parking program calls for creation of new offstreet parking facilities to care for both short-time parking and all-day parking in congested areas. Typical of plans in other cities, the program calls for fringe parking lots in the outskirts of business districts to reduce traffic congestion on busy, over-loaded streets.

Some cities are limiting special parking privileges to ease congestion. In downtown Richmond, Va., spaces reserved at the curb for various organizations and individuals are being opened to vehicles driven by the general public. Officials expect an 800 per cent increase in the number of vehicles these spaces accommodate once they are returned to public one-hour use.

R. H. O.

Milk-Coated Cans.—In place of using milk cans coated with tin we may eventually be using containers coated with an enamel made from milk itself. Facts from the annual report of the Bureau of Dairy Industry, U. S. Department of Agriculture, suggest this.

Dairymen have commonly used as milk containers cans with a protective coating of tin that covers the sheet steel of the cans, prevents rusting, and makes it easy to clean and sterilize them.

With tin in scant supply during the war, scientists sought a substitute coating which would conserve the dwindling supply of tin. It turned out that the Federal dairy scientists developed several enamel coatings that are proving satisfactory substitutes for tin. The basic material from which these enamels can be made is lactic acid, derived from milk. Thus, milk supplies the protective coatings needed in the milk containers.

"Lactic acid," says the report, "can be converted into a viscous, rather inert, resinous material by removing the water. This resinous lactic acid, combined with oils, metals, or various substances, gives products that, when coated on metals and baked are highly resistant to water, steam, and acids, and they

are sufficiently resistant to alkalies that the coated surfaces may be washed with hot solutions of soap or other common detergents." The enamels containing an oil and the lactic acid have proved the most durable. Four patents have been issued and others are pending on these processes.

Enamels somewhat similar to these new lactic enamel coatings are familiar to housewives. They are used to coat metal cans used in processing fruits and vegetables.

R. H. O.

100,000-kw. Turbine Generator, Largest of its Type in the World.—A 100,000-kw., 3600-rpm. turbine generator unit, the first machine of its size designed for 1,000° F. steam operation, has been put into full scale service in Newark, N. J., at the Essex Generating Station of the Public Service Electric and Gas Company, New Jersey.

The machine is the largest of its type in the world, measuring 77 ft. in length and 17 ft. in width at its broadest point.

The turbine, with nineteen stages in the high-pressure casing and five stages double-flow in the low-pressure section, was designed for steam conditions of 1,250 psi., 1,000° F., 1.5-in. mercury absolute back pressure.

It exhausts through 23-in. long last-stage buckets of special warped design. The last-stage buckets, which operate at 1390 ft. per second, are mounted on a larger wheel than previously had been used on 3600-rpm. machines. The 1390 rate is approximately 10 per cent higher than previous 3600-rpm. machines were capable of attaining.

Special provisions are made for the elimination of moisture and the reduction of moisture cutting of buckets.

The turbine has a newly-developed hydraulic and lubrication oiling system with the centrifugal pump mounted on the turbine shaft fed by an oil driven booster pump in a separate oil tank in the basement.

The main generator is rated at 95,000 kw., with a power factor of 0.85 and a short circuit ratio of 0.85 at 15 psi. hydrogen pressure. It is ventilated by a new-type axial-flow fan. The rotor is cooled by a system of longitudinal cooling slots between the coil slots and by longitudinal cooling tunnels located at the bottom of the coil slots under the coils.

The set also is equipped with a 7,500-kw. direct-connected house alternator. Both generators have spring-mounted stator cores.

R. H. O.

Corn Needs Nitrogen in July.—The world shortage of nitrogen fertilizers as reported by the International Emergency Food Council lends interest to the discussion of the use of nitrogen fertilizers in an article in the current Yearbook of the U. S. Department of Agriculture. The author is Dr. F. W. Parker, assistant chief of the Bureau of Plant Industry, Soils, and Agricultural Engineering, in charge of soil and fertilizer research. The Council reports that the prospect for 1947–48 is that supplies will be only a little more than two-thirds of the requirements reported by more than 100 claimant countries—1,900,000 metric tons in prospect against requests for nearly 2,750,000 tons.

Dr. Parker reviews the rapid rise in use of nitrogen consumption in the United States—from 62,000 tons in 1900 to more than 10 times that quantity

in 1944. He also indicates the probability of a continuing rise in the demand for nitrogen—even after the world has made up the current acute shortage which has followed the undersupply during war years.

Factors such as the natural fertility of the soil, rainfall, the cropping or farming system, and the value of the crop per acre influence farm use of nitrogen, Dr. Parker points out. In the West where light rainfall limits crop production, the use of nitrogen fertilizer is not likely to prove profitable. "In a good livestock system," says Parker, "legumes are grown and much nitrogen is returned to the land in farm manure." But where high value cash crops are grown farmers are likely to use much nitrogen fertilizer.

Discussing some findings of recent research, Dr. Parker says: "Large quantities of nitrogen are required for high yields of most crops. A 60-bushel corn crop contains about 95 pounds of nitrogen, 57 pounds in the grain and 38 pounds in the stover. This nitrogen must come from the soil, legumes that have been turned under, manure, or fertilizers.

"The time when the crop needs nitrogen corresponds with its rate of growth. Little nitrogen is needed in the seedling stage, but that little is highly essential. The demand is greater when growth is quite rapid. Usually this is in midsummer for spring planted crops. Corn planted on May 22 in Ohio needed only 12 pounds of nitrogen before July 1. Between July 10 and August 10 the crop absorbed 81 pounds of nitrogen—almost 60 per cent. of the nitrogen required for the 117-bushel crop. These figures indicate the corn crop needs most of its nitrogen during the one month of maximum growth."

R. H. O.

Farm Machines Save Time.—Quicker farm transportation—of both passengers and products—is the largest single item in the record of time savings for farmers revealed in a study of the progress of farm mechanization. The tractor is ordinarily thought of as the typical symbol of mechanization, but the transportation units—the farm automobile and the motor truck—save the farmers even more time.

Comparing conditions in 1944 with those in the base period of 1917–1921, an estimated total "saving" of 4.2 billion man-hours resulted from mechanization. This takes into account the labor time that would have been required if the large 1944 production had been on the same basis as the smaller annual production 25 years earlier. A third of the saving in man-hours (1.4 billion) can be traced to gains in time resulting from use of automobiles and motor trucks. There were 3,400,000 more trucks and motor cars on farms. On the average each saved the farmer more than 400 man-hours a year, "compared with the time it would take to do the same hauling with horses and mules." All the use of the trucks was considered in making this estimate but only half the automobile use was included as farm business and the other half regarded as personal use.

Comparing these years, the time saved by tractors and tractor-drawn machines adds up to nearly a billion hours a year.

A further great time saving is in chore work—the care of farm animals. More than 13 million horses and mules disappeared from the farms in these 25 years. This meant a saving of more than a billion man-hours in chore time. This however was reduced by the hours needed for farm maintenance work on

the automobiles, motor trucks, and tractors. Of the total net savings in reduction of chore time, 760 million hours is credited to the animals displaced by the tractors and 120 million hours to the animals displaced by transportation units.

R. H. O.

Nutrition-Wise Cooking.—When a housewife's cooking is needlessly wasteful of vitamins and minerals that fresh foods offer, it may take heavy toll of what the food can contribute to health and well being. To help answer the big question: "What constitutes good cooking from a nutritive standpoint?" a group of food and nutrition specialists have systematically cooked some familiar foods in family-sized quantities.

These specialists have boiled, baked, fried, and used other common cookery methods suitable to the foods under test. Before and after cooking, they checked up on twelve nutrients, including not only vitamins sensitive to heat, air, and water, but also minerals that escape into cooking water and the ash and mineral content of the foods.

Green peas, carrots, and potatoes—three vegetables commonly prepared in a wide variety of ways—were chosen for intensive tests. The findings show, for example, that although the skin of a potato is a highly protective device for holding in nutritive values during cooking, the skin of a carrot plays no such protective role. Carrots boiled whole, pared or unpared, kept about 90 per cent of their vitamin C. Potatoes boiled whole in their skins kept practically all of their vitamin C, whereas when pared before boiling, potatoes retained from 70 to 80 per cent of this most delicate of the vitamins.

It is emphasized that these tests are part of a larger effort to learn what happens to nutrients in foods all along the line from producer to consumer.

An important and difficult part of the work has been to standardize procedures and cooking conditions, so that the same type of tests can be repeated with batches of the same food or with other foods in order to draw valid conclusions. To do this, preliminary tests were made with 20 different kinds of vegetables, cereals, breads, and meats.

R. H. O.

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References with (N) are Notes from the National Bureau of Standards and references with (C) are Current Topics.